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NEW FRANKLIN

ARITHMETIC

SECOND BOOK

BY

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PREFACE.

THIS book is not merely a revised edition of the Franklin Written Arithmetic published by the authors seventeen years ago. In its general plan and method, it preserves all the essential characteristics of that favorably known and widely used text-book, but it has been constructed out of new material. All the illustrations are fresh, nearly all the examples are new, and the text has been rewritten throughout. The principles of arithmetic do not change, neither do the essential elements of good teaching change, but there is infinite room for variety in the application of principles and in the exemplification of method. In these respects the authors trust that their new book may be found fully up with the times.

The method of the book is thoroughly inductive. Every new topic is introduced, as it should be in good teaching, by easy oral questions designed to bring the subject matter clearly before the learner's mind. Then follow illustrative examples, definitions, principles and rules in true inductive order.

The number of examples provided for practice and drill is unusually large, particularly in the earlier half of the book. There appears to be a demand for an abundance of such material, which this book aims to satisfy. But the authors are very far from recommending that all the examples under one topic should be worked before a pupil or a class of pupils is permitted to take up the next topic.

Opinions may differ as to whether topics like taxes, insurance, foreign exchange, stocks and bonds, equation of payments, and average of accounts should be included in the course of study for common schools, but there seems to be no doubt that such topics should be included in a book that aims at ordinary completeness. In this book the treatment of the topics mentioned is unusually simple and practical, the illustrations being not the old-fashioned imaginary transactions, but the actual ones occurring daily in the course of mercantile business. By means of them a considerable amount of accurate and useful information is conveyed.

An important feature of the book is the anticipation of topics. In the earlier sections are found well-chosen examples which serve to familiarize the learner with matters that are to be presented in a more formal and complete shape in the later sections. Thus a knowledge of compound numbers is conveyed by the examples given to illustrate the applications of multiplication and division of simple numbers; many examples in percentage and interest are given among the applications of decimal fractions; and the simpler examples in mensuration are introduced long before the systematic treatment of that subject in the last section is reached.

It is this same principle of anticipation that has led the authors to make a somewhat free use of algebraic notation and methods in the treatment of problems in interest, proportions, square and cube root, and some other topics.

Proportion has received a somewhat broader treatment than has been usual in arithmetical text-books of late years. It is believed that proportion affords a straightforward, simple, and clear form of analysis, the training in which is highly useful for general purposes and cannot be too strongly recommended for all who are to be concerned with the further study of mathematics or with the physical sciences. In this book the range of examples is enlarged so as to include some of the simpler applications of proportion to similar geometrical figures.

The section on mensuration has been enlarged so as to contain a considerable amount of practical geometry. This, with the practical exercises suggested for the purpose of making the principles objectively clear, will, it is hoped, make mensuration a less dry and forbidding topic than children usually find it.

In conclusion, the authors desire to express their hearty acknowledgments for the favorable reception accorded to their former book, and venture to hope that the present one may be found no less worthy of public favor.

E. P. S.

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NEW
FRANKLIN ARITHMETIC.

SECOND BOOK.

SECTION I.

NUMBERS AND THEIR EXPRESSION.

ARTICLE 1. A collection of similar objects suggests the ideas of one and more than one. One of any kind is a **unit.** A unit or a collection of units is a **number.**

2. Arithmetic is the science of numbers and the art of using them in computation.

INTEGRAL NUMBERS.

3. An **integral number** is a number whose unit is a whole or undivided thing, as *seven apples,* the unit of which is one apple; *ten days,* the unit of which is one day; *fifteen dozen,* the unit of which is one dozen.

4. Small numbers are reckoned by *ones* and by *tens,* larger numbers by tens of tens, or *hundreds,* and still larger numbers by tens of hundreds, or *thousands.* Above thousands, numbers are reckoned by *ten-thousands, hundred-thousands, millions, ten-millions, hundred-millions,* and so on.

5. One, ten, a hundred, a thousand, etc., are all called units, because each is used as a single thing in reckoning other numbers.

6. To distinguish these units, *one* is called a unit of the **first order,** *ten* a unit of the **second order,** *a hundred* a unit of the **third order,** *a thousand* a unit of the **fourth order,** *ten thousand* a unit of the **fifth order,** and so on. When a unit is spoken of without the order being named, a unit of the first order, or one, is meant.

7. These units form a scale; and because ten units of any order make a unit of the next higher order, the scale is called a **scale of tens,** or a **decimal scale.**

8. A system of numbers whose successive units form a scale of tens is a **decimal system of numbers.** The system of numbers in common use is a decimal system.

NOTATION AND NUMERATION.

9. Notation is the art of writing numbers, and **numeration** is the art of reading them.

10. There are two systems of notation in common use, the Roman and the Arabic. The Roman notation employs the characters I. V. X. L. C. D. and M. and combinations of these to express numbers. It is not used in arithmetical computation.

11. The Arabic notation employs ten characters called figures.

0	1	2	3	4	5	6	7	8	9
Zero	**One**	**Two**	**Three**	**Four**	**Five**	**Six**	**Seven**	**Eight**	**Nine**

12. The first of these figures, 0, is called *zero* or *cipher,* and stands for *no number.* The others stand respectively for the numbers whose names are written beneath.

13. Numbers larger than nine require two or more figures for their notation. These are written side by side, and so the *place* occupied by each figure becomes important.

14. Tens are expressed by writing a figure to denote how many tens, and then placing a zero at the right of it. The figure so written is said to occupy the *second* place, or the *tens'* place, while the *first* or units' place is filled by the zero. Thus,

Ten (one ten), 10. Forty (four tens), 40. Seventy (seven tens), 70.

15. Numbers made up of tens and units are expressed by writing a figure in the *second* place for the tens, and a figure in the *first* place for the units. Thus,

Eleven (one ten and one), 11. Twenty-four (two tens and four), 24.

16. Hundreds are expressed by writing a figure in the *third* place, the *second* and *first* places being filled with zeros. Thus,

One hundred, 100. Five hundred, 500.

17. Numbers made up of hundreds, tens, and units are expressed by writing a figure in the *third* place for the hundreds, a figure in the *second* place for the tens, and a figure in the *first* place for the units. Thus,

Three hundred twenty-seven (3 hundreds, 2 tens, 7 units),	327.
Four hundred thirty (4 hundreds, 3 tens, 0 units),	430.
Five hundred six (5 hundreds, 0 tens, 6 units),	506.

18. Thousands are expressed by writing a figure in the *fourth* place, tens of thousands by writing a figure in the *fifth* place, and hundreds of thousands by writing a figure in the *sixth* place. The figures in these three places taken together form a group called the *thousands' group;* while the figures in the hundreds', tens', and units' places form the *units' group.* These groups are usually separated by a comma. Thus,

One thousand,	1,000.	Fifty thousand,	50,000.
Ten thousand,	10,000.	One hundred thousand,	100,000.

Ten thousand six hundred twenty,	10,620.
Nine hundred seventy-eight thousand eight hundred six,	978,806.

19. Millions, tens of millions, and hundreds of millions are expressed by writing figures in the seventh, eighth, and ninth places. These figures taken together form the *millions' group.* Thus,

Five millions,	5,000,000.
Fifty millions,	50,000,000.
Five hundred fifty-five millions,	555,000,000.

20. The above illustrations show the principles upon which all numbers are written, which are

(1) *Units of any order are expressed by writing a figure in the place corresponding to that order, and filling vacant places with zeros.*

(2) *The value expressed by a figure is increased tenfold by each removal one place to the left, and decreased tenfold by each removal one place to the right.*

21. The general method of writing integral numbers is shown by the following

TABLE.

ETC.	HUNDRED-TRILLIONS.	TEN-TRILLIONS.	TRILLIONS.	HUNDRED-BILLIONS.	TEN-BILLIONS.	BILLIONS.	HUNDRED-MILLIONS.	TEN-MILLIONS.	MILLIONS.	HUNDRED-THOUSANDS.	TEN-THOUSANDS.	THOUSANDS.	HUNDREDS.	TENS.	UNITS.	ORDER NAMES.
	15TH.	14TH.	13TH.	12TH.	11TH.	10TH.	9TH.	8TH.	7TH.	6TH.	5TH.	4TH.	3D.	2D.	1ST.	PLACES.
	4	8	0,	2	9	7,	0	3	4,	5	0	8,	6	7	2	FIGURES.
ETC.	5th group, Trillions.			4th group, Billions.			3d group, Millions.			2d group, Thousands.			1st group, Units.			GROUPS.

NOTE.—The groups above billions are seldom used. Their names are *trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, tredecillions, quatuordecillions, quindecillions, sexdecillions, septendecillions, octodecillions, novemdecillions, vigintillions,* etc.

Exercises upon the Table.

22. *a.* What places are occupied by the units' group? the thousands' group? the millions'? billions'? trillions'?

b. Write 265 as units; as thousands; as millions.

In reading and writing large numbers, the first thought should be of the groups.

23. To read integral numbers.

Illustrative Example. Read the number 46372509.

(1) Beginning at the right, point off the figures into groups of three figures each, thus:

46,372,509.

(2) Beginning at the left, name the number expressed by each group in order, thus:

"*Forty-six* million *three hundred seventy-two* thousand *five hundred nine.*"

NOTE.—In reading any number, the pupil should make a slight pause after naming a group. The name of the units' group is omitted in reading.

Exercises.

24. Read or write in words the following:

1.	415	**14.**	40762	**27.**	168410815
2.	805	**15.**	9006	**28.**	307406090
3.	2365	**16.**	300708	**29.**	1327541
4.	9078	**17.**	1094	**30.**	86410232
5.	95316	**18.**	703800	**31.**	371032654
6.	48890	**19.**	500087	**32.**	4321987654
7.	164284	**20.**	84000	**33.**	73020091
8.	304091	**21.**	510510	**34.**	206841793
9.	460203	**22.**	90007	**35.**	9876004380
10.	1703480	**23.**	700016	**36.**	2183059507
11.	2038006	**24.**	34716300	**37.**	976431807
12.	9007800	**25.**	80040704	**38.**	4806784800
13.	6400087	**26.**	38904918	**39.**	8073063980

25. To write integral numbers.

Illustrative Example. Write in figures four hundred forty-seven million one hundred ninety-eight thousand six hundred ninety-four.

(1) Think of the groups. These are

447 millions, 198 thousands, 694 units.

(2) Arrange the groups in their proper order, thus:

447,198,694.

Exercises.

26. Write in figures the following, supplying vacant places with zeros:

40. Seven thousand three hundred forty-one.

41. Eighteen thousand nine hundred eighty-four.

42. Thirty-seven thousand three hundred nineteen.

43. Nine hundred thousand five hundred sixteen.

44. Four hundred twenty thousand six hundred eighty.

45. Eight hundred ten thousand two hundred seven.

46. Two hundred fifty-nine thousand seventy.

47. One million one hundred thousand one hundred one.

48. Five million thirty-six thousand six hundred ten.

49. Forty-five million seven hundred twenty-four thousand two hundred forty-seven.

50. Eighty million seventy thousand five hundred seven.

51. Nine hundred one million two hundred eighteen thousand five hundred twenty-two.

52. Three billion thirty-seven million nine hundred six thousand two hundred.

53. Two hundred thirty-four million eight hundred sixty-three thousand three hundred eighty-nine.

54. Seventeen billion seven hundred fifty-nine million ninety thousand sixty-seven.

DECIMAL FRACTIONS.

27. As a hundred is made up of ten equal parts, each of which is *a ten,* and as a ten is made up of ten equal parts, each of which is *one,* so we may consider one to be made up of ten equal parts, each of which is *a tenth;* a tenth to be made up of ten equal parts, each of which is *a hundredth;* a hundredth to be made up of ten equal parts, each of which is *a thousandth;* and so on.

Tenths, hundredths, thousandths, etc., are *fractional units* and, as they form a decimal scale (Art. 7), collections of such units are called **decimal fractions.**

The above-named units are written as in the margin, each unit of a lower order being expressed by writing the figure 1, one place further to the right.

One hundred,	100.
One ten,	10.
One,	1.
One tenth,	0.1
One hundredth,	0.01
One thousandth,	0.001

28. The dot put at the right of the units' place is called the **decimal point.**

29. The method of writing decimal fractions is shown by the following

TABLE.

PLACE NAMES.	UNITS.	DECIMAL POINT.	TENTHS.	HUNDREDTHS.	THOUSANDTHS.	TEN-THOUSANDTHS.	HUNDRED-THOUSANDTHS.	MILLIONTHS.	ETC.
PLACES.			1ST.	2D.	3D.	4TH.	5TH.	6TH.	ETC.
FIGURES.	0	.	7	4	8	9	6	3	. . .

NOTE.—In writing decimal fractions it is well to fill the units' place with a zero when there is no other figure to be written there.

30. To read and write decimal fractions.

Illustrative Examples. Read 0.063.

First read the number as if it were an integral number, thus: "Sixty-three"; then add the name of the units of the lowest order (thousandths), thus, "Sixty-three thousandths."

Write nine hundred sixty-three thousandths.

First write the number as if it were an integral number, thus: 963, then place the decimal point so that the right-hand figure shall express the required denomination, thus, 0.963.

To read or write integral numbers and decimals together, *read or write first the integral number, then the decimal.* In reading, use the word *and* before the decimal.

Exercises.

31. Read the following:

55. 0.6	**59.** 268.9	**63.** 6.071	**67.** 400.0865
56. 0.63	**60.** 26.89	**64.** 4.007	**68.** 0.764321
57. 0.163	**61.** 2.689	**65.** 0.4867	**69.** 38.94856
58. 5.5	**62.** 51.01	**66.** 4.5385	**70.** 0.01666

Write the following:

71. Fourteen hundredths. Nine hundredths.
72. Twenty-seven thousandths. Four ten-thousandths.
73. 3 and 33 hundredths. 104 and 25 thousandths.
74. 7 and 29 thousandths. 45 hundred, and 5 tenths.
75. One hundred seventeen thousandths.
76. Four hundred and four ten-thousandths.
77. Six hundred seven, and sixty-seven thousandths.
78. Five hundred sixty-eight hundred-thousandths.

32. It is frequently convenient to separate a number into parts, each part containing only the units of a single order. Thus, the number 734.25 may be separated into 7 hundreds, 3 tens, 4 units, 2 tenths, and 5 hundredths. Such parts are called **terms** of the number.

UNITED STATES MONEY.

33. The units of United States money form a decimal scale, shown by the following

TABLE.

10 mills	= 1 cent, marked ¢, ct.
10 cents	= 1 dime.
10 dimes	= 1 dollar, marked $.
10 dollars	= 1 eagle.

34. *Dollars* and *cents* are the units chiefly used in reckoning money. Cents are hundredths of a dollar. Dollars are written as integral numbers at the left of the decimal point; cents are written in the first two places at the right, and mills in the third place.

The number 21 dollars, 43 cents, 8 mills is written thus: $21.438. The number 21 dollars, 5 cents, 3 mills is written thus: $21.053.

Exercises

35. Read the following.

79. $8.17
80. $1.354
81. $0.724
82. $0.04
83. $ 0.029
84. $34.50
85. $50.394
86. $94.902
87. $ 168.13
88. $ 406.805
89. $3000.50
90. $1909.293

Write in figures:

91. Fifteen dollars thirty-six cents.
92. Ninety-three dollars sixty two cents five mills.
93. Seventeen dollars five mills.
94. Forty-five cents four mills.
95. Nine dollars five cents two mills.
96. Five hundred dollars five cents.
97. Nine thousand seventeen dollars two cents.
98. Sixty-eight thousand four hundred eighty dollars.

SECTION II.

ADDITION.

36. *Illustrative Example.* How many books are 5 books, 3 books, and 2 books?

SOLUTION. — The numbers 5, 3, and 2 together make 10.

37. Putting numbers together to find what number they make is **adding** them.

38. The numbers to be put together are **addends.** The number found by adding two or more numbers is their **sum** or **amount.**

39. **Addition** is the process of putting two or more numbers together to find their sum or amount.

40. Only numbers of the same kind, name, or denomination can be added together, as apples with apples, pounds with pounds, units with units, tens with tens, hundreds with hundreds, etc.

41. The operations of arithmetic are indicated by signs. The sign of addition is an upright cross, $+$. It is read "and" or "plus." The sign $=$ indicates *equality*, and is read "equals" or "is equal to." The expression $5+3+2=10$ means that 5 and 3 and 2, added, make 10; it is read "5 plus 3 plus 2 equals 10."

What is the sum in the example given above? What are the addends? Define addition.

Oral Exercises in Addition.

42. I. Add each pair of numbers expressed in the columns *a*, *b*, *c*, etc., on the opposite page, till you can give the sums rapidly at sight.

	a *b*	*c* *d*	*e* *f*	*g* *h*	*i* *j*	*k* *l*
m	2 + 5	4 + 4	4 + 7	9 + 3	7 + 4	6 + 5
n	3 + 4	2 + 6	4 + 8	6 + 8	7 + 9	4 + 8
o	7 + 3	2 + 7	3 + 9	9 + 7	9 + 6	7 + 5
p	1 + 5	4 + 3	5 + 2	8 + 6	4 + 7	6 + 9
q	5 + 4	8 + 8	9 + 2	6 + 2	6 + 5	8 + 9
r	3 + 8	2 + 3	5 + 7	5 + 8	9 + 7	7 + 9

II. Add each pair expressed in lines *m*, *n*, *o*, etc.

III. Count to thirty or more,

s. By twos, beginning with 2; with 1.
t. By threes, beginning with 2.

Count to sixty or more,

u. By fours, beginning with 3; with 2.
v. By fives, beginning with 4; with 3; with 2; with 1.
w. By sixes, beginning with 5; with 4.

Count to a hundred or more,

x. By sevens, beginning with 6.
y. By eights, beginning with 7; with 6.
z. By nines, beginning with 8.

IV. Add the following as expressed in columns *a*, *b*, *c*, etc., counting from below upward and naming results only; thus, in column *a*, say 5, 7, 10, 12. To test the accuracy of the adding, add from above downwards.

	a		*b*		*c*		*d*		*e*		*f*			*g*		*h*		*i*		*j*		*k*		*l*
m	2	+	3	+	4	+	2	+	3	+	4	*q*		6	+	7	+	6	+	7	+	6	+	7
n	3	+	4	+	5	+	3	+	4	+	5	*r*		6	+	5	+	6	+	5	+	6	+	5
o	2	+	4	+	6	+	2	+	4	+	6	*s*		7	+	6	+	9	+	7	+	6	+	9
p	5	+	6	+	5	+	6	+	5	+	6	*t*		9	+	7	+	8	+	9	+	7	+	8

V. Add the above by lines from left to right.

Add the following by columns and by lines as indicated.

	a	*b*	*c*	*d*	*e*		*f*	*g*	*h*	*i*	*j*
k 27	+ 5	+ 7	+ 5	+ 7	+ 5	*r* 37	+ 9	+ 5	+ 7	+ 9	+ 3
l 38	+ 7	+ 8	+ 7	+ 8	+ 7	*s* 29	+ 6	+ 8	+ 9	+ 6	+ 8
m 55	+ 8	+ 5	+ 8	+ 5	+ 8	*t* 54	+ 8	+ 7	+ 4	+ 8	+ 7
n 49	+ 6	+ 9	+ 6	+ 9	+ 6	*u* 68	+ 9	+ 7	+ 8	+ 9	+ 8
o 26	+ 7	+ 5	+ 8	+ 9	+ 8	*v* 57	+ 4	+ 6	+ 5	+ 7	+ 5
p 35	+ 8	+ 9	+ 7	+ 5	+ 6	*w* 69	+ 2	+ 7	+ 4	+ 9	+ 8
q 46	+ 5	+ 4	+ 6	+ 8	+ 7	*x* 45	+ 7	+ 6	+ 5	+ 8	+ 9

VI. Add the following by columns upward and downward; thus, in column *a*, say "20, 50, 60, 80, 100, 110."

VII. Add by lines from left to right, and from right to left.

	a	*b*	*c*	*d*	*e*		*f*	*g*	*h*	*i*
j	10	+ 30	+ 40	+ 20	+ 40	*p*	200	+ 300	+ 400	+ 200
k	20	+ 10	+ 30	+ 40	+ 20	*q*	100	+ 200	+ 500	+ 600
l	20	+ 20	+ 20	+ 30	+ 40	*r*	300	+ 600	+ 700	+ 300
m	10	+ 10	+ 30	+ 40	+ 50	*s*	200	+ 100	+ 200	+ 800
n	30	+ 20	+ 40	+ 20	+ 20	*t*	400	+ 500	+ 800	+ 700
o	20	+ 40	+ 30	+ 40	+ 30	*u*	300	+ 700	+ 300	+ 500

VIII. Add the above again and disregard the zeros till after adding; thus, in column *a*, say "2, 5, 6, 8, 10, 11 tens, or 110"; in column *f*, say "3, 7, 9, 12, 13, 15 hundreds, or 1500."

IX. *v*. How many times does the hammer of a common clock strike from one to ten o'clock inclusive?

w. A maid counting the silver of a house, found that there were 22 teaspoons, 12 coffee spoons, 6 table spoons, 9 dessert spoons, 4 salt spoons and 2 sugar spoons, and that 2 teaspoons were missing. What number of spoons belonged to the house?

Examples for Written Work.

43. *Illustrative Example.* What is the sum of 824, 571, 278, and 636?

WRITTEN WORK.

	824
	571
	278
	636
Sum,	2309

For convenience, the numbers are written so that units of the same order shall be expressed in the same column, and a line is drawn beneath. Beginning with the units to add (thus, 6, 14, 15, 19), there are 19 units = 1 ten and 9 units. The 9 is written in the units' place, and the 1 ten is kept to add with the tens. Adding the tens (thus, 1, 4, 11, 18, 20), there are 20 tens = 2 hundreds and no tens. A zero is written in the tens' place, and the 2 hundreds are kept to add with the hundreds. Adding next the hundreds, the sum of all the numbers is found to be 2309. *Ans.* 2309.

Keeping a number to add with the numbers expressed in the next column is called **carrying.**

In working examples, use as few words as possible; thus, in the illustrative example, say merely "6, 14, 15, 19;* 1, 4, 11, 18, 20; 2, 8, 10, 15, 23; sum, 2309."

Add the following by columns; keep the answers and find their sum.

	1.	2.	3.	4.	5.	6.	7.	
8.	274 +	132 +	972 +	34 +	346 +	6079 +	8460	= ?
9.	206 +	618 +	980 +	800 +	682 +	416 +	592	= ?
10.	87 +	7 +	607 +	147 +	64 +	346 +	1848	= ?
11.	428 +	53 +	89 +	675 +	798 +	5164 +	9087	= ?
12.	536 +	964 +	563 +	398 +	499 +	9075 +	708	= ?
	? +	? +	? +	? +	? +	? +	?	= ?

Add the above numbers by lines; keep the answers and find their sum. How should the sum compare with the sum of the answers found in adding the numbers by columns?

---

* Do not stop to *say* "write 9, carry 1," but *do* it.

Add the following numbers by columns; also add them by lines.

	13.		**14.**		**15.**		**16.**		**17.**		**18.**	
19.	3072	+	5932	+	4320	+	1243	+	3201	+	9876	= ?
20.	5629	+	8275	+	7869	+	5687	+	5674	+	5941	= ?
21.	8900	+	3908	+	9541	+	3219	+	9128	+	9099	= ?
22.	2387	+	762	+	4376	+	1245	+	5463	+	8907	= ?
23.	998	+	5414	+	2408	+	987	+	9187	+	8796	= ?
24.	5734	+	9632	+	321	+	4589	+	342	+	9768	= ?
	?	+	?	+	?	+	?	+	?	+	?	= ?

ADDITION OF DECIMALS.

44. *Illustrative Example.* What is the sum of 564.18, 329.45, 207.136, 81.95, and 165.029?

WRITTEN WORK.

564.18
329.45
207.136
81.95
165.029
1347.745

The numbers are written so that units of the same order shall be expressed in the same column. The adding begins with the units of the lowest order, and proceeds briefly, thus:

Thousandths, 9, 15; write 5, carry 1.
Hundredths, 1, 3, 8, 11, 16, 24; write 4, carry 2.
Tenths, 2, 11, 12, 16, 17; write 7, carry 1, etc.
Ans. 1347.745.

Add the following by columns and by lines.

	25.		**26.**		**27.**		**28.**	
29.	321.06	+	508.7	+	0.675	+	0.045	= ?
30.	58.469	+	0.845	+	12.5	+	8.19	= ?
31.	0.25	+	75.114	+	18.435	+	124.015	= ?
32.	407.076	+	9.58	+	119.04	+	69.909	= ?
33.	98.329	+	11.875	+	365.007	+	584.64	= ?
	?	+	?	+	?	+	?	= ?

45. From the preceding examples may be derived the following

Rule for Addition.

(1) *Write the numbers to be added so that units of the same order shall be expressed in the same column. Draw a line beneath.*

(2) *Add the units of each order separately, beginning with those of the lowest order.*

(3) *When the sum of the units of any order is less than ten, write it under the line in its proper place; when ten or more, write only the units of the sum, and carry the tens to the next column.*

(4) *Write the whole sum of the last addition.*

Proof.

Repeat the work, adding downward instead of upward.

46. Forms of Examples in Addition.

34. How many are 56,789 and 9872, and 31,607?

35. Add the numbers from 95 to 104, inclusive.

36. Find the sum of \$18, \$695, \$40,817, and \$587,489.

37. How many days are there in six years, two of which have 366 days each, and the others 365 days each?

38. 3287 feet + 76,432 feet + 90,862 feet + 6 thousand 48 feet = what number of feet?

39. What is the amount of 8435 pounds, 463 pounds, 56,872 pounds, 4099 pounds, and 18,065 pounds?

40. Find the population of Boston in 1890, the number of inhabitants in 1850 having been 136,881, and the increase since 1850 being 311,596.

41. What is the sum of four hundred thousand eight hundred sixty; sixteen thousand nine hundred; twenty-seven million ninety-five thousand eight hundred ten; and one hundred forty-seven million four hundred eighty-seven thousand three hundred twenty-six?

42. \$ 38.45 + \$ 54.96 + \$ 185.40 + \$ 305.07 and ninety-four dollars more are how many dollars?

43. What is the total of the following sums of money: \$ 87.50, 97 cents, \$ 416.38, and 8 cents?

44. \$ 4.875 + \$ 3.333 + \$ 6.625 + \$ 8.30 = what?

45. Five thousand and five tenths, plus three hundred and seventy-five hundredths, plus eight thousand seven and sixty-five thousandths equals how many?

46.	**47.**	**48.**	**49.**
\$ 425.57	\$ 37,896.50	\$ 408.19	\$ 38.70
9.61	4072.31	3.48	15.25
78.32	8.65	50.72	185.07
54.18	32.19	0.38	406.504
126.73	5786.54	0.95	27.16
45.98	621.09	7.16	0.50
8.76	7958.93	24.68	1.25
53.68	8186.07	753.10	75.338

47. Mental addition — tens and units.

Illustrative Example. Add 27 and 44.

SOLUTION. — 27 and 40 are 67, and 4 more are 71. *Ans.* 71. Or say, simply, "27, 67, 71. *Ans.* 71."

a Add 17 and 36.
b Add 26 and 18.
c Add 34 and 29.

d Add 52 and 37.
e Add 64 and 28.
f Add 46 and 35.

	k		*l*		*m*	
g	21	+	18	+	32	= what?
h	32	+	44	+	26	= what?
i	28	+	26	+	45	= what?
j	35	+	38	+	24	= what?

	n		*o*		*p*		*q*	
r	25	+	18	+	26	+	15	= ?
s	33	+	27	+	44	+	17	= ?
t	18	+	45	+	19	+	26	= ?
u	26	+	27	+	48	+	39	= ?

NOTE. — Accountants often add at once the numbers expressed in two, three, or four columns, by using the method illustrated above.

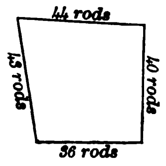
Oral Exercises.

48. *a.* To a cable road running 17 miles northerly from the post office, 8 miles were added in the opposite direction. What was the entire length of the road?

b. Sarah has 15 lilies, John has 8 more than Sarah, and Annie has 9 more than John. How many has Annie?

c. It took 24 yards to carpet a sitting-room, 13 to carpet a hall, and 9 to carpet the stairs of a house. How many yards were required for all?

d. How many rods of fencing would be required to fence the lot of land here represented?



e. It requires 37 feet of pipe to go from a spring to a lane, 9 feet to cross the lane, 12 feet more to reach the yard, and 30 feet more to reach the house. How many feet are required to reach from the spring to 20 feet within the house?

f. A flag was suspended over a street by a rope stretching from houses on opposite sides, one 15 feet back, the other flush with the street, which consisted of two sidewalks, each 9 feet wide, and a roadway between, 30 feet wide. Allowing 10 feet for tying, what length of rope was required?

g. General Ulysses S. Grant was born in 1822; he entered West Point at the age of 17, and 9 years later he was married; 12 years after his marriage, he engaged in the leather business with his father, in 4 years more he was appointed lieutenant general of the United States Army, and in another 4 years he was elected President. How old was he then, and in what year was he elected President?

h. If you add 7 to 17, and then add 30 and 8 more to the sum, how many will you have?

i. A dealer bought turnips at 45¢ a bushel. For how much must he sell them to gain 18¢ a bushel?

j. A merchant sold a hat for 55 ¢, and by so doing lost 20 ¢. How much did it cost him?

k. Having sold 19 bushels of apples and 15 bushels of pears, a marketman has 6 bushels of apples and 6 bushels of pears left. How many bushels of apples had he at first? Of pears? Of both?

l. What must you pay for 1 pound of sugar, 1 pound of coffee, 1 dozen eggs, and 6 lemons at the present prices where you live? Make an example stating prices.

49. Examples for Written Work.

50. There are 92 days in the spring months, 92 in the summer, 91 in the fall, and 90 in the winter months. How many days are there in the year?

51. The distance from Boston to Albany is 202 miles; from Albany to Buffalo, 297 miles; from Buffalo to Toledo, 296 miles; and from Toledo to Chicago, 243 miles. What is the distance from Boston to Chicago?

52. In the upper grade of a grammar school there are 29 boys and 56 girls; in the second grade, 38 boys and 60 girls; in the third grade, 42 boys and 65 girls; and in the fourth grade, 48 boys and 59 girls. How many boys are there in the school? How many girls? How many pupils?

53. After taking from a bank $72.42, I have remaining in it $289.67. How much had I in the bank at first?

54. If a history cost $1.25, a dictionary $4.35, an arithmetic $0.87, ink 13 cents, and stationery 29 cents, how much did all cost?

55. A surveyor measures four fields, and finds in the first 2.362 acres; in the second, 8.245 acres; in the third, 6.19 acres, and in the fourth, 15.784 acres. How many acres does he find in all?

56. If 189,200 bricks are required to build a house, 50,750 to build a stable, and 2364 to lay a sidewalk, how many bricks are required in all?

57. A Michigan lumber merchant sent to New England black walnut as follows: 41,872 ft., 82,974 ft., 95,480 ft. What was the amount sent?

58. What is the sum of 87 and 25 hundredths, 985 and 8 tenths, forty-three thousand eight hundred twenty-four, and 769 and 657 thousandths?

59. The capacity of 5 freight cars is as follows: 35,862 pounds, 37,570 pounds, 38,682 pounds, 39,280 pounds, and 38,049 pounds. How many pounds can all carry?

60. What is the total of the following cash sales?

Monday,	$ 94.83	Thursday,	$ 99.90
Tuesday,	103.76	Friday,	97.13
Wednesday,	126.07	Saturday,	168.58

61. After paying for a buggy $ 85, for a set of extra boxes $ 3, for labor on wheels $ 7.50, for crating and carting $ 4.50, I had $ 75 remaining. How much had I at first?

62. Mr. Grey has 15.162 acres of meadow, 12.8 acres of pasture land, 33.168 acres of woodland, a wheat field of 40.5 acres, and a house lot of 1.017 acres. How many acres has he in all?

63. If a man traveled 9.8 miles in the morning, 8.375 miles in the afternoon, and came back in the evening, how many miles did he travel during the day?

64. A man paid $ 5500 for a house, $ 250 for repairs upon it, $ 185.75 for painting, $ 89.55 for fencing, and he still owes $ 138.90 for grading and trees. What is the total cost of the estate?

65. If you should buy a sleigh for $ 62.50 and gain $ 28.50 by selling it, how much would you get for it?

66. One office on the first floor of a building rents for $ 650 a year, and another for $ 750. A, B, and C hire the second floor for $ 450, $ 125, and $ 62 a year, respectively, and an artist pays for the third floor as much as A and C together pay. For how much does the whole building rent?

Find the amounts in the following lists of items:

67. *Grocer's Bill.*

For	January,	$24.19
"	February,	25.20
"	March.	32.16
"	April,	19.68
"	May,	27.45
"	June,	30.14

68. *Bank Deposits.*

January balance,		$ 648.00
February deposit,		437.50
March	"	268.87
April	"	148.38
May	"	625.29
June	"	348.32

69. Mrs. Benson spent $ 15.63 for a dress, $ 18.68 for lace and trimmings, and $ 16 for making. She also spent $ 28.75 for a shawl, $ 8.59 for a bonnet, $ 1.25 for gloves, $ 4.75 for boots, and has $ 1.24 left. How much money had she at first?

70. My cow, Daisy, gave 1388 pounds of milk in April, 1456 pounds in May, 1440 pounds in June, 1317 in July, and 1175 in August. Jumper gave, during the same time, 1421 pounds, 1485 pounds, 1398 pounds, 1362 pounds, and 1228 pounds. How many pounds did each cow give? How many did both give?

71. New York contains 49,170 square miles; New Jersey, 7815 square miles; Pennsylvania, 45,215; Delaware, 2050; Maryland, 12,210; the District of Columbia, 70; Virginia, 42,450; and West Virginia, 24,780. Texas contains 82,020 square miles more than all these put together. How many square miles has Texas?

72. The expenses of the Worcester Normal School for 1892 were as follows: for salaries, $ 10,933.12; for janitor, $ 600; repairs, $ 1595.62; fuel, $ 772.49; stationery, $ 248.72; printing, etc., $ 448.25 + $ 33.45 + $ 47.75; telephone, $ 51.24; water, $ 79.35; apparatus, $ 348.60; books, $ 146.24; and miscellaneous, $ 95.13. What was the amount of the expenses?

73. A, B, C, and D engaged in trade; A put in $1275, and B $1350; C put in $2580 more than B, and D put in as much as A and B together. How much did all put in?

74. For the nine months preceding March 1, 1894, the receipts of the United States were $104,016,407 from customs; from internal revenue, $106,832,461; and miscellaneous, $12,268,517. What were the total receipts?

75. Find the amount of the duties paid to the United States government upon imported textile manufactures in 1893; these being as follows:

Cotton,	$11,333,605.23	Wool,	$44,608,120.95
Silk,	20,310,258.74	Flax, etc.,	18,974,839.36

76. The following were the values of the mineral productions of the United States in 1893: metallic, $304,775,379; non-metallic, $370,607,864; unspecified, $10,000,000. What was the total value?

In the report of the Commissioner of Education for 1892 are the following items:

In the North Atlantic Division there are 18,324 male teachers, 71,838 female, and $125,387,728 worth of school property; in the South Atlantic Division, 20,241 male teachers, 18,462 female, $14,208,800 school property; in the South Central Division, 29,203 male teachers, 19,512 female, $12,174,813 school property; in the North Central, 53,127 male teachers, 108,603 female, and $149,935,498 school property; in the Western Division, 4034 male teachers, 8887 female, and $21,866,693 school property.

77. How many male teachers were employed in the United States? How many female teachers?

78. What was the value of the school property?

79. Furnish yourself with a good outfit of such school-books as you use, with stationery, etc., at regular prices, and give their cost.

Add the following ledger columns:

80.	81.	82.	83.	84.
$10.82	$22.14	$53.43	$894.80	$787.49
31.94	40.56	81.71	26.51	201.16
69.30	78.95	37.20	8.49	90.75
81.68	42.12	57.69	169.12	87.12
53.09	45.08	26.75	95.04	445.73
34.70	13.03	56.07	219.20	34.24
64.13	56.30	83.96	454.66	93.76
17.62	92.06	60.40	32.33	21.07
5.46	68.47	47.05	378.74	150.81
29.55	86.54	63.88	77.63	257.24
31.37	65.45	20.69	996.84	78.13
67.26	33.87	22.97	545.37	399.25
68.91	93.69	71.99	65.98	40.45
25.94	17.16	22.61	618.23	73.34
30.18	84.88	65.41	135.75	480.56
74.29	6.13	36.54	417.82	102.68
28.37	83.54	76.82	51.19	227.17
70.93	3.26	71.91	769.86	62.43
12.25	31.42	47.12	191.50	553.34
85.81	96.64	17.35	59.08	76.46
69.85	87.30	6.89	75.88	9.78
7.68	64.84	99.99	87.50	897.62
88.72	68.83	33.67	9.98	709.62

Name the different orders of units from units to trillions; from trillions to units. Name the groups from the units' group to trillions; from trillions to units. What is the effect of removing a figure one place to the left? to the right?

What is *Addition?* What is the amount? Add orally 64 and 87. How do you write numbers to be added? Why? Add five numbers expressed by four figures each, and explain. Give the rule for addition; the proof. Illustrate adding at once numbers expressed in two or more columns.

Drill Table No. 1. (See Supplement, Art. 1.)

50. For supplementary practice in addition.

	m				*o*		*p*		*q*		*r*		*s*		*t*		*u*		*v*		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
a	2	3	2	4	2	6	5	8	4	8	6	8	7	7	9	9	9	7	9	8	*a*
b	1	2	2	2	1	3	5	5	4	2	5	2	5	3	6	9	4	1	7	8	*b*
c	2	2	2	1	3	4	5	6	4	1	5	3	7	3	7	4	8	3	8	8	*c*
d	1	5	1	6	2	5	1	7	3	5	2	9	2	5	3	7	2	4	5	9	*d*
e	2	1	2	3	3	3	5	4	4	0	9	0	6	3	3	1	7	1	8	3	*e*
f	1	3	1	9	1	5	4	5	1	7	8	4	2	8	2	7	5	4	6	7	*f*
g	2	2	2	4	3	0	6	0	5	1	8	2	7	2	9	5	8	1	8	0	*g*
h	1	4	1	8	2	8	4	9	1	9	3	4	4	7	1	9	4	6	7	6	*h*
i	2	5	2	1	2	7	6	2	4	4	9	1	6	6	7	5	8	7	9	6	*i*
j	1	1	1	8	1	9	1	8	2	6	6	4	3	9	5	6	5	8	4	8	*j*
k	2	1	2	4	3	6	6	1	5	0	8	5	7	0	9	4	9	2	9	3	*k*
l	1	2	2	3	3	2	4	3	4	3	5	7	6	5	8	9	7	9	8	6	*l*
	w				*x*				*y*				*z*				*zz*				

Exercises upon the Table.

85–104. Add the numbers expressed in columns 1 to 20.

105–114. Add the numbers expressed in the double columns *m* to *v*.

115–119. Add the numbers expressed in the fourfold columns from *w* to *zz*.

Copy and add the numbers expressed from *w* to *zz* (units to thousands), in lines

120. *a.*	**122.** *c.*	**124.** *e.*	**126.** *g.*	**128.** *i.*	**130.** *k.*
121. *b.*	**123.** *d.*	**125.** *f.*	**127.** *h.*	**129.** *j.*	**131.** *l.*

SECTION III.

SUBTRACTION.

51. *Illustrative Example.* There are 150 plum trees in an orchard; if 50 should be taken away, how many would be left?

SOLUTION.—To find how many are left, we take 50, a part of 150, away, and by counting or otherwise, find that there are 100 left.

52. Taking part of a number away to find how many are left is **subtracting.**

53. The number part of which is to be taken away is the **minuend.** The part of the minuend to be taken away is the **subtrahend.** The part of the minuend left after a part has been taken away is the **remainder.**

54. **Subtraction** is the process of taking part of a number away to find how many are left.

55. The sign of subtraction is a horizontal line, —. The expression $15 - 5 = 10$ means that if 5 of the 15 are taken away 10 are left; it is read "15 minus 5 equals 10," or "15 less 5 equals 10."

What is the minuend in the example above? What is the subtrahend? Define subtraction. Read the expression $12 - 7 = 5$.

Oral Exercises in Subtraction.

56. I. Give the remainders in each of the following pairs of numbers expressed in columns and in lines till you can give them rapidly at sight.

	a b	c d	e f	g h	i j
k	11 − 2	11 − 5	11 − 9	11 − 8	12 − 3
l	3 − 2	4 − 3	6 − 4	7 − 6	6 − 2
m	12 − 8	12 − 4	12 − 9	13 − 8	13 − 7
n	5 − 4	8 − 3	7 − 3	9 − 5	4 − 3
o	13 − 5	14 − 5	14 − 6	14 − 9	15 − 6
p	6 − 3	4 − 2	8 − 4	7 − 4	7 − 2
q	15 − 9	16 − 7	16 − 8	17 − 9	18 − 9
r	8 − 6	6 − 5	9 − 7	8 − 2	8 − 3

II. Subtract

s. By 2's from 30; from 29.
t. By 3's from 40.
u. By 4's from 50; from 49.
v. By 5's from 50; from 49; from 48; 47; 46.
w. By 6's from 100; from 99.
x. By 7's from 100.
y. By 8's from 100; from 99.
z. By 9's from 100.

III. From

a. 11 take 2, 4, 7, 6, 3, 8, 9, 5.
b. 12 take 3, 5, 8, 9, 4, 6, 7.
c. 13 take 5, 4, 8, 6, 7, 9.
d. 14 take 6, 5, 9, 8, 7.
e. 15 take 9, 7, 8, 6.
f. 16 take 9, 8, 7.
g. 17 take 8, 9.
h. 18 take 9.

IV. From each number expressed in columns *i*, *j*, *k* below, subtract 2, and notice the units of the remainders.

In the same way subtract 3; 4; 5; 6; 7; 8; 9.

	i	j	k	l	m	n	o	p	q
r	10	11	12	13	14	15	16	27	28
s	20	21	32	33	24	45	46	67	48
t	40	41	42	43	54	35	66	57	88
u	70	61	62	63	74	65	56	77	78
v	90	81	52	83	94	85	96	87	98

V. From the numbers expressed in column *k* above, subtract any number from 3 to 9; in column *l* from 4 to 9; in column *m* from 5 to 9, and so on.

VI. *a* 49 – 7 – 3 – 5 – 4 – 9 – 8 – 7 = how many?
b 54 – 5 – 2 – 3 – 7 – 9 – 8 – 6 = how many?
c 60 – 4 – 7 – 6 – 2 – 3 – 9 – 9 – 8 – 5 – 2 = ?
d 72 – 8 – 5 – 4 – 9 – 7 – 3 – 6 – 7 – 9 – 8 = ?

VII. Give the remainders in the following examples, both in the columns and in the lines.

	e *f*	*g* *h*	*i* *j*	*k* *l*	*m* *n*
o	40 – 30	50 – 30	70 – 50	80 – 50	90 – 60
p	20 – 10	20 – 10	50 – 10	40 – 30	80 – 30

57. Mental subtraction — tens and units.

Illustrative Example. From 42 take 25.

SOLUTION. — 42 less 20 are 22, less 5 more are 17.

VIII. In the following, subtract first by columns, then by lines, giving results only.

	a *b*	*c* *d*	*e* *f*	*g* *h*	*i* *j*
k	50 – 41	61 – 37	80 – 67	81 – 53	71 – 42
l	28 – 19	38 – 19	54 – 28	44 – 39	54 – 29
m	62 – 33	72 – 65	90 – 55	61 – 45	102 – 73
n	34 – 15	36 – 28	66 – 37	48 – 29	84 – 17
o	83 – 47	91 – 44	93 – 74	91 – 42	105 – 88
p	66 – 19	52 – 37	75 – 26	33 – 18	77 – 29

IX. *q*. From 30 take 20; 10; 15; 27; 3; 12; 18; 21; 9; 26; 5; 23; 19.

r. From 60 take 40; 20; 15; 45; 12; 48; 24; 36; 54; 6; 37; 49.

s. From 100 take 70; 30; 65; 35; 25; 75; 17; 83; 33; 66; 45; 87; 54.

t. From 125 take 15; 17; 13; 12; 5; 8; 7; 16; 9; 11; 6; 14; 16; 18.

Examples for Written Work.

58. *Illustrative Example.* John started to walk a mile, which is 5280 feet; after walking 4326 feet, how many more feet had he to go?

WRITTEN WORK.

Minuend,	5280
Subtrahend,	4326
Remainder,	954
Proof,	5280

The minuend and subtrahend are written as in the margin, units under units, tens under tens, and so on. The subtracting begins with the units.

As there are no units in the minuend, one of the tens must be changed to 10 units; 6 units from 10 units leaves 4 units, which is written in the units' place; 2 tens from the 7 tens remaining leaves 5 tens, which is written in the tens' place.

As there are but 2 hundreds in the minuend, one of the thousands must be changed to 10 hundreds, making, with the 2, 12 hundreds; 3 hundreds from 12 hundreds leaves 9 hundreds; 4 thousands from the 4 thousands remaining leaves no thousands. The remainder is 954. *Ans.* 954 feet.

The operation may be expressed briefly, thus: 6 from 10, 4; 2 from 7, 5; 3 from 12, 9; 4 from 4, 0. *Ans.* 954. In practice simply think of results and write them down in order.

To prove the work, add together the remainder and subtrahend. Their sum equals the minuend.

Perform and prove the following examples:

1.	2.	3.	4.	5.	6.
1131	1161	1191	1141	1181	1252
−312	−629	−844	−935	−773	−436
7.	**8.**	**9.**	**10.**	**11.**	**12.**
1292	1232	1282	1052	1192	1270
−668	−715	−279	−343	−407	−947
13.	**14.**	**15.**	**16.**	**17.**	**18.**
1181	1260	1051	1383	1363	1373
−537	−905	−435	−904	−507	−726

19.	**20.**	**21.**	**22.**	**23.**	**24.**
1353	1373	1434	1474	1444	1575
−809	−665	−615	−807	−619	−907
25.	**26.**	**27.**	**28.**	**29.**	**30.**
1585	1676	1626	1787	1797	1858
−648	−958	−719	−809	−968	−829
31.	**32.**	**33.**	**34.**	**35.**	**36.**
1386	1545	2067	2894	2670	3013
−747	−906	−658	−985	−708	−1909
37.	**38.**	**39.**	**40.**	**41.**	**42.**
1819	1612	1528	1625	1737	2478
−923	−891	−753	−973	−875	−488
43.	**44.**	**45.**	**46.**	**47.**	**48.**
1537	1936	1845	1744	1351	2579
−664	−982	−992	−751	−980	−594
49.	**50.**	**51.**	**52.**	**53.**	**54.**
1853	2169	2368	2281	2660	3142
−891	−675	−885	−2099	−2376	−2365
55.	**56.**	**57.**	**58.**	**59.**	**60.**
3654	7654	4820	7443	5652	3544
−2897	−6666	−3945	−2548	−4938	−1735
61.	**62.**	**63.**	**64.**	**65.**	**66.**
2052	2456	4278	5387	5615	7667
−1104	−1979	−1586	−4789	−3224	−6759
67.	**68.**	**69.**	**70.**	**71.**	**72.**
8096	6320	3333	8116	6666	9710
−1897	−3741	−2345	−1807	−5877	−4908

59. *Illustrative Example.* Mr. Graves received a salary of $ 3000, and his son received $ 268. What was the difference in their salaries?

WRITTEN WORK.

```
 (2) (9) (9) (10)
$ 3  0   0   0
     2   6   8
$ 2  7   3   2
```

To find the difference, a part of 3000 equal to 268 must be taken away.

As there are no units, no tens, and no hundreds in the minuend, one of the 3 thousands is taken (leaving 2 thousands) and changed to hundreds; one of the 10 hundreds is taken (leaving 9 hundreds) and changed to tens; and one of the 10 tens is taken (leaving 9 tens) and changed to units. 3000 is thus changed to 2 thousands 9 hundreds 9 tens and 10 units, from which, taking 2 hundreds 6 tens and 8 units, 2732 remains. *Ans.* $ 2732.

In practice simply think "8 from 10, 2"; "6 from 9, 3"; "2 from 9, 7"; "0 from 2, 2"; and write the results in order.

Perform and prove the following examples:

73.	**74.**	**75.**	**76.**	**77.**	**78.**
1407	4000	9081	5015	7026	2003
−769	−192	−483	−364	−857	−648

79.	**80.**	**81.**	**82.**	**83.**	**84.**
5000	6507	8040	3005	6000	9180
−921	−5028	−1816	−1907	−5654	−2498

85.	**86.**	**87.**	**88.**	**89.**
60,320	70,067	80,000	100,000	500,000
−16,804	−14,789	−29,056	−57,035	−302,009

90. 648,305 − 57,937.

91. 440,073 − 155,986.

92. 671,504 − 285,279.

93. 750,006 − 664,087.

94. 1,510,367 − 946,879.

95. 1,009,090 − 999,999.

96. 5,037,256 − 264,000.

97. 13,750,000 − 9,445,382.

SUBTRACTION OF DECIMALS.

60. *Illustrative Example.* From 16.5 miles take 3.875 miles.

WRITTEN WORK.

16.5
3.875
12.625

The numbers are written as before, so that units of the same order shall be expressed in the same column. Beginning with the units of the lowest order to subtract: 5 from 10, 5; 7 from 9, 2; 8 from 14, 6; etc. *Ans.* 12.625 miles.

98. From 3.667 take 1.145.

99. From 32.05 take 17.125.

100. Subtract 0.095 from 1.

101. Subtract 2.083 from 10.

102. Find the difference between 16 and 16 thousandths.

Subtract the following in lines and in columns:

	103.		**104.**
105.	458.09	–	28.365
106.	167.76	–	7.584

	107.		**108.**
109.	902.68	–	28.145
110.	191.87	–	10.728

61. From the preceding examples may be derived the following

Rule for Subtraction.

(1) *Write the minuend and underneath write the subtrahend, so that units of the same order shall be expressed in the same column. Draw a line beneath.*

(2) *Begin with the units of the lowest order to subtract, and proceed to the highest, writing each remainder under the line in its proper place.*

(3) *If any term of the minuend is less than the corresponding term of the subtrahend, add ten to it and then subtract; but consider that the next term of the minuend has been diminished by one.*

Proof.

Add the remainder to the subtrahend: the sum should equal the minuend.

62. Forms of Examples in Subtraction.

111. From 9460 take 5466; from the remainder take 1284.

112. From ten thousand one hundred subtract four thousand fifteen; from the remainder subtract 397.

113. How many more is 3685 than 1857?

114. What number added to 1763 will make 1822?

115. The sum of two numbers is 745; the greater one is 569. What is the smaller one?

116. Find the difference between 202,020 and twenty thousand two hundred two.

117. A minuend being 1828 inches, a subtrahend 934 inches, what is the remainder?

118. 2789 plus what number equals 4280?

119. 7280 − ? = 291.

120. A minuend being 5280 feet, and a remainder 2789 feet, find the subtrahend.

121. After a certain number had been taken out of 500, there remained 184. What was the number taken out?

122. From 980 take 98; from the remainder take 98, and so continue till nothing remains. How many 98's are subtracted?

123. 17.24 less 9.16 equals what?

124. 40.7 rods − 5.25 rods = how many?

125. Subtract 1 tenth from 10.

126. Take 426 thousandths from 1.

127. $ 35.60 added to what will make $ 500?

128. If 91 and 7 hundredths is the minuend and 13.102 the remainder, what is the subtrahend?

129. Mr. Lee bought a farm for $ 3650.80, and sold it for $ 4525. How much did he gain?

130. Mr. Fitz bought a house for $ 975. After spending upon it $ 46.30, he sold it for $ 862.50. How much did he lose?

Examples for Oral or Written Work.

63. *a.* In July, during 17 days the wind blew from the southwest, during 6 from the west; for the other days, the winds were variable. How many days were they variable?

b. Mr. Wood had 38 tomato plants, and sold 12 to one man and 8 to another. How many had he left?

c. Alvin had 27 peaches and 25 pears. He gave 9 peaches to Bennie and 8 pears to John. How many peaches had he left? How many pears?

d. Having 80 miles to travel, I rode 20 miles in the morning, 15 in the afternoon, and 12 in the evening. How many miles of the journey remained?

e. Harry was to be away from home 63 days. After 25 days had passed, how many more days had he to remain away?

f. If James buys a knife for 35 cents and loses 12 cents by selling it, for how much does he sell it?

g. Nellie's cash account for a week was as follows: On hand, 20 cents; allowance, 25 cents; earned, 15 cents; spent, 13 cents. What was the balance of her account?

h. If you have 23 cents, how many more cents must you get to buy a sled worth 62 cents?

i. A man bought a horse for $ 75 and sold him for $ 54. How much money did he lose?

j. Out of 50 butterflies observed by a boy, all but 17 alighted upon objects approaching their own color. How many alighted on such protecting surfaces? How many more did alight on such surfaces than did not?

k. A caterer bargained to pay for a set of fixtures $ 90 in board. The board for four successive months was $ 12, $ 10, $ 15, and $ 11. What still remained due?

h. The roadway over the Natural Bridge in Virginia is 122 feet high from the creek below, and 40 feet above the top of the arch. What is the distance from the top of the arch to the creek below?

m. The first Sunday school in New England was established in 1793. How many years is it since then?

n. Franklin was born in 1706 and died in 1790. What was his age at the time of his death?

o. In 1830 the population of the United States was 13 millions; in 1890 it was 63 millions. How many millions did the population increase in that time?

p. If sound travels 4768 feet a second in salt water, and 4714 feet a second in fresh water, how many more feet a second does it travel in salt water than in fresh?

How much money would be left, and what might the coins be, if a dollar were given in payment for

q. A hat, 25¢; a slate, 15¢; ink, 8¢?

r. A dictionary, 67¢; a dozen pencils, 18¢?

s. A microscope, 25¢; a case to put it in, 13¢?

t. 50 one-cent letter stamps; paper, 7¢; envelopes, 8¢?

u. A collar, 13¢; tie, 18¢; a handkerchief, 19¢?

v. "Robinson Crusoe," 27¢; "Grandfather's Chair," 36¢; "Black Beauty," 24¢?

64. Examples for Written Work.

131. The first savings bank was established in 1778 at Hamburg; the first established in New England was in 1816. What time elapsed between?

132. The year's earnings of a family were $1925. If their expenses were $1136, how much was saved?

133. A and B together invested $ 5740 in business. If B put in $2575, how much did A put in?

134. The earnings of a coöperative bank for the second six months after its establishment were $1874.53, which was $ 576.34 more than the earnings for the first six months. What were the earnings for the first six months?

135. The Amazon River is 3750 miles long; the Mississippi is 4300 miles long. What is the difference in their lengths?

136. The sailing distance from New York to Queenstown is 2890 miles. When a steamer has run 1368 miles on her course from New York, how far has she still to go?

137. Iron melts at a temperature of 3980 degrees Fahrenheit, and gold at 2590 degrees. How much higher temperature is required to melt iron than gold?

138. In one week a grain elevator received 984,560 bushels of grain; of this 769,386 bushels were delivered. How much remained in the elevator?

139. A pound of lead, avoirdupois weight, equals 7000 grains; a pound of gold, troy weight, equals 5760 grains. How many more grains are there in a pound of lead than in a pound of gold?

140. What is the difference in value between a half eagle ($5) of United States money and a pound of English money valued at $4.8665?

141. On the 1st of January I had $216.35 in a bank; on the 3d I drew out $31.50, and on the 4th, $62.75. How much remained in the bank?

142. A man having $918 kept $381 and bought with the remainder a horse and carriage, paying for the horse $387. How much did he pay for the carriage?

143. Two vessels sailing towards each other are 108 miles apart. If one sails 35.4 miles and the other 47.7 miles, how far apart will they be?

144. The depth of rainfall for the summer months was found to be 15.5 inches; for June and July it was 9.875 inches. What was the depth of rainfall in August?

145. The several items of an account amount to $1968.24; of this amount $1174.68 has been paid. Find the balance.

Some Mountain Heights in Feet.

Mt. Blanc, Switzerland,	15,810	Mt. Olympus, Greece,	9745
Mt. Everest, Hindostan,	29,062	Pike's Peak, Colorado,	14,147

Mt. Vesuvius, Italy, 3948

146. Mt. Washington is 9522 feet lower than Mt. Blanc How high is Mt. Washington?

147. How much does Olympus exceed a mile (5280 feet)?

148. How much does Vesuvius fall short of a mile?

149. How much does Pike's Peak exceed 2 miles?

150. If Vesuvius could be put upon Olympus, how much lower would its peak be than that of Mt. Blanc?

151. Compare Mt. Blanc with Pike's Peak; state which is higher, and how much?

152. Compare also Mt. Olympus and Mt. Blanc.

153. How much higher than Mt. Everest is a balloon at a height of 32,786 feet?

154. The estimated height attained by Glaisher in 1862 was 37,000 feet. How much does this exceed 32,786 feet?

65. Miscellaneous Examples.

155. Out of a barrel of flour, 196 pounds, there were sold 14 pounds, 28 pounds, and 56 pounds. How much remained?

156. Two men start at the same place and ride in opposite directions around a park till they meet, one traveling 598.7 rods and the other 1746.8 rods. How many rods is it around the park?

157. Two vessels sailed from port by the same course at the same time. At the end of four days one had sailed 354 miles and the other 423 miles. How far apart were they?

158. John Adams and Thomas Jefferson died on the fiftieth anniversary of the Declaration of Independence. In what year did they die? Jefferson was born in 1743. How old was he when he died?

159. In 1850 my grandfather was 87 years old. In what year was he born? How old was he when the independence of the United States was declared?

160. A man owing \$726.30 paid \$93.75 of the debt in cash and \$84 by check. How much did he then owe?

161. Having \$675 in the bank, Mr. Black drew as follows: \$37.50, \$18.68, \$125.87, and \$48.63. How much money was left in the bank?

162. Previous to a fire, the contents of a store were estimated to be worth \$14,500; after the fire to be worth \$5485.75; the difference was to be paid by an insurance company. How much was the company to pay?

163. By making part payment of a note, the interest upon it, which had been \$125 a year, was reduced to \$97.75. How much interest was saved by the payment?

164. A speculator bought a house for \$3400, paid \$1800 for repairs, and \$545 for grading the lot, after which he sold it for \$8000. Did he gain or lose, and how much?

165. The polar diameter of the earth is 7898.8809 miles. How much greater is the equatorial diameter, which is 7924.9111 miles?

166. In 1892 Paris contained 2,447,957 inhabitants, and London 1,773,495 more than Paris. How many inhabitants had London?

167. In 1890 the United States numbered 62,622,250 inhabitants; of these, 5,067,130 males and 4,182,417 females were foreign born. How many were native born?

168. At the World's Fair in Chicago in 1893, the total attendance was 23,529,400; this was 8,824,711 less than the attendance at the Paris Exposition in 1889. What was the attendance at the Paris Exposition?

169. The salaries of clerks at the Chicago Fair amounted to \$387,001.88; of officers, \$323,685.48; of musicians, \$128,674.85; of architects, \$133,830.23. What was the sum of all the salaries?

170. The largest day's attendance at each of three great World's Expositions was as follows: Paris, October 13, 1889, 397,150; Philadelphia, September 28, 1876, 217,526; Chicago, October 9, 1893, 729,203. What was the excess of Chicago's attendance over each of the others?

171. The sum of three numbers is 4875; one of the numbers is 255.6, and another is 25.58. What is the third number?

172. From the sum of 98.39 and 116.7, take their difference.

173. A cubic inch of mercury in a solid form weighs 0.567 of a pound, in liquid form, 0.078 of a pound less. What is the weight of a cubic inch in liquid form?

Find the balance of the following accounts:

174.

Dr. *	*Cr.* †
$ 0.78	$ 16.20
1.125	36.15
4.09	0.25
1.50	10.38
18.625	4.16

175.

Dr.	*Cr.*
$ 3.185	$ 59.00
41.07	18.05
63.98	121.48
0.44	62.18
37.50	49.56

176.

Dr.	*Cr.*
$ 9.64	$ 8.28
17.18	13.63
26.73	5.41
19.48	7.86
0.29	28.45
30.48	4.18

177.

Dr.	*Cr.*
$ 41.875	$ 50.50
98.21	5.84
3.67	17.63
58.42	2.075
8.625	27.68
7.34	18.28

What is *Subtraction?* What is the minuend? The subtrahend? The remainder? Take orally 36 from 84. Find the difference between 469 and 5007, and explain. Give the rule; the proof. When the minuend and difference are given, how can you find the subtrahend? When the subtrahend and difference are given, how can you find the minuend?

* Dr. is read "debtor," and means that the sums written beneath are owed.

† Cr. is read "creditor," and means that the sums written beneath have been paid by the debtor or are due from him.

DRILL TABLE No. 2. (See Supplement, Art. 1.)

66. For supplementary practice in subtraction.

	m		n		o		p		q		r		s		t		u		v		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
a	2	3	2	4	2	6	5	8	4	8	6	8	7	7	9	9	9	7	9	8	*a*
b	1	2	1	4	1	3	5	5	4	2	5	2	5	3	6	9	4	1	7	8	*b*
c	2	2	2	1	3	4	5	6	4	1	5	3	7	3	7	4	8	3	8	8	*c*
d	1	5	1	6	2	5	1	7	3	5	2	9	2	5	3	7	2	4	5	9	*d*
e	2	1	2	3	3	3	5	4	4	0	9	0	6	3	3	1	7	1	8	3	*e*
f	1	3	1	6	1	5	4	5	1	7	8	4	2	8	2	7	5	4	6	7	*f*
g	2	2	2	4	3	0	6	0	5	1	8	2	7	2	9	5	8	1	8	0	*g*
h	1	4	1	8	2	8	4	9	1	9	3	4	4	7	1	9	4	6	7	6	*h*
i	2	5	2	1	2	7	6	2	4	4	9	1	6	6	7	5	8	7	9	6	*i*
j	1	1	1	8	1	9	1	8	2	6	6	4	3	9	5	6	5	8	4	8	*j*
k	2	1	2	4	3	6	6	1	5	0	8	5	7	0	9	4	9	2	9	3	*k*
l	1	2	1	3	3	2	4	3	4	3	5	7	6	5	8	9	7	9	8	6	*l*
							w				x				y						

Exercises upon the Table.

178–189. In each line *a, b, c,* etc., from 100 take $m + n + o$.

190–201. From each number in *w*, take 1728.

202–213. From 9234 take each number in *w*.

214–225. Take each number in *x* from 10,300.

226–237. Take each number in *y* from 1,234,567.

238–243. In *w*, take *b* from *a*; *d* from *c*, etc.

244–249. In *x*, take *b* from *a*; *d* from *c*, etc.

250–255. In *y*, take *b* from *a*; *d* from *c*, etc.

256–267. In each line *a, b, c,* etc., from *x* plus *y* take *w*.

SECTION IV.

MULTIPLICATION.

67. *Illustrative Example.* Find the number of days in 6 months of 31 days each.

(1) *Added.*	(2) *Multiplied.*	
31	Multiplicand,	31
31	Multiplier,	6
31	Product,	186
31		
31		
31		
186		

The number of days in 6 months of 31 days each can be found by adding, as in the margin, or by taking six 1's, which is six, for the units of the number, and six 3's, which is 18, for the tens of the number. The result is 186. *Ans.* 186 days.

68. When equal numbers are united by uniting at once the units of the same order, the numbers are said to be **multiplied.**

69. One of the equal numbers to be united is the **multiplicand.** The number that shows how many equal numbers are to be united is the **multiplier.** The result obtained by multiplying is the **product.**

70. **Multiplication** is the process of uniting equal numbers to find their product.

71. The multiplicand and multiplier are called **factors** (makers) of the product. The product is a **multiple** of the factors making it; thus, 15 is a multiple of 5 and of 3.

NOTE.—The units of the product are of the same kind as the units of the multiplicand. Thus, if days are multiplied, the product is a number of days; if tens are multiplied, the product is a number of tens; if tenths are multiplied, the product is a number of tenths.

72. The sign of multiplication is an oblique cross $\times$. The expression $40 \times 5 = 200$ means that five 40's are 200. It is read "40 multiplied by 5 equals 200." It may also be read "five 40's equal 200," or "five times 40 equals 200."

In the example above, what is the multiplicand? the multiplier? the product? Name two factors of 24. Name two other factors. Name another two. Read the expression $15 \times 3 = 45$.

73. How many are 5×3? 3×5? $2\times4\times3$? $4\times2\times3$? The *product of two or more factors is the same, whatever the order in which the factors are taken.*

74. To find the product of two factors in the Multiplication Table, find one of the factors in the left-hand column, the other in the top line; the product will be found in the same line with the first factor and in the same column with the second.

MULTIPLICATION TABLE.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
2	**4**	6	8	10	12	14	16	18	20	22	24
3	6	**9**	12	15	18	21	24	27	30	33	36
4	8	12	**16**	20	24	28	32	36	40	44	48
5	10	15	20	**25**	30	35	40	45	50	55	60
6	12	18	24	30	**36**	42	48	54	60	66	72
7	14	21	28	35	42	**49**	56	63	70	77	84
8	16	24	32	40	48	56	**64**	72	80	88	96
9	18	27	36	45	54	63	72	**81**	90	99	108
10	20	30	40	50	60	70	80	90	**100**	110	120
11	22	33	44	55	66	77	88	99	110	**121**	132
12	24	36	48	60	72	84	96	108	120	132	**144**

75. The numbers 1, 4, 9, etc., in the diagonal of the table are the products of two equal factors, and are called the **squares** of the numbers 1, 2, 3, etc., respectively.

Oral Exercises in Multiplication.

76. I. *a.* Repeat the multiplication table of 2's; thus, "Two 2's are 4, three 2's are 6, four 2's are eight," to "twelve 2's are 24."

b. Repeat the same in the reverse order.

c. In the same way repeat the multiplication table of 3's; 4's; 5's; 6's; 7's; 8's; 9's; 10's; 11's; 12's.

II. *d.* Name the squares of the numbers from 1 to 12; thus, 1, 4, 9, etc. (Art. 75.)

III. *e.* Turn to page 44 and multiply the number expressed by each figure in line *a* by 2; by 3; by 4; 5; 6; 7; 8; 9; 10; 11; 12.

IV. Give at sight the products in the following examples in the columns and in the lines.*

	a b	*c d*	*e f*	*g h*	*i j*	*k l*
m	2 × 5	4 × 4	4 × 7	7 × 4	7 × 5	11 × 2
n	3 × 4	2 × 6	4 × 8	7 × 9	6 × 9	3 × 12
o	7 × 3	9 × 3	3 × 9	9 × 7	8 × 9	11 × 4
p	1 × 5	2 × 5	8 × 6	8 × 8	7 × 9	5 × 12
q	2 × 7	6 × 2	9 × 2	6 × 7	11 × 6	11 × 8
r	4 × 3	5 × 8	5 × 7	4 × 8	7 × 12	9 × 12
s	5 × 4	8 × 6	6 × 5	9 × 6	11 × 10	12 × 7
t	3 × 8	2 × 3	6 × 7	4 × 7	11 × 12	12 × 9

* Thus in columns *a*, *b*, etc., do not say "three 2's are 6; four 5's are 20"; but simply "6, 20," etc.

V. *a.* Multiply each of the following numbers by 2 and add 1 to the product, naming only final results.

1 3 6 4 7 2 8 5 9

Multiply the numbers above

b. By 3 and add 2.

c. By 4 and add 2; 3.

d. By 5 and add 3; 4.

e. By 6 and add 4; 5.

f. By 7 and add 5; 6.

g. By 8 and add 6; 7.

h. By 9 and add 7; 8.

i. By 12 and add 8; 9.

How many cents equal in value

j. 3 two-cent pieces + 1 cent?
k. 8 ten-cent pieces + 2 cents?
l. 6 five-cent pieces + 3 cents?
m. 9 ten-cent pieces + 5 cents?

77. *Illustrative Example.* Multiply 60 by 4.

Solution. — 4 times 6 tens is 24 tens, or 240. *Ans.* 240.

Naming results only,

n. Multiply 40, 70, 20, 60, 50, 30, 80, 90, each by 2; by 3; 4; 5; 6; 7; 8; 9; 10.

o. Multiply 600, 400, 200, 700, 500, 300, 800, 900, each by 2; by 3; 4; 5; 6; 7; 8; 9; 10.

78. *Illustrative Example.* What is the cost of 4 wagons at $ 70 each?

Solution. — Since 1 wagon costs $ 70, 4 wagons will cost 4 times $ 70, or $ 280. *Ans.* $ 280.

p. What is the cost of 2 horses at $ 200 each?

q. The original value of shares in the Washington mills was $ 1000 each. What was the value of 10 shares?

r. At 60 pounds to a bushel, how many pounds are there in 6 bushels of wheat? In 9 bushels of potatoes?

s. At 100 pounds to each, how many pounds are there in 5 quintals of dried fish? In 10 kegs of nails?

t. It is 30 rods from my home to school. How far do I travel in going and returning for 5 school sessions?

u. It is 500 feet across the bridge at Lawrence, Mass. How many feet does a person travel who crosses over it and returns 3 times?

UNITED STATES MONEY.

79. The legal coins in common use are:

Gold.		Silver.	
Double-eagle	= $20.00	Dollar	= $1.00
Eagle	= 10.00	Half-dollar	= 0.50
Half-eagle	= 5.00	Quarter-dollar	= 0.25
Quarter-eagle	= 2.50	Dime	= 0.10
Nickel.		**Bronze.**	
Five-cent piece.		One-cent piece.	

Bank bills and United States Treasury notes are largely used in place of coins. These represent the values of $1, $2, $5, $10, $20, $50, $100, $500, and $1000.

a. Change 3 eagles to dollars. 8 half-eagles to dollars.

b. How many cents are 3 dimes 4 cents? 8 dimes 6 cents? 2 eagles and 2 five-dollar bills?

c. How many cents are $8? $5 and 20 cents?

How much money have you if you have

d. 3 ten-dollar bills, 4 twos, 1 five, and 6 ones?

e. 2 five-dollar bills, 1 twenty, 4 fives, and 5 twos?

f. 7 ten-dollar bills, 6 ones, with 1 half-dollar, 1 quarter, and 2 dimes?

g. 2 twenty-dollar bills, 3 tens, with 3 half-dollars, 2 quarters, and 4 dimes?

h. 5 half-dollars, 2 quarters, 3 dimes, and 1 five-cent piece?

Examples for Written Work.

80. *Illustrative Example.* How many days are there in 3 years of 365 days each?

WRITTEN WORK.	
Multiplicand,	365
Multiplier,	3
Product,	1095

Writing the multiplicand and multiplier as in the margin, multiply each term separately, beginning with the units.

Three 5's are 15;* 15 units equal 1 ten and 5 units; write 5 under the line in the units' place, and keep the 1 ten to add to the product of the tens.

Three 6's are 18; 18 tens with the 1 ten kept, equal 19 tens, or 1 hundred and 9 tens. Write 9 under the line in the tens' place, and keep the 1 hundred to add to the product of the hundreds.

Three 3's are 9. 9 hundred and the 1 hundred kept equal 10 hundred, or 1 thousand and no hundreds. Write 0 under the line in the hundreds' place, and 1 in the thousands' place. The entire product is 1095. *Ans.* 1095.

1.	**2.**	**3.**	**4.**	**5.**
2736	2543	3352	5123	6234
× 2	× 3	× 4	× 5	× 6
6.	**7.**	**8.**	**9.**	**10.**
3524	4829	7826	6705	6384
× 3	× 2	× 4	× 5	× 6
11.	**12.**	**13.**	**14.**	**15.**
6537	4689	7529	3445	9814
× 2	× 3	× 6	× 4	× 5

16. 7435 × 7 **18.** 6838 × 9 **20.** 7459 × 9 **22.** 3598 × 8

17. 2368 × 8 **19.** 6249 × 8 **21.** 8302 × 7 **23.** 4965 × 7

* For the sake of rapid working, use as few words as possible. Thus, in the example above, say "fifteen," "eighteen," "nineteen," "nine," "ten." While saying fifteen, write 5; while saying nineteen, write 9; and while saying ten, write 10.

24. 1257×9 **26.** 3798×12 **28.** $62{,}436 \times 10$ **30.** $54{,}965 \times 11$
25. 2806×11 **27.** 7381×11 **29.** $43{,}578 \times 12$ **31.** $74{,}261 \times 12$

Multiply	How many are
32. 2581 days by 4	**36.** 7682 sheep × 7 ?
33. 3428 hours by 5	**37.** 3085 books × 9 ?
34. 7398 miles by 8	**38.** 7692 rods × 11 ?
35. 9402 eggs by 6	**39.** 8548 feet × 12 ?

Find the sum of the first two products in each of the following examples, and see if it agrees with the third product.

40.	**41.**	**42.**	**43.**
2468 × 3	3568 × 2	3579 × 3	52479 × 2
2468 × 7	3568 × 9	3579 × 7	52479 × 9
2468 × 10	3568 × 11	3579 × 10	52479 × 11

44.	**45.**	**46.**	**47.**
2468 × 5	2468 × 4	3579 × 5	73579 × 4
2468 × 6	2468 × 8	3579 × 6	73579 × 8
2468 × 11	2468 × 12	3579 × 11	73579 × 12

81. Multiplication — tens and units.

Illustrative Example. At $ 36 each, how much will 7 coats cost?

SOLUTION. — For oral work multiply first the tens, then the units; thus 7 times $ 30 is $ 210, and 7 times $ 6 is $ 42, which, added to $ 210, makes $ 252. *Ans.* $ 252.

Find the cost

48. Of 6 mugs at 15 ¢ each. **50.** Of 10 hats at 64 ¢ each.
49. Of 4 caps at 75 ¢ each. **51.** Of 9 coats at $ 34 each.

52. Of 7 cows at $ 56 each, and 4 oxen at $ 100 each.
53. Of 10 sleighs at $ 75 each, and 10 robes at $ 8 each.

54. If 3 boys can weed a row of beets in 18 minutes, how long will it take 1 boy to weed it?

55. How many lines are there in two magazine articles, one of 7, and the other of 5 pages of 48 lines each?

56. If Charles and William earn 35 cents an hour each, how much will both earn in 8 hours?

82. *Illustrative Example.* How many pounds of butter are there in 47 boxes of 5 pounds each?

SOLUTION. — 47 boxes of 5 pounds each, will contain 47 times 5 pounds. But 47 times 5 equals 5 times 47, which is 235 (Art. 73). *Ans.* 235 pounds.

NOTE. — In explaining problems, follow the logical process; in doing the written work, take the shortest way, disregarding the denomination; but with the result, give the denomination required by the statement of the problem.

What must be paid for the following purchases:

57. 32 spools of cotton at 4¢?
58. 50 yards of muslin at 9¢?
59. 48 yards of gimp at 8¢?
60. 76 feet whitewood at 8¢?
61. 80 feet pine at 7¢?
62. 100 feet walnut at 11¢?

63. A dealer bought two horses at $135 each. The next week he sold the span for $348. Did he gain or lose, and how much?

64. How much is gained by a broker in exchanging foreign coins for $235 United States money, if he makes 2 cents on a dollar?

83. In 1 whole thing of any kind, how many halves are there? How many thirds? fifths? eighths? etc.

65. 12 wholes are how many halves? sixths? ninths?

66. 30 wholes are how many thirds? fifths? tenths?

67. How many halves are 16 and 1 half? 62 and 1 half?

68. How many thirds are 33 and 1 third? 66 and 2 thirds? 16 and 2 thirds?

69. How many fourths are 31 and 1 fourth? 18 and 3 fourths?

84. Examples for Written Work.

70. How many miles can be traveled in 6 days, the rate being 345 miles per day?

71. Three heirs inherited $5468 each. How much did all inherit?

72. How many kernels of corn, 5 kernels to a hill, are planted in 8 rows of 127 hills each?

73. What must be paid for 4 barrels of sugar, each containing 331 pounds, at 5 cents a pound?

74. In one year Texas produced 62,152 pounds of rice. What was its value at 6 cents a pound?

75. What will 825 sheep cost at $9 apiece? at $8 apiece?

76. What must be paid for the use of $3695 for a year, if 7 cents is paid on every dollar?

77. What must be paid for insuring a house worth $12,560, at the rate of 2 cents on a dollar?

78. When 718 bricks are required for each rod of sidewalk, how many will be required for 11 rods? for 12 rods?

85. *Illustrative Example.* At 56 pounds to a bushel how many pounds are there in 10 bushels of corn? in 100 bushels? in 1000 bushels?

Solution. — Ten 56's equal 56 tens (Art. 73), or 560; one hundred 56's equal 56 hundreds, or 5600; one thousand 56's equal 56 thousands, or 56,000. *Ans.* 560 pounds; 5600 pounds; 56,000 pounds.

In multiplying by 10, 100, 1000, etc., the product may be immediately expressed *by annexing to the expression for the multiplicand as many zeros as the multiplier has.*

Find the sum of the products in each example below:

79.	80.	81.
42 × 10	689 × 100	4692 × 100
9 × 100	83 × 1000	4692 × 1000
25 × 100	125 × 10	4692 × 10,000

Find the total value of

82. 75 town bonds at \$ 100, 100 R.R. tickets at \$ 9, 450 eagles at \$ 10.	**83.** 100 cords of wood at \$ 8, 10 saddles at \$ 42, 600 acres of land at \$ 100.

86. *Illustrative Example.* There are 640 acres in 1 square mile. How many acres are there in 200 square miles?

WRITTEN WORK.

```
  640
  200
-------
128,000
```

$640 = 64 \times 10$, and $200 = 2 \times 100$; hence 640×200 is the same as $64 \times 2 \times 10 \times 100$ (Art. 73).

When the multiplicand and multiplier, one or both, have zeros at the right, ***the zeros may be disregarded in multiplying, but there must be annexed to the expression for the product as many zeros as were disregarded.***

84. Multiply 475 by 20; by 300; by 4000; and add the products.

85. Multiply 7500 by 500; by 6000; by 70; and add the products.

86. Multiply 6740 by 8000; by 110; by 1200; and add the products.

87. Multiply 869 by 10,000; by 9000; by 900; and add the products.

88. There are 60 seconds in a minute, and 60 minutes in an hour. How many seconds are there in an hour?

89. To allow 50 cubic feet of air per minute for each child, how many cubic feet of air per hour should pass through a school hall containing 840 pupils?

90. During the World's Fair in Chicago there were 21,530,854 paid admissions. At 50 cents each, what amount was paid for admissions?

91. Multiply 282 by 8; by 70; and by 900; and add the products.

92. Multiply 312 by 3; by 40; by 200; and add the products.

87. *Illustrative Example.* Multiply 312 by 243.

WRITTEN WORK.

(1)	(2)
312	312
243	243
936	936
12480	1248
62400	624
75816	75816

For convenience, the multiplicand, 312, is multiplied by each term of the multiplier separately. The only difference between forms (1) and (2) of the written work is, that in form (2) the zeros in the partial products are omitted.

In the second form, care must be taken to place the first or right-hand figure of each partial product directly under the term of the multiplier used.

Multiply the following as indicated in lines and columns, and prove each example by multiplying the multiplier by the multiplicand (Art. 73).

		95.	96.		99.	100.		103.	104.
Multiply	**93.**	237	× 321	**97.**	243	× 542	**101.**	645	× 761
by	**94.**	432	× 243	**98.**	576	× 342	**102.**	342	× 524

		107.	108.		111.	112.		115.	116.
Multiply	**105.**	806	× 871	**109.**	1065	× 925	**113.**	4059	× 956
by	**106.**	528	× 634	**110.**	5670	× 573	**114.**	2133	× 874

		119.	120.		123.	124.		127.	128.
Multiply	**117.**	596	× 639	**121.**	788	× 928	**125.**	7912	× 395
by	**118.**	842	× 978	**122.**	837	× 842	**126.**	6998	× 876

88. *Illustrative Example.* Multiply 7643 by 604.

WRITTEN WORK.

```
  7643
   604
 30572
45858
4616372
```

129. 7338 × 305 = ?

130. 9560 × 7080 = ?

131. 30972 × 289 = ?

132. 44002 × 6007 = ?

133. 13487 × 1060 = ?

134. 69080 × 7508 = ?

MULTIPLICATION OF DECIMALS.

89. *Illustrative Example.* Multiply 307.48 by 9.

WRITTEN WORK.	
307.48 9 2767.32	Since the lowest order of units in the multiplicand is hundredths, the lowest order in the product must be hundredths, which is expressed by putting the decimal point two places from the right. *Ans.* 2767.32.

135. Multiply 3.018 by 8.
136. Multiply 76.91 by 43.
137. Multiply 0.489 by 39.
138. Multiply 6.0981 by 40.
139. Multiply 0.0004 by 131.
140. Multiply 0.625 by 325.

141. There are 16.5 feet in a rod. How many feet are there in 23 rods?

142. If an express train goes 40.75 miles an hour for 15 hours, what distance does it go?

90. From the preceding examples may be derived the following

Rule for Multiplication.

(1) *Write the multiplicand, and underneath write the multiplier. Draw a line beneath.*

(2) *If the multiplier consists of one term only, multiply each term of the multiplicand by the multiplier, beginning with the term of the lowest order, and carrying as in addition.*

(3) *If the multiplier consists of more than one term, multiply by each term of the multiplier separately, writing the partial products so that units of the same order shall be expressed in the same column.*

(4) *Add the partial products thus obtained, and the result will be the entire product.*

Proof.

Multiply the multiplier by the multiplicand. If the work is correct, the two products will be equal.

143. $59,780 \times 8004$

144. $67,093 \times 3096$

145. $30,600 \times 4009$

146. $69,997 \times 8706$

147. $4120 \times 17,800$

148. $57,000 \times 90,200$

149. Find the cost of 17 pounds of chickens at 22 cents per pound, and 15 pounds of turkey at 18 cents per pound.

150. What will be the cost of 5 gallons of syrup at 65 cents a gallon, 30 barrels of apples at $3.75 a barrel, and 17 barrels of pears at $4.25 a barrel?

91. *Illustrative Example.* Multiply 5.789 by 10; by 100; by 1000; by 10,000.

$5.789 \times 10 \quad = 57.89$
$5.789 \times 100 \quad = 578.9$
$5.789 \times 1000 \quad = 5789.$
$5.789 \times 10,000 = 57.890.$

Since moving the decimal point to the right has the effect of moving the figures the opposite way (Art. 20):

To multiply a decimal by 10, 100, 1000, etc., *move the decimal point as many places to the right as the multiplier has zeros.*

151. Multiply 48.654 by 10; by 100; by 1000; and add the products.

152. Multiply 3.709 by 100; by 10; by 10,000; and add the products.

153. What is the cost of 10 shares of Merchants' Bank stock, at $157.75 per share, and 100 shares of Union Pacific stock at $104.25 per share?

154. One meter equals 1.0936 yards. How many yards do 10 meters equal?

155. In one mile there are 1.60931 kilometers. How many kilometers are there in 100 miles?

156. $5.625 are how many cents? how many mills?

157. $3.87 are how many cents? how many mills?

158. Change $43 to cents; to mills.

159. At 100 centimes per franc, how many centimes are there in 48 francs?

92. Forms of Examples in Multiplication.

160. Multiply 509,367 by 9, by 8, and by 7, and add the products.

161. Multiply 78 by 10, by 3000, and by 600, and add the products.

162. Find the product of 368 by 59.

163. How many are 416 times 985?

164. Two thousand four hundred ninety-nine × seven hundred eight = ?

165. 48,967 × 67 plus 1900 × 800 = what number?

166. 98,765 × 432 are how many?

167. Take 407,063 for a multiplicand and 159 for a multiplier, and find the product.

168. A multiplier is 14,506 and the multiplicand thirty-six thousand sixty-nine; what is the product?

169. One factor being 318,459 and the other 6005, find the product.

170. What product will be obtained by using 25 as a factor twice? three times?

171. 58,000,007 × 506 = ? **172.** Multiply 0.625 by 6.

NOTE. — When a zero occurs at the right of a decimal fraction, it may be disregarded. In the answer to the 172d example, 750 thousandths is the same as 75 hundredths.

173. Multiply $1.08 by 15; $5.165 by 3; $0.125 by 5; and add the products.

174. What is the product of 18 thousandths multiplied by 14?

175. Multiply 6.458 by 10, by 1000, by 100; and add the products.

176. At $0.17 apiece for tumblers, what is the cost of 5 lots of 100 tumblers each?

177. At 30 cents an hour, how much can a person earn in 15 days of 9 hours each?

178. What is the cost of 16 yards of silk at $1.125 a yard, and 25 yards of cambric at $0.07 a yard?

WEIGHTS AND MEASURES IN COMMON USE.

93. LIQUID MEASURE.

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).

94. DRY MEASURE.

2 pints = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

95. NUMBERS.

12 ones = 1 dozen (doz.).
12 dozen = 1 gross.
12 gross = 1 great gross.
20 ones = 1 score.

96. PAPER.

24 sheets = 1 quire.
20 quires = 1 ream.
2 reams = 1 bundle.
5 bundles = 1 bale.

97. AVOIRDUPOIS WEIGHT.

Common Weights.

16 ounces (oz.) = 1 pound (lb.).
2000 pounds = 1 ton (T.).

Gross Weights.

112 pounds = 1 hundred weight (cwt.).
20 cwt. (2240 lb.) = 1 long ton.

NOTE. — In weighing some articles, as iron and coal at the mines, and goods on which duties are paid at the United States custom houses, the long ton of 2240 pounds is used; when the kind of ton is not designated, the common ton of 2000 pounds is understood.

Oral Exercises.

98. *a.* Repeat the table of liquid measure; of dry measure; of numbers; of paper; of avoirdupois weight.

b. How many things are there in 9 dozen? in a gross? in a great gross?

c. 4 gallons of oil are how many quarts? pints? gills?

d. 2 bushels of corn are how many pecks? quarts?

e. A ream of paper is how many quires? sheets?

f. 6 gallons 3 quarts equal how many quarts?

SOLUTION. — Since 1 gallon equals 4 quarts, 6 gallons will equal 6 times 4 quarts, or 24 quarts, which with 3 quarts added equal 27 quarts. *Ans.* 27 quarts.

g. 6 dozen and 4 equal how many?

h. How many sheets of letter paper are 2 quires 4 sheets?

What must be paid for

i. 2 bushels 1 peck of potatoes at 40 cents a peck?

j. 3 pecks of berries at 6 cents a quart?

k. 4 bushels of walnuts at 10 cents a quart?

l. 3 quarts 1 pint of cream at 20 cents a pint?

m. 1 pint 3 gills of cologne at 25 cents a gill?

Examples for Written Work.

99. *Illustrative Example.* How many ounces are 35 lb. 7 oz.?

WRITTEN WORK.

```
35 lb. 7 oz.
16
———
210
35
———
567 oz.
```

Observe that the 7 ounces are added with the partial products to make 567.

179. How many pounds are there in 6 loads, each containing 2 tons 750 pounds?

180. A gardener had 18 pounds of seeds which he put into bags, using 4 bags for every ounce. How many bags did he use?

181. How many papers of matches are 48 great gross and 10 gross?

182. How many lines has a person written who has filled a quire of letter paper both sides, writing 24 lines to each page?

183. How many letters can be written on a ream of letter paper, using one half sheet for each letter?

184. What is the difference in pounds between 250 long tons and 280 short tons?

185. How much will a dealer receive for 6 gross of clothespins at 10 cents a dozen, 2 gross of slates at 4 cents apiece, and a great gross of buttons at 11 cents a dozen?

186. Find the total cost of 4 gross packages of tacks at $1.44 a gross, 2 gross at $1.80, 1 gross at $2.16, and 7 gross at $2.25.

100. TIME MEASURE.

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (h.).
24 hours = 1 day (d.).
7 days = 1 week (w.).
365 days, or 52 weeks 1 day } = 1 common year (y.).
366 days = 1 leap year.
100 years = 1 century.

The year begins with the first of January, and is divided into four **seasons** of three months each, as follows:

The **winter** months are December, January, and February.
The **spring** months are March, April, and May.
The **summer** months are June, July, and August.
The **autumn** months are September, October, and November.

"Thirty days hath September,
April, June, and November,
All the rest have thirty-one
Excepting February alone,
To which we twenty-eight assign
Till leap year gives it twenty-nine."

NOTE.—For explanation of leap year, see Supplement, Art. 39.

Oral Exercises.

101. *a.* How many months are there in 1 y.? in 3 y. 4 mo.? in 8 y. 4 mo.? in 16 y. 8 mo.?

b. Which months have 30 d. each? Which have 31 d. each?

Allowing 30 days to a month, change

c. 2 mo. 3 d. to days.
d. 3 mo. 3 d. to days.
e. 5 mo. 10 d. to days.
f. 4 mo. 17 d. to days.

g. Change 4 wk. 6 d. to days; 5 d. 10 h. to hours.

h. Change 3 h. 20 min. to minutes; 24 h. to minutes; 1 h. to seconds.

i. What date is 30 days later than April 5? than May 5?

j. What date is 30 days earlier than April 5? than May 5?

How many days are there

k. From June 1 to July 1? *m.* From Apr. 1 to May 10?

l. From Nov. 25 to Dec. 5? *n.* From Apr. 20 to June 10?

o. How many minutes are there between 6 o'clock 45 min. A.M. and 8 o'clock 55 min. A. M.? between 11 o'clock 20 min. A.M. and 2 o'clock 52 min. P.M.?

MEASURES OF LENGTH.

102. ______1 inch______. How many inches long is this book? How many inches wide?

Long Measure.

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
5 and 1 half yards, or 16 and 1 half feet } = 1 rod (rd.).
320 rods, or 5280 feet = 1 mile (m.).

103. Examples for Written Work.

187. What is the price of 18 yards of rubber hose at 16¢ a foot?

188. To find the width of his house lot, Mr. Allen applied a yard stick 57 times, and 2 feet then remained. How wide was the lot in feet?

189. What is the cost of netting for the two ends of a tennis court, each measuring 18 yards, at 75¢ per running foot?

190. How many rods does a person travel in going 12 miles? In going 300 miles?

191. If there are 1.0567 quarts of milk in a liter, how many quarts are there in 10 liters? in 100 liters?

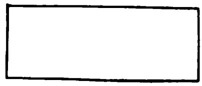
192. Mt. St. Elias is 3 miles and 2060 feet high. What is the height in feet?

MEASURES OF SURFACE.

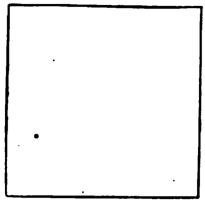
104. A flat surface, as the surface of a slate or the top of a table, is a **plane** surface. Such a surface bounded by lines is called a **plane figure.**

105. Each of the figures in the margin is bounded by how many straight lines? How many square corners has each? A plane figure bounded by four straight lines and having square corners is a **rectangle.**

RECTANGLES.



An Oblong.



A Square.

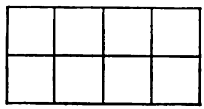
Which of the two rectangles has all its sides equal? Such a rectangle is a a **square.**

106. A square each of whose sides is 1 inch long is a square inch.

What is a figure called each of whose sides is 1 foot long? 1 yard long?

The Areas of Rectangles.

107. If a rectangle is 4 inches long and 1 inch wide, how many square inches does it contain?



The cut represents a rectangle 4 inches long and 2 inches wide, scale 1 fourth of an inch to 1 inch; how many times 4 square inches does the rectangle contain?

How many times 4 square inches will a rectangle contain that is 4 inches long and 3 inches wide? 4 inches long and 4 inches wide?

How many square feet will a rectangle contain that is 4 feet long and 3 feet wide? 5 feet long and 4 feet wide?

108. The **area** of a surface is its contents reckoned in square units.

To find the area of a rectangle, *multiply the number of units in the length by the number of like units in the width. The product is the number of square units in the area.*

Oral Exercises.

109. *a.* How many square inches are there in a rectangle 6 inches long and 4 inches wide? 8 inches long and 5 inches wide? 12 inches (1 foot) long and 12 inches (1 foot) wide? Then how many square inches are there in a square foot?

b. How many square feet are there in a rectangle 3 feet (1 yard) long and 3 feet (1 yard) wide? Then how many square feet are there in a square yard?

110. SQUARE MEASURE.

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).

c. Measure the length and width of the top of your desk, and find how many square inches its surface contains.

d. How many square inches of writing surface are on your slate? On four leaves of your writing book?

e. How many square feet of land has a strip 20 feet long and 4 feet wide? Has a court 20 feet square (20 feet long and 20 feet wide)?

f. If you know your steps are 2 feet long, how can you find the number of square feet in a rectangular lot of land?

g. Pace off a portion of the school yard, and find its area.

h. Find the number of square feet in a school yard 200 feet long and 100 feet wide. In a park 200 feet square. What is the difference between 4 square feet and 4 feet square? Make diagrams to show the difference.

111. Examples for Written Work.

193. A sitting room is 15 feet long and 14 feet wide. How many square feet does the floor contain?

194. How many square yards of oilcloth will be required to cover a floor that is 4 yards long and 3 yards wide, and how much will it cost at 62¢ per square yard?

195. How many more square feet has a grass plot 50 feet square than a plot 100 feet long and 25 feet wide?

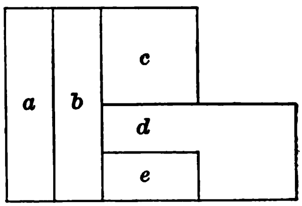
196. How many feet of fencing will be required to surround a lot 50 feet square? To surround a rectangular lot 100 feet long and 25 feet wide?

197. What must I pay for a building lot 90 feet on the street, and 120 feet deep, at $ 1.75 a square foot?

198. What would be the price of the above lot at $ 210 per running foot on the street?

199. How many square feet of painting will be required to cover a floor 5 yards square, and both sides of 5 doors, each 7 feet high and 4 feet wide?

Here is the plan of a garden drawn on a scale of a quarter of an inch to a rod, and planted as follows:



a. pease; *b.* lettuce; *c.* cabbages; *d.* potatoes; *e.* tomatoes.

200. How many square rods are there in the garden?

201. There are 6 rows of pease running lengthwise. What is the entire length of all the rows? If each row yields 2 bushels of pease, which sell at $ 1.80 per bushel, what is the value of the whole yield?

202. There are 16 rows of lettuce, 64 plants in a row. At 3¢ each, what are all worth?

203. Cabbages are set 65 heads to a square rod. At 15¢ each, what are all worth?

204. If early potatoes are worth $ 1.40 a peck, and 2 bushels 2 pecks are produced to a square rod, what is the value of the potatoes?

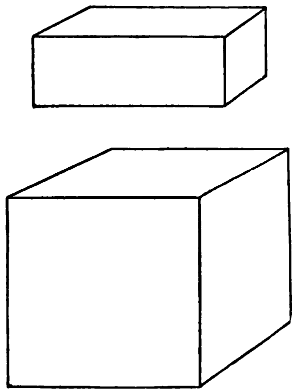
205. If tomato plants are set 9 to a square rod, and each yields a peck of tomatoes, what is the value of the tomatoes, at 8 cents a quart?

206. If for a second crop, the entire garden is sown with turnips, which yield 2.25 bushels to the square rod, what will the yield be worth at 28¢ per bushel?

MEASURES OF VOLUME.

112. What two dimensions do surfaces have? Name some things that have another dimension. A figure that has three dimensions, length, breadth, and thickness, is a **solid.** A solid that is bounded on all sides by rectangles is a **rectangular solid.**

RECTANGULAR SOLIDS.



A Cube.

Name a rectangular solid. How many rectangles does it have?

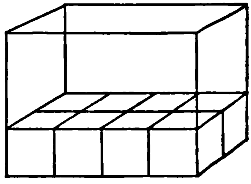
The rectangles are the **faces** of the solid and together make its **surface.** The lines formed by the meeting of the rectangles are the **edges** of the solid.

113. A solid that is bounded by six equal squares is a **cube.** If the edges are 1 inch long the solid is a **cubic inch.** What is the solid called if each edge is 1 foot long? 1 yard long?

114. The **volume** of a solid is its contents reckoned in cubic units.

The Volume of Rectangular Solids.

115. If a block of marble is 4 feet long and 2 feet wide, how many square feet does its lower surface contain?



If the block is 4 feet long, 2 feet wide, and 1 foot thick, how many cubic feet does it contain? How many times 8 cubic feet will it contain if it is 2 feet thick? if it is 3 feet thick?

The volume of any rectangular solid is found *by multiplying the number of units in the length by the number of like units in*

the breadth, and this product by the number of like units in the thickness. This is expressed, for brevity, as *multiplying together the length, breadth, and thickness.*

Oral Exercises.

116. *a.* What is the number of cubic inches in a cube 12 inches long, 12 inches wide, and 12 inches thick, or in 1 cubic foot? How many cubic inches are in a cubic foot?

b. How many cubic feet are in a cube 3 feet long, 3 feet wide, and 3 feet thick, or in 1 cubic yard?

117. CUBIC MEASURE.

1728 cubic inches	=	1 cubic foot (cu. ft.).
27 cubic feet	=	1 cubic yard (cu. yd.).
128 cubic feet	=	1 cord (cd.), used in measuring wood.

c. How many cubic feet has a rectangular block of marble that is 3 feet square at the end and 9 feet long? 4 feet square by 8 feet long?

d. How many blocks, each of whose edges is 1 inch, can be made from a cubic foot of wood, no allowance to be made for waste in sawing?

e. When a number is used three times as a factor, the product is called the **cube** of the number. Name the cubes of the numbers from 1 to 10.

118. Examples for Written Work.

207. Yellowstone Park is a rectangle 65 miles long and 55 miles wide. What is its area in square miles?

208. How many cubic feet of earth must be removed in digging a cellar 25 feet long, 16 feet wide, and 11 feet deep?

209. How many cubic inches would 1000 bricks occupy, each brick measuring 8 inches by 4 inches by 2 inches?

210. How much must be paid for 12 drain tiles at 25 cents each, 6 hours of masons' work at 40 cents an hour, and 7 hours' digging at 20 cents an hour?

211. It is 90 miles from New York to Philadelphia. How many miles will be traveled by a conductor who makes two trips daily for 313 days?

212. In one season, the heaviest peach train to Boston contained 40 carloads. At 540 baskets a load, and $ 1.25 a basket, what was the value of the peaches?

213. Mr. Gray sold hops for a farmer to the amount of $ 365, taking for selling them 10 cents on each dollar he sold them for. How much money did he take, and how much was due the farmer?

214. A invested $ 200 and B $ 590 in business. They lost 45 cents on each dollar invested. How much did each lose?

215. A capitalist sold for $ 78 each, 1000 shares of stock which cost him $ 14 a share. How much did he gain?

216. Two steamers started at the same time from the same place, and sailed in opposite directions, one at the rate of 18 miles an hour, and the other at 22 miles an hour. How far apart were they at the end of 6 hours?

217. If the above steamers had been 680 miles apart, and sailing towards each other, how far apart would they have been at the end of the same time?

218. One horse-power can raise 550 pounds 1 foot in a second; how many pounds can a 50 horse-power engine raise 1 foot in a second? how many pounds can it raise 10 feet in a second?

219. A charity collection contained 2 fifty-dollar bills, 9 tens, 11 fives, 4 twos, 27 ones, 2 half-eagles, 16 half-dollars, 26 quarters, 46 dimes, and 19 nickels. What was the value of the whole collection?

220. What must be paid for 250 crowns, Austrian money, at $ 0.203 per crown?

221. A steel company in Harrisburg advertises to make door mats of any size for 75 cents a square foot. How much must I pay for 6 mats, each 3 feet long and 2 feet wide, and one dozen, each 5 feet long and 3 feet wide?

222. The telegraphic plateau beween Newfoundland and Ireland is 1640 miles long, with an average width of 400 miles. How many square miles is its area?

223. What is the cost of 50 pounds sterling at $4.8665 per pound?

224. A square mile is 640 acres. How many acres has the District of Columbia, which contains 70 square miles?

225. Grace has $ 3.42, Charles has 5 times as much as Grace, and Delia has 8 times as much as Charles and Grace together. How much money has Delia?

226. How many square rods are there in a rectangular field 14.5 rods long and 13 rods wide?

227. Mr. Mead has 123.35 acres of land in one lot, 4 times as much in another lot, and 17.125 acres in a third lot. How many acres has he in all?

228. At 18 cents per pound, find the cost of butter in 43 firkins of 58 pounds each.

229. A man invested $ 150 in potatoes, and by selling them gained 25 cents on every dollar invested. What was his gain? What were his total receipts?

230. A bazaar was attended by 5326 persons, 135 of whom were admitted free; the remainder paid 75 cents each for tickets. What were the receipts for admission?

231. I buy each year 10 tons of stove coal and 8 tons of furnace coal. Last year stove coal was $ 5.75 a ton, and furnace coal $ 5.25; this year each kind costs $ 6.25 a ton. How much more does my coal cost this year than last?

232. A dealer has 225 cords of pine wood, 540 cords of oak, and 164 cords of maple. He sells 138 cords of pine at $ 8 a cord, 125 cords of oak at $ 9.50 a cord, and 88 cords of maple at $ 9.00 a cord. How many cords has he left? How much money does he receive for what he sells?

233. A man receives $ 40 a month as night watchman, and works besides 5 hours a day for 24 days of the month at 20 cents an hour. How much does he earn in a year?

234. Some oil fields in Ohio are said to yield 5370 barrels of oil daily. At this rate, how much will they yield in the last six months of the year?

235. At the rate of $ 15.45 on a thousand dollars, what will be the tax on $ 5000 worth of real estate, and $ 12,000 of personal property?

236. The owner of a wood lot valued at $ 5850, paid each year 3 cents on a dollar to insure himself against loss by its taking fire. What sum did he pay in 5 years?

237. At 60 cents per square yard, find the cost of flagging to lay two walks, one 220 yards long, and the other 560 yards, each 3 yards wide.

238. How many pickets, 5 to a yard, will be required to fence a rectangular field 69 yards long and 65 yards wide?

239. If in one yard of cloth there are 580 fibers of warp and 432 of filling, and each fiber of warp contains 32 strands, and each of filling 48, how many strands are there in the yard?

240. In 1850, J. M. Hosmer's ranch in California yielded 35,000 bushels of potatoes at $ 5 per bushel, 40,000 pounds of onions at $ 0.40 per pound, 80,000 pounds of pumpkins at $ 0.06 per pound. Find the total value of the yield.

241. In a Vermont marble quarry were moved upon a truck 3 blocks of marble, each 6 feet long and 4 feet square at the end. If a cubic foot of marble weighs 169.875 pounds, what was the entire weight?

What is *Multiplication?* What is the multiplicand? the multiplier? the product? What are factors? Multiply orally 45 by 6. Perform and explain an example in which the multiplier has at least two terms. Give the rule; the proof. How do you multiply by 10, 100, 1000, etc.? How do you proceed if there are zeros at the right of the expression of the multiplicand or the multiplier, or both?

Tens × units = what? Units × tens? Thousands × tens? Tens × hundreds? Ten-thousands × hundreds?

DRILL TABLE No. 3. (See Supplement, Art. 1.)

119. For supplementary practice in multiplication.

	m		*n*		*o*		*p*		*q*		*r*		*s*		*t*		*u*		*v*		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
a	2	3	2	4	2	6	5	8	4	8	6	8	7	7	9	9	9	7	9	8	*a*
b	1	2	1	4	1	3	5	5	4	2	5	2	5	3	6	9	4	1	7	8	*b*
c	2	2	2	1	3	4	5	6	4	1	5	3	7	3	7	4	8	3	8	8	*c*
d	1	5	1	6	2	5	1	7	3	5	2	9	2	5	3	7	2	4	5	9	*d*
e	2	1	2	3	3	3	5	4	4	0	9	0	6	3	3	1	7	1	8	3	*e*
f	1	3	1	6	1	5	4	5	1	7	8	4	2	8	2	7	5	4	6	7	*f*
g	2	2	2	4	3	0	6	0	5	1	8	2	7	2	9	5	8	1	8	0	*g*
h	1	4	1	8	2	8	4	9	1	9	3	4	4	7	1	9	4	6	7	6	*h*
i	2	5	2	1	2	7	6	2	4	4	9	1	6	6	7	5	8	7	9	6	*i*
j	1	1	1	8	1	9	1	8	2	6	6	4	3	9	5	6	5	8	4	8	*j*
k	2	1	2	4	3	6	6	1	5	0	8	5	7	0	9	4	9	2	9	3	*k*
l	1	2	1	3	3	2	4	3	4	3	5	7	6	5	8	9	7	9	8	6	*l*
							w				*x*				*y*						

Exercises upon the Table.

In each of the twelve lines *a, b, c,* etc., multiply

242–253. *w* by 6.

254–265. *x* by 8.

266–277. *y* by 9.

278–289. \$ *p*, *q*¢ by 7.

290–301. \$ *r*, *s*¢ by 11.

302–313. $(m + n)$ by *o*.

314–325. *y* by 12.

326–337. *p* by *q*.

338–349. *r* by *s*.

350–361. *m* by *n* by *o*.

362–373. *t* by *u* by *v*.

374–385. *w* by 34.

386–397. \$0.56 by *w*.

398–409. \$0.78 by *y*.

410–421. \$9.02 by *x*.

422–433. *x* by *w*.

434–445. *y* by *x*.

446–457. *y* by *u*.

SECTION V.

DIVISION.

120. *Illustrative Example.* Separate 48 children into equal groups of 12 children each. How many such groups will there be?

SOLUTION. — By multiplication we know that four 12's are 48, so 48 children can be separated into *four* equal groups of 12 children each.

121. Separating a number into equal parts is **dividing.**

122. The number to be separated or divided is the **dividend.** The number to divide by is the **divisor.** The result obtained by dividing is the **quotient.**

123. **Division** is the process of separating, or dividing a number into equal parts.

124. In the example, "Divide 50 by 12," the quotient is 4, with 2 left undivided. The number left after the product of the equal parts is subtracted is the **remainder.**

125. The usual sign of division is $\div$. The expression $48 \div 12 = 4$, means, and is read, "48 divided by 12 equals 4," or "12 in 48, 4 times," or "12's in 48, 4."

Division is also indicated thus, 48 : 12, and $\frac{48}{12}$.

Oral Exercises in Division.

126. I. How many

How many	**Divide**	**Divide**
a. 2's in 14? 18? 24? 22?	*g.* 30 by 6.	*m.* 55 by 11.
b. 3's in 21? 18? 27? 36?	*h.* 27 by 9.	*n.* 54 by 9.
c. 4's in 16? 24? 36? 28?	*i.* 80 by 10.	*o.* 36 by 9.
d. 5's in 25? 45? 60? 55?	*j.* 49 by 7.	*p.* 72 by 8.
e. 6's in 24? 42? 54? 18?	*k.* 64 by 8.	*q.* 88 by 11.
f. 7's in 42? 84? 63? 56?	*l.* 77 by 7.	*r.* 96 by 8.

II. Give rapidly the quotients in the following examples in the columns and in the lines.

	a b	c d	e f	g h	i j
k	18 ÷ 6	66 ÷ 33	90 ÷ 10	72 ÷ 24	64 ÷ 32
l	9 ÷ 3	22 ÷ 11	30 ÷ 5	36 ÷ 12	16 ÷ 4
m	48 ÷ 6	40 ÷ 4	66 ÷ 6	60 ÷ 15	60 ÷ 10
n	4 ÷ 2	20 ÷ 2	22 ÷ 2	30 ÷ 3	15 ÷ 5
o	32 ÷ 8	108 ÷ 9	72 ÷ 9	120 ÷ 60	72 ÷ 36
p	16 ÷ 2	12 ÷ 3	24 ÷ 3	40 ÷ 4	24 ÷ 4
q	48 ÷ 8	80 ÷ 8	100 ÷ 20	144 ÷ 12	70 ÷ 7
r	12 ÷ 4	40 ÷ 8	50 ÷ 10	72 ÷ 6	35 ÷ 7
s	80 ÷ 20	72 ÷ 12	90 ÷ 9	84 ÷ 42	28 ÷ 7
t	40 ÷ 5	36 ÷ 6	45 ÷ 9	21 ÷ 7	14 ÷ 7
u	96 ÷ 12	120 ÷ 12	88 ÷ 22	120 ÷ 60	56 ÷ 8
v	8 ÷ 4	10 ÷ 2	44 ÷ 11	30 ÷ 6	28 ÷ 4

III. Divide by 2 each number expressed in columns *a* and *b* below, naming quotients and remainders at sight:

a	b	c	d	e	f	g	h	i	j	k	l
4	16	29	39	58	71	76	90	99	118	124	142
7	15	35	46	61	70	79	88	104	111	130	135
5	21	33	42	55	66	77	91	100	121	123	140
3	25	28	49	51	73	81	87	110	117	125	139
6	20	31	43	54	64	78	89	102	112	129	136
8	22	38	47	59	69	80	94	106	115	133	145
13	17	34	40	52	63	75	97	101	114	128	137
11	23	27	45	57	68	85	93	107	119	131	141
9	18	30	41	62	65	82	95	103	113	126	138
10	19	32	44	53	67	83	92	105	116	127	144
12	24	36	48	56	72	84	96	108	120	132	143
14	26	37	50	60	74	86	98	109	122	134	146

Divide as follows the numbers in the columns under Exercise III., previous page.

m. *a* to *c* by 3.	*p.* *b* to *f* by 6.	*s.* *b* to *i* by 9.
n. *b* to *d* by 4.	*q.* *b* to *g* by 7.	*t.* *b* to *k* by 11.
o. *b* to *e* by 5.	*r.* *b* to *h* by 8.	*u.* *b* to *l* by 12.

IV. *a.* How many 3's are there in 240? in 2400?

SOLUTION. — 240 equals 24 tens. 3's in 24, 8; in 24 *tens*, 8 *tens*, or 80. *Ans.* 80 threes in 240; 800 threes in 2400.

Tell at sight, how many times

b. 2 in 80; in 120; 160; 140.	*f.* 4 in 800; 2400; 3600; 2800.
c. 3 in 90; in 210; 270; 180.	*g.* 7 in 140; 4200; 2100; 3500.
d. 6 in 120; in 480; 360; 540.	*h.* 5 in 450; 1500; 3500; 5500.
e. 8 in 240; in 160; 560; 320.	*i.* 9 in 720; 540; 6300; 8100.

127. *j.* When car fares are 5 cents each, how many rides can be had for 35 cents? for 42 cents?

SOLUTION. — As many rides can be had for 35 cents as there are 5's in 35, which is 7. *Ans.* For 35 cents, 7 rides; for 42 cents, 8 rides, and 2 cents remain.

k. At $ 6 per square for tinning a roof, how many squares can be tinned for $ 42? for $ 75?

l. The loss on the sale of some cattle was $ 7 each. If the total loss was $ 84, how many were sold?

m. How many steps, 9 inches high, are required to ascend 6 feet? 12 feet?

How many	How many
n. Pints are 48 gi.? 50 gi.?	*r.* Bushels are 60 pk.? 34 pk.?
o. Quarts are 50 pt.? 25 pt.?	*s.* Feet are 36 in.? 74 in.?
p. Gallons are 44 qt.? 50 qt.?	*t.* Yards are 30 ft.? 210 ft.?
q. Pecks are 96 qt.? 100 qt.?	*u.* Weeks are 60 d.? 72 d.?

v. How many wholes are 27 halves? 36 fourths?

w. How many wholes are 32 thirds? 55 eighths?

SHORT DIVISION.

128. *Illustrative Example.* Divide 358 by 4.

WRITTEN WORK.

Dividend.
Divisor 4)358 — 2, Remainder.
89, Quotient.

Writing the dividend and divisor as in the margin, and beginning at the left, as far as possible divide each term separately. As 4 is not contained in 3 hundred any hundreds of times, begin with 35 tens, thus: 4's in 35 tens, 8 tens, and 3 tens remain. Write 8 under the tens of the dividend, and unite the 3 tens remaining with the 8 units, making 38 units.

4's in 38 units, 9 units, and 2 remain; write 9 under the units of the dividend, and the 2 remaining at the right of the dividend, as in the margin. *Ans.* 89, Remainder 2.

In dividing, simply say, "4's in 35, 8, and 3 over; 4's in 38, 9, and 2 over"; or abbreviating still more, "4's in 35, 8; in 38, 9, and 2 remain."

To prove the work, *muitiply the divisor by the quotient, and add the remainder;* if the work is right the result will equal the dividend; thus, $89 \times 4 + 2 = 358$.

Examples for Written Work.

129. Perform and prove the following examples:

1.	**2.**	**3.**	**4.**	**5.**
2)254	2)439	2)695	2)500	2)3524
6.	**7.**	**8.**	**9.**	**10.**
3)671	3)1505	3)1070	3)3111	3)1235
11.	**12.**	**13.**	**14.**	**15.**
4)3224	4)1185	5)1135	5)3140	5)9433

Divide the numbers given below as indicated:

By 6	By 7	By 8	By 9
16. 2725	**21.** 2208	**26.** 19,291	**31.** 19,142
17. 2184	**22.** 3822	**27.** 72,982	**32.** 38,106
18. 5007	**23.** 82,564	**28.** 49,889	**33.** 54,725
19. 9991	**24.** 66,881	**29.** 34,278	**34.** 90,895
20. 2186	**25.** 58,800	**30.** 43,896	**35.** 42,008

36. $8459 \div 6$	42. $90,327 \div 7$	48. $70,001 \div 9$
37. $43,372 \div 7$	43. $83,363 \div 6$	49. $72,353 \div 8$
38. $70,699 \div 6$	44. $83,458 \div 4$	50. $62,186 \div 6$
39. $50,000 \div 7$	45. $40,314 \div 7$	51. $630,008 \div 4$
40. $70,007 \div 8$	46. $68,005 \div 8$	52. $289,873 \div 9$
41. $90,893 \div 9$	47. $70,907 \div 9$	53. $170,079 \div 9$

54. 3865 pounds of tea were put up in 5-pound packages. How many packages were there?

55. To hold 264 quarts of milk, how many 6-quart cans will be required? How many 2-gallon cans?

56. A man employed an agent to purchase a lot of cows, paying for the service $ 7 a head. If $ 1743 was paid the agent, how many cows were bought?

57. At 11 tons a carload, how many cars are needed to transport 4092 tons of coal?

58. A dealer sold a quantity of wood for $ 9 a cord, and received $ 6858. How many cords did he sell?

59. In 4824 soldiers, how many squads are there of 8 each? of 9 each?

60. The distance across the Atlantic Ocean being 3000 miles, in what time will it be crossed by a ship sailing at the rate of 4 miles an hour? By a ship sailing at the rate of 12 miles an hour?

61. In 2745 days how many weeks are there, and how many days remain?

62. How many yards are 482 feet? 5374 feet?

63. Change 6284 months to years. 5424 inches to feet.

PARTITIVE FORM OF DIVISION.

130. *Illustrative Example.* Separate 48 children into 4 equal groups; how many children are there in each group? *Ans.* 12 children.

Solution. — In this example the number of the groups, *four*, is given, and it is required to find the *size* of each group which is one

fourth part of 48 children, or 12 children. This is the **partitive form** of division.

In the example in Art. 120, "Separate 48 children into equal groups of 12 children each," the size of each group, *twelve*, is given, and it is required to find the *number* of the groups. This is the **measuring form** of division.

All examples in division are either of the **measuring** or of the **partitive** form. In the "measuring" form of division, the dividend and divisor are of the same denomination; in the "partitive" form, the dividend and quotient are of the same denomination.

131. What is one of the equal parts called, when a thing or number is divided into 2 equal parts? into 3? 4? 5? 6? 10? 20? 100? 1000?

These parts are written thus:

one half, $\frac{1}{2}$	one fourth, $\frac{1}{4}$	one sixth, $\frac{1}{6}$
one third, $\frac{1}{3}$	one fifth, $\frac{1}{5}$	one tenth, $\frac{1}{10}$, etc.

The partitive form of division is denoted thus:

$\frac{1}{2}$ of 26; $\frac{1}{3}$ of 36; $\frac{1}{100}$ of 480; $\frac{1}{1000}$ of 1876. These expressions are read "one half of 26; one third of 36," etc.

Examples for Written Work.

132. *Illustrative Example.* What is 1 twelfth of 161 miles?

WRITTEN WORK.

$$\begin{array}{r} 12)\underline{161} \text{ miles} \\ 13\frac{5}{12} \text{ miles, } \textit{Ans.} \end{array}$$

1 twelfth of 161 miles is found by dividing 161 by 12.

1 twelfth of 16 tens is 1 ten; 4 tens remain. 1 twelfth of 41 units is 3 units, and 5 units remain. If 1 twelfth of each of the 5 units is taken, 5 twelfths will be taken. This may be expressed as in the margin. The entire quotient is then $13\frac{5}{12}$. *Ans.* $13\frac{5}{12}$ miles. In doing the work, say simply, "1 twelfth of 16, 1; of 41, 3 and $\frac{5}{12}$."

Find the quotients of the following:

64. $\frac{1}{2}$ of 3007	**67.** $\frac{1}{4}$ of 168,305	**70.** $\frac{1}{6}$ of 490,607
65. $\frac{1}{3}$ of 14,684	**68.** $\frac{1}{5}$ of 651,734	**71.** $\frac{1}{6}$ of 876,543
66. $\frac{1}{4}$ of 85,470	**69.** $\frac{1}{5}$ of 199,017	**72.** $\frac{1}{7}$ of 760,863

73. $\frac{1}{7}$ of 432,760
74. $\frac{1}{7}$ of 338,607
75. $\frac{1}{8}$ of 200,788
76. $\frac{1}{8}$ of 474,324
77. $\frac{1}{9}$ of 674,830
78. $\frac{1}{9}$ of 298,979
79. $\frac{1}{2}$ of 380,268,412
80. $\frac{1}{4}$ of 926,003,971
81. $\frac{1}{8}$ of 1,156,704,794
82. $\frac{1}{5}$ of 7,123,476,543
83. $\frac{1}{6}$ of 1,740,080,666
84. $\frac{1}{8}$ of 1,952,017,550
85. $\frac{1}{7}$ of 9,970,572,641
86. $\frac{1}{9}$ of 1,746,908,460
87. $\frac{1}{8}$ of 9,588,710,957
88. $\frac{1}{9}$ of 16,009,006,314

Oral Exercises.

133. *Illustrative Example.* In 3 bins there were 27 bushels of corn. How many bushels was this to a bin?

Solution. — Since in 3 bins there were 27 bushels, in 1 bin there was $\frac{1}{3}$ of 27 bushels, which is 9 bushels. *Ans.* 9 bushels.

a. A rent was $ 84 a year. How much was this a month?

b. How many yards of single track railroad can be laid with 120 yards of rail? How many yards of double track can be laid with 800 yards?

c. 3 score equals 5 dozen. How many units is 1 score?

d. The errors made by 4 boys in spelling were severally 6, 7, 5, and 10. Find the average number of errors to a boy.

Note. — The average of two or more quantities is their sum divided by the number of quantities added. The whole number of errors is 28. $\frac{1}{4}$ of 28 is 7. *Ans.* 7 errors.

e. A mountain party of 7 had expenses as follows: refreshments $ 6, hotel $ 15, teams $ 14. What was the expense to each person?

f. Find the average earnings per week of 2 weavers, each earning $ 13, and 5 carders, each earning $ 6.

g. If 4 feet 6 inches of wire are cut into 9 equal pieces, what is the length of 1 piece?

h. If a piece of work can be done by one man in 63 hours, how long will it require 7 men to do the same work?

i. If 2 bunches of 100 laths each will lay 12 square yards, how many laths will be required to lay 1 square yard?

134. Examples for Written Work.

89. Six persons shared equally in the profits from the freight of a vessel. The net profits for one year were $ 1435. How much was each one's share?

90. There are 2394 pupils in the town of Grafton, and 7 schools, 9 classes to a school. What is the average number of pupils to a school? to a class?

91. One third of an estate of $ 85,650 falls to a widow, and the remainder is left to 7 children. How much do all the children receive? How much does each receive?

92. From November 1 to April 1 there were burned in a furnace 6 long tons of coal; in a cooking range 4 long tons. What was the average number of pounds burned per month in the furnace? in the range?

93. When a grocer's bill for 6 months is $ 196.86, what does it average a month?

94. If 8 miles of the Union Pacific railroad cost $ 64,688, what was the average cost per mile?

95. John has $ 25.20, James has 1 fifth as much, and Harry has 1 eighth as much. How much have James and Harry together?

96. The number 9296 is 9 times what number? 11 times what number?

DIVISION OF DECIMALS.

135. *Illustrative Examples.* (1) What is 1 eighth of 409.12? (2) 1 eighth of 947? (3) 1 seventh of 17.4?

(1)	(2)	(3)
WRITTEN WORK.	WRITTEN WORK.	WRITTEN WORK.
8)409.12	8)947.000	7)17.400
51.14	118.375	$2.4857\frac{1}{7}$
		or 2.4857 ...

In dividing a decimal by an integral number, ***insert the decimal point in the quotient when the decimal point in the dividend is reached.***

In example (2), the remainder, 3 units, is changed to tenths and divided, leaving a remainder which is changed to hundredths and divided, and so on.

In example (3), after the division has been carried to thousandths, there is still a remainder, which may be changed to ten-thousandths, and the division be continued; or, *to show that the division is incomplete, dots may be written in the quotient.*

In the following examples, where the division is incomplete, the answers may be carried to thousandths.

97. Divide 86.436 by 3.

98. Divide 22.328 by 4.

99. Divide 244.034 by 7.

100. Divide 89.075 by 5.

101. Divide 734.8 by 11.

102. Divide $ 1.68 by 6.

103. A quart dry measure contains 67.2 cubic inches. How many cubic inches are there in a pint? in a gill?

104. A bushel contains 2150.42 cubic inches. How many cubic inches are there in a peck?

105. When coal is $ 7.50 a ton, how much will half a ton cost? How much will 1 fourth of a ton cost?

106. Shrimps at $ 3.24 per dozen cans are how much per can?

107. Three partners share equally a profit of $ 40,450. What is each one's share?

108. A house rent of $ 1875 for 9 months is how much per month?

109. The President's salary of $ 50,000 a year is how much a month?

LONG DIVISION.

136. When the divisor is not larger than 12, as in the examples thus far given, it is customary to divide without expressing the entire operation. The process is then called **short division.**

When the divisor is larger than 12, it is convenient in dividing to express the entire operation. The process is then called **long division.**

137. *Illustrative Example.* Divide 29,515 by 72.

WRITTEN WORK.

$$\begin{array}{l} 72)29515(409\frac{67}{72} \\ \quad \underline{288} \\ \qquad 715 \\ \qquad \underline{648} \\ \qquad \;\; 67 \end{array}$$

WRITTEN WORK, SECOND FORM.

$$\begin{array}{l} 72)29515(409.93\ldots \\ \quad \underline{288} \\ \qquad 715 \\ \qquad \underline{648} \\ \qquad 670 \text{ tenths} \\ \qquad \underline{648} \\ \qquad \;\; 220 \text{ hundredths} \\ \qquad \;\; \underline{216} \\ \qquad \qquad 4 \end{array}$$

Write the dividend and divisor as for short division.

Since 72 is a larger number than 2 or than 29, first divide 295 hundreds by 72.

295 divided by 72 will give about the same quotient as 29 divided by 7,* which is 4. The first term of the quotient is then 4 hundreds, which is written at the right of the dividend. Multiplying 72 by 4 hundred, and subtracting the product, 7 hundreds remain; these 7 hundreds with the 1 ten of the dividend make 71 tens.

71 tens divided by 72 will give no tens in the quotient. Write 0 in the tens' place of the quotient, and unite with the 71 tens the 5 units of the dividend, making 715 units.

715 units divided by 72 (using 7 as a trial divisor) gives 9 units in the quotient, which write. Multiplying and subtracting as before, 67 units remain. Dividing 67 units by 72 gives $\frac{67}{72}$. The entire quotient is $409\frac{67}{72}$.

In the second form of written work, after dividing the whole number, the remainder 67 is changed to 67.0 (670 tenths). This is divided as before, and the division is thus continued as far as desirable, or until there is no remainder (Art. 135, illustrative examples 2 and 3). Here the division is stopped at hundredths, and dots are written to show that the work is incomplete.

$72 \times 2 = 144$
$72 \times 3 = 216$
$72 \times 4 = 288$
etc.

NOTE. — When the divisor is large and used many times, it saves labor to form a table of the multiples of the divisor. Thus, in performing the illustrative example above, a table may be made as in the margin. Having these multiples, the quotient figures may be seen at once.

* 7 is therefore the trial divisor.

In the following examples, answers may be given in either or in both forms shown on page 81. If the second form is used, continue the division to thousandths.

In each example below, what number will you take as a trial divisor?

110. Divide 3864 by 21.
111. Divide 6438 by 31.
112. Divide 9607 by 42.
113. Divide 4076 by 62.
114. Divide 5081 by 51.
115. Divide 1357 by 52.
116. Divide 3579 by 63.
117. Divide 2543 by 74.
118. Divide 3857 by 81.
119. Divide 5278 by 93.
120. Divide 70.96 by 24.
121. Divide 438.5 by 41.
122. Divide 87.09 by 71.
123. Divide 9.067 by 62.
124. Divide 436.2 by 53.
125. Divide 6438 by 32.
126. Divide 5.782 by 54.
127. Divide 173.9 by 82.

138. *Illustrative Example.* Divide 1508 by 197.

WRITTEN WORK.

197)1508(7 $\frac{129}{197}$, *Ans.*
1379
129

As 197 is nearly 200, 1508 divided by 197 will give about the same quotient as 1500 divided by 200, or as 15 divided by 2. Therefore make 2 a trial divisor. *Ans.* $7\frac{129}{197}$.

128. Divide 68,751 by 89.
129. Divide 30,742 by 78.
130. Divide 48,685 by 27.
131. Divide 50,871 by 192.
132. Divide 76,035 by 281.
133. Divide 30,571 by 698.
134. Divide 743,054 by 56.
135. Divide 938,601 by 29.
136. Divide 208,754 by 283.
137. Divide 1,728,564 by 579.
138. Divide 8,076,308 by 394.
139. Divide 6,207,006 by 886.
140. Divide 7,214,532 by 178.
141. Divide 3,062,707 by 481.
142. Divide 5,930,872 by 1845.
143. Divide 4,273,659 by 3951.

139. From the preceding examples may be derived the following

Rule for Division.

1. *Write the dividend; at the left draw a curved line; and at the left of this line write the divisor.*

2. *Divide the highest term or terms of the dividend by the divisor.*

3. *In long division write the result for the first term of the quotient at the right of the dividend; in short division write it beneath.*

4. *Multiply the divisor by this term.*

5. *Subtract the product thus obtained from the part of the dividend used.*

6. *Unite the next term of the dividend with the remainder for a new partial dividend; divide, multiply, and subtract as before; and so continue till all the terms of the dividend are used.**

7. *Express the division of the final remainder, should there be any, in the fractional form.*

Or, change the remainder to tenths, hundredths, thousandths, etc., and continue the division as far as desirable.

Proof.

Find the product of the quotient and divisor, and add to it the remainder, if there is one. The result should equal the dividend.

140. Division by tens, hundreds, etc.

Illustrative Example. Divide 5487 by 10; by 100; by 1000.

$$5487 \div 10 = 548.7$$
$$5487 \div 100 = 54.87$$
$$5487 \div 1000 = 5.487$$

Moving the decimal point to the left has the effect of moving the figures the opposite way (Art. 20).

Hence, to divide a number by 10, 100, 1000, etc., *move the decimal point as many places to the left as the divisor has zeros.*

144. A meter contains 39.37 inches. How many inches are there in a decimeter, which is 1 tenth of a meter?

---

* If at any time the divisor is not contained in a partial dividend, write a zero for the next figure of the quotient, and unite with the partial dividend the next term of the given dividend.

145. In 18,645 cents, how many dimes are there? how many dollars?

146. 35,255 mills are how many cents? how many dollars?

147. If 38,645 cents are to be put up in packages of 100 each, how many packages will there be, and how many cents remaining?

148. If a mileage ticket for 1000 miles sells for $25, what is the cost per mile?

149. When 100 shares of railroad stock sold for $1830, what was the price per share?

150. 5741 pounds of nails will fill how many 100-pound kegs?

151. How many centals (100 lb.) of grain are 84,320 pounds?

152. Divide 36,875 by 10, by 100, by 1000, and add the quotients.

153. Divide 48,623 by 100, by 10, by 10,000, and add the quotients.

141. *Illustrative Examples.* (1) Divide 50,876 by 4200. (2) Divide 57,688 by 1200.

NOTE. — When a divisor has zeros at the right, the process of dividing may be shortened, as in the written work below.

(1) WRITTEN WORK.

```
42|00)508|76(12 476/4200
      42
      88
      84
       476
```

(2) WRITTEN WORK.

```
12|00)576.88
      48.073...
```

(1) Since $4200 = 42 \times 100$, we first divide by 100, which is indicated by a vertical line drawn so as to cut off two figures of the dividend, and then divide the result by 42; this gives for a quotient 12, with a remainder of 476. *Ans.* $12\frac{476}{4200}$.

(2) In the second example, the division by 100 is indicated by the placing of the decimal point (Art. 140). This gives 576.88. Dividing this by 12, the answer is 48.073...

154. Divide 6842 by 200.

155. Divide 3872 by 15,000.

156. Divide 78,638 by 3200.

157. Divide 18,360 by 7000.

158. Divide 96,843 by 840.

159. Divide 100,000 by 650.

160. How many customers can be supplied by using a ton of ice (2240 lb.) if each customer receives 60 lb.? 80 lb.?

161. When 40 cubic feet of coal weigh 1 long ton, how many pounds does 1 cubic foot weigh?

162. Change 72,000 seconds to minutes. To hours.

163. At 210 pounds of salt to a barrel, how many barrels of salt were produced in the salt works of New York in the year 1892, the product, as reported, being 1,603,420,000 pounds?

142. To divide one sum of money by another.

Illustrative Example. At $ 1.25 each, how many chairs can be bought for $ 32, and how much money will remain?

WRITTEN WORK.

```
1.25)32.00(25
     250
     ---
      700
      625
      ---
       75
```

To divide one sum of money by another (measuring form of division) *both dividend and divisor must be expressed in the same denomination.* Here the divisor being cents, the dividend must be changed to cents (Art. 130). Dividing 3200 cents by 125 cents, the quotient is 25, and there are 75 cents remaining.

Ans. 25 chairs, and 75 cents remain.

164. I have $ 5 with which to purchase railroad tickets at 21 cents each. How many can I buy and how much money will remain?

165. At $ 8.25 each, how many trunks can be bought for $ 100?

166. At $ 2.50 each, how many Smyrna rugs can a dealer buy for $ 50? for $ 75?

167. A merchant has sent $ 75 to be spent for flour, at $ 6.75 per barrel. How many barrels can he buy and how much money will remain?

168. Divide $ 57 by $ 1.18.

169. Divide $ 32.50 by $ 1.65.

170. $ 18 ÷ 87¢ = ?

171. $ 170 ÷ 43¢ = ?

172. $ 48 ÷ $ 13.28 = ?

173. $ 521 ÷ $ 1.98 = ?

174. The city of Holyoke was taxed for school purposes \$73,500 on property valued at \$24,500,000. What was the tax on a dollar?

175. The city of Brockton was taxed \$76,196.04 on property valued at \$19,049,010. What was the tax on a dollar?

143. Forms of Examples in Division.

176. Divide 72,864 by 11.

177. If a dividend is 46,008 and the divisor is 24, what is the quotient?

178. $72,864 \div 6624 = ?$ $3456 \div \text{what} = 144$?

179. \$85 multiplied by what number gives a product of \$20,825?

180. What number multiplied by 59 gives a product of \$8673?

181. 5460 is 52 times what number? 100 times what?

182. 71 times what sum of money equals \$2556?

183. What is the quotient of $49,008 \div 73$?

184. Divide 684,000 by 10, by 1000, by 100, and add the quotients?

185. How many are $3,654,321 \div 39,000$?

186. The product of two factors is \$7448, one of them is 98. What is the other?

187. The product of three factors is 11,904; two of them are 24 and 31. What is the third?

188. What is 1 eighth of \$13?

189. Find 1 eleventh of \$7.062.

190. $\frac{1}{6}$ of $864.42 = \text{what number}$?

191. Divide \$8.75 by \$1.25.

192. $\$545 \div \$3.87 = ?$ $\$1468.46 \div \text{what} = \$86.38$?

193. A quotient is 16 men; the dividend 15,552 men. What was the divisor?

194. A dividend being 58,329 miles, the remainder 249 miles, and the quotient 11, what was the divisor?

144. Examples for Written Work.

195. How many sheets of paper are required for a book of 768 pages, 24 pages to a sheet?

196. How many square miles are there in a township of 5168 acres, 640 acres making 1 square mile?

197. Mr. Pratt put 1980 bushels of apples into barrels each holding 2 bushels 3 pecks. How many barrels did he use?

198. Mr. Granger received \$2548 in payment of a debt, took \$275 from the savings bank, and added to this \$177 of his earnings. With the money he bought 160 acres of land. What was the price per acre?

199. Mr. Adams bought a house for \$6200, paid \$1500 down, and the remainder in four equal payments. How much was each payment?

200. If an oil well yields 6664 barrels of oil in 8 weeks, what is its daily yield?

201. Mr. Prince has 56,168 acres of pasture land, 1 seventh as much meadow land, and 1 eighth as much wood land as pasture. How many acres has he in all?

202. Charles is to keep 1 twentieth of all he receives from the sale of peaches. How much in dollars and cents will he keep if he sells \$35 worth on Friday and \$47 worth on Saturday?

203. A broker let a store for \$240, and received for his services \$19.20. How much did he get on a dollar?

204. In 1892 there were 133,683 students in the different colleges in the United States, with 9326 teachers. Find the average number of students to a teacher.

205. The United States in 1892 maintained schools for Indians, accommodating 13,588 pupils, at a total cost of \$1,600,313. Find the cost for each pupil.

206. An operative can weave 30,000 yards of sheeting in 52 weeks. How much is that a week?

207. What is the sum of 36.84 ÷ 4 and 1 tenth of 568.72?

208. The readings of a thermometer kept by a school boy in Lowell for 4 successive days were as follows:

Monday	6 A.M.	56°	12 M.	64°	4 P.M.	70°
Tuesday	"	48°	"	69°	"	62°
Wednesday	"	45°	"	63°	"	59°
Thursday	"	53°	"	70°	"	62°

Find the averages for 6 A.M., 12 M., and 4 P.M.; also the highest average for one day.

209. Very heavy cannonading has been heard at a distance of 475,000 feet. What is the distance in miles and feet?

210. If the earth is 92,800,000 miles from the sun, and the moon at its full is 224,000 miles farther on, and if light travels at the rate of 186,350 miles a second, how many seconds is it in passing from the sun to the moon and back to the earth? (Answer to tenths.)

211. The limit of perpetual snow at Quito, on the equator, is 15,800 feet above the level of the sea. Find the height in miles, carrying the answer to hundredths.

212. The National debt of the United States in 1890 was $891,960,104, and the number of inhabitants was 62,622,250. How much was the debt to each inhabitant?

What is *Division?* What is the dividend? the divisor? the quotient? the remainder? Perform and explain an example in short division; prove the work. Perform and explain an example in long division. Give the rule; the proof. How do you divide by 10, 100, 1000, etc.? How do you divide when the expression of the divisor contains zeros at the right?

Make examples to illustrate the measuring form of division; the partitive form.

When the dividend and quotient are given, how can you find the divisor? When the divisor and quotient are given, how can you find the dividend? When the multiplier and product are given, how can you find the multiplicand? When the multiplicand and product are given, how can you find the multiplier?

Drill Table No. 4. (See Supplement, Art. 1.)

145. For supplementary practice in division.

	m		*n*		*o*		*p*		*q*		*r*		*s*		*t*		*u*		*v*		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	
a	2	3	2	4	2	6	5	8	4	8	6	8	7	7	9	9	9	7	9	8	*a*
b	1	2	1	4	1	3	5	5	4	2	5	2	5	3	6	9	4	1	7	8	*b*
c	2	2	2	1	3	4	5	6	4	1	5	3	7	3	7	4	8	3	8	8	*c*
d	1	5	1	6	2	5	1	7	3	5	2	9	2	5	3	7	2	4	5	9	*d*
e	2	1	2	3	3	3	5	4	4	0	9	0	6	3	3	1	7	1	8	3	*e*
f	1	3	1	6	1	5	4	5	1	7	8	4	2	8	2	7	5	4	6	7	*f*
g	2	2	2	4	3	0	6	0	5	1	8	2	7	2	9	5	8	1	8	0	*g*
h	1	4	1	8	2	8	4	9	1	9	3	4	4	7	1	9	4	6	7	6	*h*
i	2	5	2	1	2	7	6	2	4	4	9	1	6	6	7	5	8	7	9	6	*i*
j	1	1	1	8	1	9	1	8	2	6	6	4	3	9	5	6	5	8	4	8	*j*
k	2	1	2	4	3	6	6	1	5	0	8	5	7	0	9	4	9	2	9	3	*k*
l	1	2	1	3	3	2	4	3	4	3	5	7	6	5	8	9	7	9	8	6	*l*
							w				*x*				*y*						

Exercises upon the Table.

In each of the lines *a*, *b*, *c*, etc., divide.

213–224. *w* by 5.
225–236. *x* by 6.
237–248. *x* by 7.
249–260. *x* by 8.
261–272. *y* by 9.
273–284. *y* by 11.
285–296. *y* by 12.
297–308. *w* by 23.
309–320. *w* by 41.
321–332. *w* by 857.
333–344. *x* by 689.
345–356. *x* cents by \$ 0.17.
357–368. *x* feet by 19 feet.
369–380. \$ *p*, *q* ¢ by 4.
381–392. \$ *r*, *s* ¢ among 9 men.
393–404. \$ *r*, *s* ¢ by \$ 0.90.
405–416. \$ *p*, *q* ¢ by \$ 0.72.
417–428. $x \times t$ by *u*.

SECTION VI.

MISCELLANEOUS AND REVIEWS.

SYMBOLS OF OPERATION.

146. The signs indicating the different arithmetical processes are sometimes used in the same expression.

Thus $8 - 3 \times 2 = 2$. $12 \times 2 + 8 \div 4 = 26$.

In every such case *the multiplying and dividing indicated should be done first,* the adding and subtracting afterwards. If, however, the terms to be operated upon are joined by a parenthesis ( ) or vinculum $\overline{\quad}$, the expression joined is treated *as if it denoted a single number.*

To illustrate: $8 - 3 \times 2$ means that 3×2 is to be taken from 8; result 2. But $(8 - 3) \times 2$, or $\overline{8 - 3} \times 2$, means that 3 is to be taken from 8, and the remainder multiplied by 2; result 10.

Again, $8 + 3 \times 2 = 14$; but $(8 + 3) \times 2$, or $\overline{8 + 3} \times 2 = 22$.

So $(8 + 2) \div 5$, $\overline{8 + 2} \div 5$, or $\frac{8 + 2}{5} = 2$.

$[(4 + 8) \times 3 - 6] \div 5$ means that the sum of 4 and 8 is to be multiplied by 3, the product to be diminished by 6, and the remainder divided by 5.

$24 \div 4 \times 2 = 12$; but $24 \div (4 \times 2) = 3$.

147. In performing a series of operations indicated by signs,

First, *operate on the numbers that are joined by a parenthesis or vinculum as indicated by the signs.* Next, *multiply and divide, using the signs* $\times$ *and* $\div$ *in order as they occur.* Finally, *add and subtract as indicated by the signs* $+$ *and* $-$.

148. Exercises for Oral or Written Work.

1. $(12+8)\times 4=?$
2. $12+8\times 4=?$
3. $(20-8)\div 2=?$
4. $20-8\div 2=?$
5. $4\times 6\div 2\times 3=?$
6. $4\times 6\div(2\times 3)=?$
7. $\dfrac{5+3}{2}+\dfrac{17-5}{3}=?$
8. $(3+8)\times 4-(8-3)\div 5=?$
9. $(7+11)\div(54\div 9)=?$
10. $28+\dfrac{15\times 4-6\times 3}{7}=?$
11. $[(16-5)\times 6+8\times 5]\div 2=?$

In solving problems, the pupil will be aided by expressing the required operations by using the proper symbols, as shown in the following examples.

12. How many square yards are there in a roadway 200 feet long and 40 feet wide? $\dfrac{200\times 40}{9}=$ *Ans.*

13. From the freezing to the boiling point in the Centigrade thermometer is 100°; in Fahrenheit's it is 180°. How many degrees of Fahrenheit's equal 40 of Centigrade? $\dfrac{180\times 40}{100}=$ *Ans.*

14. How many degrees Centigrade equal 36° Fahrenheit? $\dfrac{100\times 36}{180}=$ *Ans.*

15. When 360 hours of the month of July have passed, how many days remain? $31-\dfrac{360}{24}=$ *Ans.*

16. A and B, two candidates for office, received a total of 2584 votes, A receiving 430 less than B. How many votes did each receive? $\dfrac{2584-430}{2}=$ A's.

A's + 430 = B's.

17. If 19,200 coins can be stamped by 8 presses in 1 hour, how many can be stamped by 1 press in 40 minutes? $\dfrac{19200\times 40}{8\times 60}=$ *Ans.*

18. For how much must 8 tons of hay, which cost \$17.50 a ton, sell, to gain \$3.75 per ton? $(\$17.50+\$3.75)\times 8=$ *Ans.*

19. $2500 was paid for 10 acres of land. For how much must it be sold to gain $200 per acre?

$$\$2500 + \$200 \times 10 = Ans.$$

20. A merchant sold 40 yards of cloth for $140, having bought it for $2.50 a yard. What was his gain per yard?

$$\frac{\$140}{40} - \$2.50 = Ans.$$

21. The charges of a student in a preparatory school last year were, for tuition, $75, and for incidental expenses $9. This year in college the charges were twice as much for tuition, and 5 times as much for incidentals. What was the total cost for the two years? $\$75 + \$9 + \$75 \times 2 + \$9 \times 5 = Ans.$

22. I owe John Brent $600, due 6 months hence. If I pay him 1 half now, he is willing to take off 5 cents on a dollar of the other half of the debt. What will the balance be? $\$600 \div 2 - \$0.05 \times 300 = Ans.$

23. Find the cost of storage of the following lots of flour at 3 cents on a barrel per month.

200 barrels, 2 months.	175 barrels, 3 months.
150 barrels, 1 month.	280 barrels, 2 months.

$$[(200 + 280) \times 2 + 150 + 175 \times 3] \times 3 \text{ cents} = Ans.$$

Miscellaneous Oral Examples.

149. *Illustrative Example.* *a.* If 8 yards of cotton cost 72 cents, how much will 5 yards cost?

SOLUTION. — If 8 yards cost 72 cents, 1 yard will cost 1 eighth of 72 cents, or 9 cents, and 5 yards will cost 5 times 9 cents, or 45 cents.

Ans. 45 cents.

b. If $3.50 is paid for 7 days' work, how much will be paid for 10 days' work?

c. How many hours' work @ $0.12 an hour will pay for 3 loaves of bread @ $0.08 a loaf and 4 lb. of beef at $0.09 a lb.?

d. If 9 rows of corn yield 36 bushels, how many bushels will 20 similar rows yield?

e. If an express train runs 114 miles in 3 hours, how far can it go in 4 hours?

f. If $2 will buy 1 bushel 2 pecks of pease, how many pecks will $12 buy?

g. If 6 boys can put the schoolroom in order in 10 minutes, how many boys can do the work in 12 minutes?

h. A field of rye was reaped by 12 men in 5 days. What length of time would be required for 15 men to reap the same amount?

i. If a cistern can be emptied in 20 minutes by 3 pipes of the same size, in what time can it be emptied when only 2 of the pipes are open?

j. A sea breeze was felt on the coast at half-past four. If its velocity was 4 miles an hour, at what time was it felt 9 miles inland?

k. The Israelites left Egypt in the year 1491 B.C. In what year did they enter Canaan 40 years later?

l. If it takes a dozen lemons to make lemonade for 24 persons, how many lemons must be allowed for 300 persons?

m. When a floor is 11 feet wide, what must be its length to contain 132 square feet?

NOTE. — Since the number of feet in length multiplied by the number of feet in width equals the area, to find the length, 132 must be divided by the number of feet in the width (Art. 108).

n. A grocer has 4 grades of coffee worth 29 cents, 21 cents, 40 cents, and 42 cents, which he mixes, using equal quantities of each. At what price can he afford to sell the mixture?

o. It took 12 yards of carpeting 3 quarters of a yard wide to carpet a room. How much would it take of carpeting that is 4 quarters wide?

p. A newsboy sells papers for an agent at 5 cents each, and is allowed 2 cents a copy for every paper sold. If his allowance is 24 cents, how many papers does he sell, and what does the agent receive?

q. What is the profit in buying 10 shares of stock at $93 per share and selling them at $115 a share?

150. Miscellaneous Examples for Written Work.

24. If 6 yards of plush cost $13.50, what is the cost of 13 yards?

25. Five hundred passengers were carried by special train for $65.00. How much should be paid for carrying a family of 7 persons at the same rate?

26. A crew of 12 persons had provisions for 36 days, when they took in 9 more persons from a wreck. How many days' provision had they for all?

27. How many strips of flooring 4 inches wide must be used to lay a floor which requires 144 strips 6 inches wide?

28. How many loaves of bread, 7 ounces of flour to a loaf, can be made from a barrel of flour (196 pounds)?

29. At 17 cents a yard for paper bordering, what will be the cost of bordering for a room 16 feet long and 14 feet wide?

30. What must be the length of a lot of land 80 feet wide to contain 10,000 square feet?

31. A cheese weighing 24 pounds cost $3.60. For how much a pound should it be sold to give a profit of 10 cents a pound?

32. What shall I receive for selling $7800 worth of goods if I receive $7 on each $100?

33. When taxes are $16 on $1000, how much is a person taxed who has $13,000 worth of real estate and $28,000 of personal property?

34. The bell of St. Peter's in Rome weighs 18,600 pounds; that of the Kremlin in Moscow, the largest in the world, weighs 448,000 pounds. How many times as heavy as the former is the latter?

35. 18 acres of land were bought for $4050 and sold for $5000. What was the gain per acre?

36. If the land above had been sold for $3900, what would have been the loss per acre?

37. What must be paid for insuring my house worth \$2500, at \$1.50 on each \$100?

38. It is estimated that in 1890 there were 77,875,901 people in North America, 34,440,619 in South America, and 366,571,583 in Europe. How many more people were there in Europe than in North and South America together?

39. Mr. Jaynes bought 41 acres of land at \$60 an acre, and sold it for \$3000. Did he gain or lose, and how much? How much per acre?

40. One person can make by hand 40 pairs of sleeve buttons in a day, another with a machine can make 9000 pairs in a day. How many men working by hand would be required to do a day's work of one man with a machine?

41. Dora deposited in the savings bank \$3.20 in May, twice the amount in June, \$4.75 in July, \$5.80 in August; she took out \$8 in September, deposited \$18.48 in October, \$4.78 in November, and took out \$15.50 in December. How much remained in the bank?

42. What is your ice bill for the year if you commence to take ice in May and pay \$9 from May through September, and 30 cents a hundred for the remainder of the year, using 260 pounds in October, 155 in November, and 85 in December?

43. A dealer sells a bushel of berries in quart boxes for which he receives \$0.15 a box. If he pays \$3 for his berries and 2 cents for every box he uses, what are his profits?

44. Four children amused themselves by counting watermelon seeds on their plates. They counted 25, 70, 143, and 212. Suppose these were 1 third of the seeds in the whole melon, how many seeds did the melon contain?

45. In 1880 Boston had 362,839 inhabitants; in 1890 there were 448,477 inhabitants. What was the increase in 10 years, and the average increase per year?

46. By the U.S. census of 1890, the ages of seven very old persons, were 128, 125, 123, 122, 118, 115, and 113 years respectively. What was their average age?

47. Mary is 6 years 2 months old, Charles is 11 years 3 months, Ruth 11 years 1 month, and Horace 13 years 2 months. What is the sum of their ages in months? What is their average age in months? In years and months?

48. On five successive mornings the barometer stood as follows: 29.65 inches, 29.73 inches, 29.8 inches, 30.02 inches, and 30.254 inches. What was the average height?

49. A grocer had 2 pounds of tea worth 35 cents a pound, 4 pounds worth 40 cents a pound, 1 pound worth 74 cents, and 10 pounds worth 75 cents a pound, what was the value of the entire lot? If he should mix the teas, for how much per pound could he afford to sell the mixture?

50. A grocer mixed together 3 pounds of coffee worth 34 cents a pound, 4 pounds worth 29 cents a pound, 6 pounds worth 18 cents a pound, and 2 pounds worth 32 cents a pound. What was the value of 1 pound of the mixture?

51. In the Lincoln school there were three classes as follows: first class, 27 pupils, average age 14 years; second class, 65 pupils, average age 10 years; third class, 106 pupils, average age 7 years. Find the average age of the pupils in the whole school. (Answer to tenths.)

52. A man had a stock of goods worth $10,000; these he expected to sell at a gain of $15 on every $100 worth. How much did he expect to receive for them?

53. The above goods became damaged by fire, and were sold at a loss of $40 on every $100. How much did the dealer receive for them?

54. A man sells books for a publisher at $4.50 each. For selling he is to receive $1.50 on every book sold. If he receives $24, how many books are sold? How much is due the publisher?

55. If a steamer can cross from New York to Southampton in 6 days 13 hours, a distance of 3,119 miles, what is her average speed per hour?

56. If the front wheel of a bicycle is 7.065 feet around the outer edge, and it turns 8,960 times in an hour's run, how many feet does it go? How many miles, and how many feet over?

57. How many square miles (640 acres) are there in the forests of Russia, which are estimated at 460,000,000 acres?

58. New South Wales has produced the largest piece of gold ever found. Its weight being 640 pounds and value $ 148,000, what was its value per pound?

The area and population of the New England states by the census of 1890 were as follows:

States.	Area in sq. miles.	Population.	States.	Area in sq. miles.	Population.
Maine	33,040	661,086	Massachusetts	8,315	2,238,943
New Hampshire	9,305	376,530	Rhode Island	1,250	345,506
Vermont	9,565	332,422	Connecticut	4,990	746,258

59. What is the area of all the New England states?

60. How much smaller is all New England than the state of Missouri, which has 69,415 square miles?

61. How many less square miles are there in Maine than in all the other New England states?

62. How many inhabitants were there in New England in 1890?

63. How many more inhabitants were there in Massachusetts than in Maine, Vermont, and New Hampshire?

64. In 1890 Boston had 448,477 inhabitants. How many inhabitants had Massachusetts exclusive of Boston?

65. How many more inhabitants were there in Boston than in the state of New Hampshire?

How many inhabitants to a square mile were there

66. In Maine?*
67. In New Hampshire?
68. In Vermont?
69. In Massachusetts?
70. In Rhode Island?
71. In Connecticut?

72. It is estimated that there are 1,479,486,192 people in the world, and 52,361,115 square miles of land. What is the average number of inhabitants to a square mile?

The area of the surface of the system of great lakes of North America is as follows:—

Lake Superior,	31,200 sq. m.	Lake St. Clair,	410 sq. m.
Lake Michigan,	22,450 sq. m.	Lake Erie,	9,960 sq. m.
Lake Huron,	23,800 sq. m.	Lake Ontario,	7,240 sq. m.

73. What is the sum of the areas of these lakes?

74. How much larger is Lake Ontario than Rhode Island and Connecticut together?

75. How much smaller is Massachusetts than Lake Erie?

76. Which is larger, all the great lakes together or New England, and how much?

77. The number of inhabitants in the United States in 1880 was 50,155,783; in 1890 it was 62,622,250. What was the average increase per year?

78. The area of the United States being 3,668,167 square miles, what was the number of inhabitants to a square mile in 1880?* In 1890?*

79. October 1, a gas meter registered 122,300 feet consumed. January 1, it registered 128,300 feet. What was the cost of the gas for the quarter at $2.25 per thousand feet?

80. Telegraphic rates to Canton being $2.09 per word of ten letters or less, with double rate for each word exceeding ten letters, what is the charge for the following telegram? "Bronson, Canton: River and harbor appropriation bill passed senate Wednesday by bare majority."

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* Answer to hundredths.

ACCOUNTS AND BILLS.

151. Below is a record by a painter and glazier of articles sold and services rendered by him to Mr. Edward Crane.

NEW YORK, Dec. 1, 1894.

Mr. Edward Crane

To H. H. HUNT, Painter and Glazier, *Dr.*

1894.			$	¢
Nov.	8	To Shellac 25¢, 1 hour's work 45¢		70
"	28	" Oil 9¢, Spirits 5¢, 4 lbs. Paint 67¢		81
"	"	" Setting glass	2	00
"	"	" 4 hours' work, @ 30¢	1	20
"	30	" Painting piazza roof	7	00
"	"	" Oiling floor	1	75
"	"	" Draperies for parlor	53	80

152. Such a record as the above is called an **account.**

153. The person who owes a debt is a **debtor** (Dr.). The person to whom a debt is owed is a **creditor** (Cr.). A written statement of sums due on account rendered, by the creditor to the debtor, is a **bill.** (See examples 81, 82 and 83.)

Who is debtor in the account above? Who is creditor? Which of the items are charges for work done? Which are charges for material furnished?

154. When the bill is paid, the creditor, or some one authorized by him, signs the bill, after the words "Received payment" or "Paid." The bill is thus **receipted.** (See examples 82 and 83.)

Examples for Written Work.

155. Find the cost of each article in the following bills, and their several amounts :

81.

NEW YORK, May 1, 1895.

Mr. Charles Otis

To WOOD, BARKER & CO., *Dr.*

1895.				
Mar.	17	To 28 ft. Walnut,	@ 11¢	
"	"	" 119 ft. Pine,	" 7¢	
"	23	" 88 ft. Sash Cord,	" 2¢	
Apr.	23	" 53 hours' painting,	" 35¢	
"	"	" 28 hours' work,	" 30¢	

Received payment,

82.

CHICAGO, MAY 19, 1896.

Mr. M. V. Bowen

Bought of L. D. COBB & CO.

30 lb. Coffee,	@ 28¢	
46 " Formosa Tea,	" 75¢	
5 " Cocoa,	" 52¢	
227 " Granulated Sugar,	" 6¢	
12 cans Tomatoes,	" 13¢	
10 " Pease,	" 14¢	
10 " Corn,	" 9¢	

Received payment, $

L. D. COBB & Co.,

Per J. L. C.

83.

PHILADELPHIA, April 22, 1895.

Mrs. J. H. Kimball

To M. DEWEY, *Dr.*

Mar.	20	To making Violet Silk	$16	00
"	"	To Ribbon $1.50, Lace 25¢, Silk 35¢, Canvas 25¢		
"	"	" Twist 10¢, Whalebone 55¢, Bone casing 15¢		
"	"	" Ruche for Skirt 75¢, Belting 15¢, Tulle $1.00		
"	"	" Skirt Lining 55¢, Waist Lining 80¢, Braid 18¢		
"	"	" Tape 10¢, Express 25¢		
			$	
		Cr.		
Mar.	10	By 3 yards Silk @ $1.42 $		
"	20	" Cash 15.00		
		$		

Balance due $

Received payment,

M. DEWEY.

Find the amounts due in the following examples, and make out the bills, supplying dates, etc., when wanting.

84. Louis J. Kendall furnished ice to Mr. Fred. Fox from Oct. 1, 1894 to May 1, 1895, 1750 pounds at 30¢ a hundred, and supplied him with ice from May 1 to Oct. 1, 1895, for $9. Date the bill Oct. 1.

85. Nov. 3d, 1894, Mrs. J. H. Dole bought of Smith and Felton, New York, 5 yards of velvet at $3.50, 8 yards of cashmere at 87¢, 4 yards of silesia at 25¢. Nov. 8, she bought 7 yards of flannel at 42¢, 43 yards of sheeting at 28¢, and 4 yards of cambric at 37¢. Date the bill the first of the following January.

86. Joseph Sears bought of Mr. S. H. Allen, June 4, 1895, 2 pounds of steak at 23¢, squash 25¢, lettuce 15¢; June 7, 10 pounds of veal at 17¢; June 11, 2 pounds of steak at 28¢, lettuce 10¢; June 18, 7 pounds of lamb at 18¢, beets 20¢, pease 25¢; June 28, 8 pounds fowl at 17¢, pease 50¢.

87. April 1, Mr. M. H. Humphrey repaired for Mr. David French, a sweeper for 50¢ and a hose for 15¢, using 2 clamps at 10¢ each; June 3, he sold him a kettle for 90¢ and a pail for 42¢; June 5, 4 bolts at 3¢, 8 pounds lead pipe at 7¢, and charged for 3 hours' labor at 35¢.

88. Bonney and Rogers shipped to Thomas Durgan & Co. from Cincinnati,

2 oxen, 1246 lb., 1318 lb., at 6¢;
2 steers, 922 lb., 893 lb., at 5¢;
3 hogs, 319 lb., 366 lb., 384 lb., at 4¢.

89. Martha Jones made a dress for Mrs. Mary Fitch and furnished 6 yards cambric at 7¢, braid 8¢, silk and twist 50¢, 3 bones at 32¢, 2 yards silesia at 25¢, buttons 38¢, 5 yards ribbon at 20¢; the work was valued at $12.

90. Purdy & White cleaned carpets for Mr. Seth Granger as follows: April 24, 23 yards at 3¢ and 82 yards at 4¢; April 25, 64 yards at 4¢ and a stair carpet $1.50; April 29, 28 yards at 4¢; May 2, 30 yards at 3¢.

91. Sell four different articles from a grocery at current prices and make out the bill, giving credit for 4 dozen eggs at the current price.

92. Sell five articles from a meat and vegetable market and make out the bill with credit for 2 bushels of pease at $3.00 a bushel.

TEST EXERCISES.

Sight Work.

156. In each column below commence at the bottom and add or subtract as indicated.

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
+ 8	+ 6	− 6	+ 5	− 8	− 2	− 9	− 8
− 7	+ 9	− 7	− 9	+ 7	+ 4	− 8	+ 4
+ 9	− 7	+ 4	− 5	+ 9	+ 9	+ 7	− 9
− 6	− 9	− 8	+ 9	− 7	− 3	+ 9	+ 2
+ 8	+ 5	+ 7	+ 4	− 9	+ 5	− 5	− 9
+ 8	+ 6	+ 4	− 3	+ 8	− 8	+ 8	− 5
+ 5	+ 17	+ 25	+ 32	+ 46	+ 54	+ 69	+ 98

Written Work.

157. Add the following across the page, without writing the numbers in columns:

93. $4.25, $0.62, $1.54, $106.60, $70.13, $2.17.

94. $7.63, $37.57, $0.85, $924.80, $482.74, $806.80.

95. $36.84, $74.63, $9.99, $0.88, $3689.09, $300.30.

96. $490.86, $40.53, $3648.13, $17406, $1.79, $2345.97.

158. Subtract the following without writing the numbers, but preserve the remainders. Add all the minuends and then all the subtrahends, and find the difference of their sums. Compare this difference with the sum of the remainders:

97.	$3,274.00	−	$192.56	= ?
98.	$6,368.42	−	$4,639.25	= ?
99.	$623.44	−	$563.55	= ?
100.	$4,000.00	−	$1,926.18	= ?
101.	$10,536.23	−	$7,685.99	= ?
102.	$765.31	−	$97.16	= ?
103.	$80,341.05	−	$9,087.58	= ?
104.	?	−	?	= ?

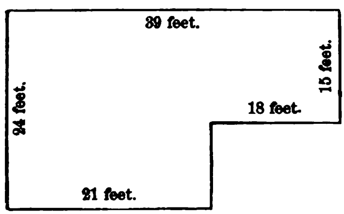
159. Supply the missing numbers in place of x in the following examples:

105.	$98765 + x = 136542.$	**109.**	$x + 193826 = 248701.$
106.	$26784 - x = 9789.$	**110.**	$x - 82394 = 340786.$
107.	$59341 \times x = 712092.$	**111.**	$x \times 2681 = 115283.$
108.	$43416 \div x = 1809.$	**112.**	$x \div 681 = 29283.$

160. Copy the following table and fill out with the sum, difference, product, and quotient of each pair of numbers, A. B., examples 113 to 117, and find their totals as indicated:

	A.	B.	Sum.	Difference.	Product.	Quotient.	Total.
113.	6552	28					
114.	6786	39					
115.	7896	47					
116.	8904	56					
117.	8415	99					

118. At 54 cents per square yard, what will be the cost of cementing the bottom of a cellar, the dimensions of which are shown in the following diagram?



119. What will be the cost of a trench wall to surround the above cellar at $1.85 per running foot, an allowance of 16 feet being added for corners?

SECTION VII.

FACTORS.

161. *Illustrative Example.* What numbers multiplied together will produce 12?

Ans. $2 \times 6 = 12$; $3 \times 4 = 12$; $1 \times 12 = 12$. 2 and 6, 3 and 4, 1 and 12, are **factors** (makers) of 12 (Art. 71).

162. A **factor** of a number is a number that may be used either as multiplicand or multiplier to produce or make the number. A factor of a number is always a **divisor** or **measure** of the number.

NOTE. — The word *factor* is here used to denote only such factors as are integral numbers.

163. A number that has no integral factors besides itself and one is a **prime number.** A number that has other integral factors besides itself and one is a **composite number.** A factor that is a prime number is a **prime factor.**

NOTE. — In naming the factors of a number, the number itself and one are not usually included.

Name a prime number; a composite number; a prime factor of 12.

Oral Exercises.

164. *a.* Which of the numbers from 1 to 20 are prime?

b. Which of the numbers from 1 to 20 are composite?

c. What are the prime factors of 6? 8? 9? 10? 12? 14?

d. What are the prime factors of 16? 18? 21? 22? 24?

e. What are the prime factors of 25? 27? 28? 30? 32?

165. When the same number is used several times as a factor of a number, the number of times it is so used is indicated by a small figure called an **exponent.**

Thus, $5^2 = 5 \times 5$. 5^2 is read "Five to the second power"; $2^5 = 2 \times 2 \times 2 \times 2 \times 2$. 2^5 is read "Two to the fifth power."

DIVISIBILITY OF NUMBERS.

166. Numbers divisible by 2 are **even** numbers; all other numbers are **odd** numbers. All prime numbers except 2 are odd.

The factors of a number may be found by division. The divisibility of numbers by some factors can be determined by the following tests:

1. A number is divisible by 2 if the units are divisible by 2.
2. A number is divisible by 3 if the sum of its digits* is divisible by 3. Thus, 285 is divisible by 3, for $2 + 8 + 5 = 15$ is divisible by 3.
3. A number is divisible by 4 if its tens and units together are divisible by 4. Thus, 6724 is divisible by 4, while 6731 is not.
4. A number is divisible by 5 if the units' figure is either 0 or 5.
5. A number is divisible by 6 if it is divisible by 2 and by 3.
6. A number is divisible by 8 if its hundreds, tens, and units together are divisible by 8. Thus, 6728 is divisible by 8, while 6724 is not.
7. A number is divisible by 9 if the sum of its digits is divisible by 9.
8. A number is divisible by 11 if the sums of its alternate digits are equal, or if their difference is divisible by 11. Thus, 1782 and 1859 are divisible by 11, while 4987 is not.
9. A number is divisible by a *composite number*, if it is divisible by each of the factors of the composite number. Thus, 3825 is divisible by 15, for it is divisible by 3 and by 5.

NOTE. — Reasons for these tests are given in the Supplement, Art. 6.

167. Exercises for Written Work.

1. Turn to page 89 and, using the tests above, write the numbers expressed in *w*, lines *a*, *b*, *c*, etc., that contain as a factor, 2; 3; 4.

---

* A digit here means the number denoted by a figure independently of its place or order.

2. Write the numbers expressed in x, lines a, b, c, etc., that contain as a factor, 5; 6; 8.

3. Write the numbers expressed in y, lines a, b, c, etc., that contain as a factor 9; 10; 11.

Find by inspection, or by trial when necessary, all the factors not greater than 12 that are contained in the numbers given below:

4. 25,785	**7.** 39,490	**10.** 90,273	**13.** 80,661
5. 38,948	**8.** 25,404	**11.** 50,809	**14.** 22,891
6. 27,645	**9.** 82,936	**12.** 39,872	**15.** 56,784

168. To find the prime factors of a number.

Illustrative Examples. (1) What are the prime factors of 735? (2) What are the prime factors of 409?

WRITTEN WORK.

(1) $3 \vert \underline{735}$	(2) 19)409(21	23)409(17
$5 \vert \underline{245}$	$\underline{38}$	$\underline{23}$
$7 \vert \underline{\ 49}$	29	179
7	$\underline{19}$	$\underline{161}$
	10	18

(1) Applying the tests (Art. 166) to the given number, we find that 2 is not a factor of 735, but that 3 is, and by dividing, we see that $735 = 3 \times 245$.

Continuing this process, we find that $245 = 5 \times 49$, and that $49 = 7 \times 7$. Therefore $735 = 3 \times 5 \times 7 \times 7$, and the prime factors are 3, 5, 7, and 7.

(2) Applying the tests (Art. 166) to the second example, we find that 409 is not divisible by 2, 3, or 5. We then try to divide by the other prime numbers in order until we reach 23, when we see that the quotient is less than the divisor. There can then be no prime factor in 409 greater than 23, for if there were, there would be another factor (the quotient) less than 23, which we should have found before reaching 23. The number 409 is, therefore, prime.

As 735 is found to equal the product of all its prime factors, so will it always be found that *a composite number equals the product of all its prime factors.*

169. From the preceding examples may be derived the following

Rule.

To separate a number into its prime factors:

1. *Divide the given number by one of its prime factors.*

2. *Divide the quotient thus obtained by one of its prime factors; and so continue dividing until a quotient is obtained that is a prime number.*

3. *This quotient and the several divisors are the prime factors sought.*

Proof.

Multiply together the prime factors thus found. The product should equal the given number.

NOTE. — If no prime factor is readily found by which to divide, try the several prime numbers in order. If no prime factor is found before the quotient becomes less than the trial divisor, the given number is prime.

Examples for Written Work.

170. Separate into prime factors the following numbers:

16. 116	**19.** 204	**22.** 342	**25.** 567
17. 164	**20.** 252	**23.** 364	**26.** 644
18. 176	**21.** 270	**24.** 363	**27.** 684

Select the prime numbers and find the prime factors of the composite numbers among the following:

28. 357	**32.** 490	**36.** 627	**40.** 908
29. 372	**33.** 497	**37.** 711	**41.** 972
30. 377	**34.** 499	**38.** 754	**42.** 1728
31. 367	**35.** 570	**39.** 760	**43.** 1830

What is a *factor* of a number? What is a composite number? a prime number? a prime factor?

What is an even number? an odd number? What numbers are divisible by 2? 3? 4? 5? 6? 8? 9? 10? 11?

How can you find the *prime factors* of a number? A composite number equals what product?

CANCELLATION.

171. *Illustrative Examples.* (1) Multiply 5 by 3 and divide the product by 3. (2) Divide the product of 5 and 6 by 2.

WRITTEN WORK.

(1)

$$\frac{5 \times \cancel{3}}{\cancel{3}} = 5.$$

(1) If any given number is multiplied by a number and the product divided by the same number, the two operations offset, or cancel, each other, and the result is the given number. Hence, both operations may be omitted. *Ans.* 5.

(2)

$$\frac{5 \times \overset{3}{\cancel{6}}}{\cancel{2}} = 15.$$

(2) $6 = 2 \times 3$; therefore 2 is a factor of both the dividend and divisor, and may be struck out or canceled, and 5 may be multiplied by 3 only. *Ans.* 15.

172. Cancellation is the process of striking out equal factors in dividend and divisor.

NOTE.—The pupil should shorten his work by canceling, whenever it is possible.

Examples for Written Work.

173. Divide:

44. $6 \times 12 \times 9$ by 3×4.

45. $8 \times 14 \times 5$ by 4×7.

46. $30 \times 5 \times 8$ by 15×4.

47. $4 \times 44 \times 2$ by 33×8.

Divide:

48. $50 \times 16 \times 3$ by 10×24.

49. $48 \times 32 \times 5$ by 12×40.

50. $52 \times 34 \times 12$ by 17×13.

51. $38 \times 56 \times 30$ by 14×19.

52. $\frac{14 \times 27 \times 84}{56 \times 21} = ?$

53. $\frac{64 \times 35 \times 18 \times 6}{63 \times 8} = ?$

54. $\frac{9 \times 8 \times 22 \times 46}{44 \times 18 \times 2} = ?$

55. $\frac{169 \times 51 \times 33}{9 \times 26 \times 11} = ?$

56. $\frac{125 \times 42 \times 38}{10 \times 31 \times 7} = ?$

57. $\frac{132 \times 16 \times 56}{84 \times 24 \times 2} = ?$

58. If a bar of iron 8 feet long weighs 36 pounds, what will a bar of iron of the same thickness, and 100 feet long, weigh?

59. If a tree 69 feet high casts a shadow 90 feet, what length of shadow will a tree 92 feet high cast at the same time of day?

60. If the work of 6 men is equal to the work of 9 boys, how many men's work will equal the work of 21 boys?

61. A yachting party has provisions for 12 persons 60 days. How long will it last 21 persons?

62. A merchant exchanged 126 pounds of sugar at 6 cents a pound for eggs at 28 cents a dozen. How many dozen did he receive?

63. How many yards of cloth 24 inches wide will be required to line six yards of cloth 28 inches wide?

64. A rectangular plat of ground 63 feet by 48 feet is turfed with sods 6 feet long and 1 foot wide. How many sods does it take?

65. 100 rolls of dimes, 20 to a roll, were given in exchange for a roll of $5 bills. How many bills were there in the roll?

66. How many shares of stock worth $175 per share will pay for 750 shares at $77 per share?

67. The freight on 19,000 pounds of coal was $7.60. What is the freight on 100 long tons at the same rate?

68. How many years will be required to pay a debt of $3,900 at the rate of $650 in 5 years?

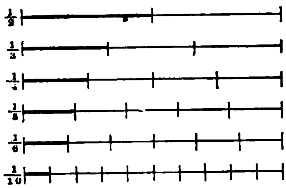
69. I sold 20 barrels of apples at $2.50 a barrel and spent the money thus obtained for cloth at $0.50 a yard. This I afterwards sold at 60 cents a yard and bought a horse with the proceeds. How much did I pay for the horse?

70. On a certain railroad, a passenger counted 25 telegraph poles passed in 3 minutes. If the poles were set 88 yards apart, how many miles per hour was the train going?

SECTION VIII.

COMMON FRACTIONS.

174. When a unit is divided into two equal parts, what is each part called? What is each part called when the unit is divided into three equal parts? into four? five? six? ten? twenty? one hundred?



175. One of the equal parts of a unit is a **fractional unit.** A collection of fractional units is a **fractional number.** By common usage fractional units and fractional numbers are all called **fractions.**

176. A **fraction** is one or more of the equal parts of a unit.

177. The unit which is divided to make a fraction is called the **unit of the fraction.**

178. The number of equal parts into which the unit of the fraction is divided, is the **denominator** (namer) of the fraction. The number of equal parts taken is the **numerator** (numberer) of the fraction. The numerator and denominator are called the **terms** of the fraction.

In two thirds of an inch what is the fractional unit? What is the unit of the fraction? The denominator? The numerator?

179. Tenths, hundredths, thousandths, etc., are **decimal fractions** (Art. 27). All other fractions are called **common fractions.**

180. To write common fractions.

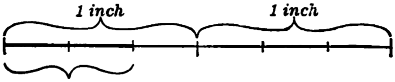
The terms of a fraction are written, the numerator above and the denominator below a line. Thus, *two thirds of an inch* is written as in the margin.

Numerator, $\dfrac{2}{3}$ inch.
Denominator,

The form of writing fractions as shown above, is the same as one of the forms used to indicate division (Art. 125).

The expression $\frac{2}{3}$ may mean *two thirds of one* or *one third of two.*

By the following illustration, the two quantities are shown to be equal.



$\frac{2}{3}$ of 1 equals $\frac{1}{3}$ of 2.

181. *a.* Name a fraction that has for its denominator four; eight; eleven. Name a fraction that has for its numerator six; twelve; nine.

b. Which is the greater part of a thing, $\frac{1}{4}$ or $\frac{1}{5}$? $\frac{1}{8}$ or $\frac{1}{10}$?

c. What is meant by the expression $\frac{5}{12}$ of a mile?

Ans. It means 5 of the 12 equal parts into which the unit 1 mile is divided, or it means 1 twelfth of 5 miles.

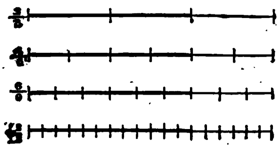
d. What is meant by the expression $\frac{5}{6}$ of a foot? $\frac{7}{10}$ of 1?

e. Show by a diagram that $\frac{3}{4}$ of 1 equals $\frac{1}{4}$ of 3.

182. To change a fraction to larger or smaller terms.

By the following illustration it will be seen that the fractions $\frac{2}{3}$, $\frac{4}{6}$, $\frac{6}{9}$, and $\frac{12}{18}$ of the same unit are equal.

In changing $\frac{2}{3}$ to $\frac{4}{6}$, both terms of the fraction $\frac{2}{3}$ are made twice as large; in changing $\frac{2}{3}$ to $\frac{6}{9}$, both terms of $\frac{2}{3}$ are made 3 times as large.



So, on the other hand, in changing $\frac{12}{18}$ to $\frac{6}{9}$, both terms of $\frac{12}{18}$ are made 1 half as large, and in changing $\frac{6}{9}$ to $\frac{2}{3}$ both terms of $\frac{6}{9}$ are made 1 third as large.

In changing $\frac{2}{3}$ to larger terms, the *number* of parts taken is increased as the *size* of the parts is diminished; in changing $\frac{12}{18}$ to smaller terms, the *number* of the parts is diminished as the *size* is increased. Hence,

183. *If both terms of a fraction are multiplied or divided by the same number, the value of the fraction will not be changed.*

184. To change a fraction to an equivalent fraction of the smallest terms, *strike out all the factors common to both terms.*

Oral Exercises.

185. Change to their smallest terms:

a. $\frac{4}{20}$; $\frac{20}{30}$; $\frac{20}{24}$; $\frac{36}{42}$; $\frac{36}{60}$; $\frac{24}{36}$; $\frac{12}{72}$; $\frac{48}{64}$; $\frac{25}{100}$.

b. $\frac{16}{22}$; $\frac{24}{28}$; $\frac{16}{40}$; $\frac{18}{48}$; $\frac{45}{60}$; $\frac{27}{30}$; $\frac{36}{72}$; $\frac{15}{75}$; $\frac{50}{100}$.

c. $\frac{8}{28}$; $\frac{28}{36}$; $\frac{20}{48}$; $\frac{18}{30}$; $\frac{35}{56}$; $\frac{30}{45}$; $\frac{2}{50}$; $\frac{50}{75}$; $\frac{75}{100}$.

d. $\frac{24}{32}$; $\frac{21}{28}$; $\frac{26}{52}$; $\frac{28}{56}$; $\frac{12}{30}$; $\frac{45}{54}$; $\frac{30}{50}$; $\frac{4}{100}$; $\frac{150}{300}$.

e. $\frac{12}{36}$; $\frac{12}{40}$; $\frac{18}{54}$; $\frac{15}{60}$; $\frac{10}{30}$; $\frac{54}{60}$; $\frac{12}{64}$; $\frac{24}{64}$; $\frac{270}{1000}$.

f. $\frac{2}{26}$; $\frac{22}{44}$; $\frac{25}{75}$; $\frac{56}{63}$; $\frac{15}{55}$; $\frac{60}{80}$; $\frac{56}{64}$; $\frac{36}{81}$; $\frac{125}{500}$.

NOTE. — When the common factors of the terms are not readily seen, the greatest common factor may be found by the method shown in the Supplement, Art. 7, and both terms may then be divided by it. This process is seldom required.

Examples for Written Work.

186. Change to equivalent fractions of smallest terms:

1. $\frac{90}{120}$.	**5.** $\frac{120}{180}$.	**9.** $\frac{225}{900}$.	**13.** $\frac{792}{880}$.
2. $\frac{48}{128}$.	**6.** $\frac{144}{162}$.	**10.** $\frac{147}{273}$.	**14.** $\frac{660}{2145}$.
3. $\frac{60}{144}$.	**7.** $\frac{75}{300}$.	**11.** $\frac{209}{638}$.	**15.** $\frac{864}{3456}$.
4. $\frac{75}{105}$.	**8.** $\frac{84}{420}$.	**12.** $\frac{288}{360}$.	**16.** $\frac{735}{1260}$.

187. To change fractions to larger terms.

Illustrative Example. Change $\frac{3}{4}$ to 12ths.

WRITTEN WORK.

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}.$$

To change 4ths to 12ths the denominator must be multiplied by 3. To preserve the value of the fraction the numerator also must be multiplied by 3 (Art. 183). *Ans.* $\frac{9}{12}$.

And, generally, to change a fraction to larger terms, ***multiply both terms by such a number as will change the denominator to the required denominator.***

NOTE. — If the number to multiply by is not readily seen, it can be found by dividing the required denominator by the denominator of the given fraction.

Oral Exercises.

188. *a.* Change $\frac{2}{3}$ to equivalent fractions having for denominators 9, 12, 24, 30, 48, 60, 120, 150.

b. Change $\frac{3}{4}$ to equivalent fractions having for denominators 16, 24, 28, 36, 40, 48, 60, 96, 100, 120.

c. Change $\frac{4}{5}$ to equivalent fractions having for denominators 15, 20, 30, 45, 55, 60, 100, 125, 200.

Change:

d. To 12ths: $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{2}{3}$.

e. To 16ths: $\frac{1}{2}$, $\frac{1}{8}$, $\frac{3}{8}$, 1, $\frac{3}{4}$.

f. To 24ths: 2, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{7}{8}$, $\frac{3}{4}$.

g. To 30ths: $\frac{2}{3}$, $\frac{2}{5}$, $\frac{5}{6}$, $\frac{5}{10}$, $\frac{2}{15}$.

h. To 36ths: $\frac{1}{6}$, $\frac{3}{4}$, $\frac{5}{12}$, $\frac{10}{9}$, $\frac{1}{6}$.

i. To 40ths: $\frac{1}{5}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{10}$, $\frac{3}{5}$.

j. To 42ds: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{4}{7}$, $\frac{1}{14}$.

k. To 48ths: $\frac{1}{4}$, $\frac{2}{3}$, $\frac{5}{8}$, $\frac{11}{12}$.

l. To 56ths: $\frac{1}{8}$, $\frac{3}{7}$, $\frac{3}{14}$, $\frac{1}{28}$.

m. To 72ds: $\frac{1}{8}$, $\frac{2}{9}$, $\frac{7}{12}$, $\frac{5}{24}$.

n. To 80ths: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{5}{8}$, $\frac{7}{16}$.

o. To 100ths: $\frac{3}{4}$, $\frac{9}{10}$, $\frac{9}{20}$, $\frac{4}{25}$.

p. To 120ths: $\frac{2}{3}$, $\frac{5}{6}$, $\frac{1}{12}$, $\frac{1}{24}$.

q. To 150ths: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{2}{25}$, $\frac{4}{50}$.

189. A **proper fraction** is a fraction that is less than an integral unit. A fractional number that equals or exceeds an integral unit, as $\frac{3}{3}$, $\frac{9}{8}$, is called an **improper fraction.**

190. A **mixed number** is a number consisting of an integer and a fraction, as $3\frac{2}{5}$.

191. To change improper fractions to integers or to mixed numbers.

Illustrative Example. Change $\frac{560}{7}$ and $\frac{750}{7}$ to integers or to mixed numbers.

WRITTEN WORK.

$$7)\underline{560} \qquad 7)\underline{750}$$
$$80 \qquad\qquad 107\tfrac{1}{7}$$

Since 7 sevenths equal a unit, in $\frac{560}{7}$ there are as many units as there are 7's in 560, or 80.

Ans. $\frac{560}{7} = 80$; $\frac{750}{7} = 107\frac{1}{7}$.

192. From this illustration may be derived the following

Rule

To change an improper fraction to an integer or to a mixed number: *Divide the numerator by the denominator.*

Oral Exercises.

193. Change to integers or to mixed numbers:

a. $\frac{45}{9}$; $\frac{54}{6}$; $\frac{56}{7}$; $\frac{63}{7}$; $\frac{81}{9}$; $\frac{144}{12}$; $\frac{121}{11}$; $\frac{140}{14}$; $\frac{100}{5}$; $\frac{75}{5}$; $\frac{96}{6}$.

b. $\frac{91}{7}$; $\frac{96}{4}$; $\frac{98}{2}$; $\frac{104}{8}$; $\frac{128}{8}$; $\frac{135}{9}$; $\frac{1000}{10}$; $\frac{75}{12}$; $\frac{87}{11}$; $\frac{125}{8}$; $\frac{100}{9}$.

Examples for Written Work.

194. Change the following to integers or mixed numbers, and express the resulting fractions in their smallest terms:

17. $\frac{144}{18}$	**22.** $\frac{400}{19}$	**27.** $\frac{615}{25}$	**32.** $\frac{967}{82}$	**37.** $\frac{914}{68}$
18. $\frac{172}{14}$	**23.** $\frac{806}{20}$	**28.** $\frac{818}{26}$	**33.** $\frac{906}{91}$	**38.** $\frac{792}{99}$
19. $\frac{1475}{78}$	**24.** $\frac{1584}{29}$	**29.** $\frac{4457}{76}$	**34.** $\frac{3968}{142}$	**39.** $\frac{9862}{187}$
20. $\frac{1062}{49}$	**25.** $\frac{2844}{47}$	**30.** $\frac{4827}{128}$	**35.** $\frac{4562}{162}$	**40.** $\frac{2848}{298}$
21. $\frac{2223}{58}$	**26.** $\frac{3792}{56}$	**31.** $\frac{4625}{186}$	**36.** $\frac{2847}{176}$	**41.** $\frac{1001}{242}$

195. To change an integer or a mixed number to an improper fraction.

Illustrative Example. Change $48\frac{3}{4}$ to fourths.

WRITTEN WORK.

$$48\tfrac{3}{4} = \tfrac{195}{4}.$$
$$\underline{4}$$
$$195$$

Since in 1 there are 4 fourths, in 48 there are 48 times 4 fourths, which with 3 fourths added are 195 fourths. *Ans.* $\frac{195}{4}$.

196. From the foregoing illustrations may be derived the following

Rule

To change an integer or a mixed number to an improper fraction: *Multiply the integer by the denominator of the fraction, and to the product add the numerator; the result will be the numerator of the required fraction.*

Oral Exercises.

197. Change the following to 7ths; 8ths; 9ths; 12ths.

8; 3; 5; 2; 4; 7; 6; 9; 11; 10.

Change to improper fractions:

a. $4\frac{1}{8}$; $3\frac{1}{7}$; $9\frac{2}{5}$; $7\frac{5}{7}$; $3\frac{8}{9}$; $4\frac{7}{12}$; $12\frac{7}{8}$; $6\frac{8}{9}$; $11\frac{5}{12}$.

b. $3\frac{1}{4}$; $5\frac{2}{7}$; $7\frac{3}{4}$; $4\frac{5}{8}$; $4\frac{5}{9}$; $3\frac{11}{12}$; $4\frac{11}{12}$; $7\frac{7}{8}$; $10\frac{10}{11}$.

c. $5\frac{2}{3}$; $8\frac{3}{4}$; $6\frac{5}{6}$; $6\frac{7}{8}$; $8\frac{7}{8}$; $7\frac{8}{9}$; $8\frac{2}{9}$; $7\frac{11}{12}$; $12\frac{9}{11}$.

d. $6\frac{3}{5}$; $4\frac{6}{7}$; $9\frac{7}{10}$; $8\frac{6}{7}$; $8\frac{5}{6}$; $9\frac{5}{9}$; $7\frac{8}{9}$; $11\frac{10}{12}$; $12\frac{8}{12}$.

Examples for Written Work.

198. Change the following to improper fractions:

42. $31\frac{1}{4}$	**46.** $28\frac{6}{9}$*	**50.** $96\frac{6}{7}$	**54.** $333\frac{1}{8}$	**58.** $384\frac{18}{20}$
43. $83\frac{1}{8}$	**47.** $68\frac{4}{7}$	**51.** $88\frac{2}{9}$	**55.** $272\frac{1}{4}$	**59.** $567\frac{7}{16}$
44. $62\frac{1}{2}$	**48.** $79\frac{9}{12}$*	**52.** $87\frac{16}{28}$*	**56.** $166\frac{4}{6}$*	**60.** $264\frac{4}{25}$
45. $37\frac{1}{2}$	**49.** $89\frac{5}{8}$	**53.** $99\frac{7}{9}$	**57.** $453\frac{4}{5}$	**61.** $897\frac{56}{700}$*

What is a fractional unit? A fractional number? What name is applied to both? Name and define the terms of a fraction. Explain the expression $\frac{4}{5}$. How do you change fractions to smaller terms? To larger terms? When is a fraction expressed in its smallest terms? How do you change improper fractions to integers or mixed numbers? How do you change integers or mixed numbers to fractions?

* First change the fractional part to smallest terms.

ADDITION OF FRACTIONS.

199. Like fractions are like parts of the same or like units.

Thus $\frac{1}{3}$ apple and $\frac{2}{3}$ apple are like fractions, while $\frac{2}{3}$ apple and $\frac{2}{3}$ pear are unlike fractions, and so are $\frac{2}{3}$ apple and $\frac{3}{4}$ apple.

Only like fractions can be added together.

a. Add $\frac{3}{7}$, $\frac{4}{7}$, and $\frac{5}{7}$.

b. Add $\frac{6}{13}$, $\frac{2}{13}$, and $\frac{4}{18}$.

c. Add $\frac{9}{25}$, $\frac{4}{25}$, and $\frac{7}{25}$.

d. Add $\frac{7}{20}$, $\frac{8}{20}$, and $\frac{5}{20}$.

e. Add $\frac{19}{75}$, $\frac{8}{75}$, and $\frac{5}{75}$.

f. Add $\frac{51}{365}$, $\frac{17}{365}$, and $\frac{5}{365}$.

200. Like fractions have the same denominator, which, because it belongs to several fractions, is called a **common denominator.**

How do you add fractions that have a common denominator?

201. To add fractions not having a common denominator.

Illustrative Example. Add $\frac{4}{9}$ and $\frac{5}{12}$.

WRITTEN WORK.

$$\frac{4}{9} = \frac{4 \times 4}{9 \times 4} = \frac{16}{36};$$

$$\frac{5}{12} = \frac{5 \times 3}{12 \times 3} = \frac{15}{36}. \quad \textit{Ans. } \frac{31}{36}.$$

Or $$\frac{4}{9} + \frac{5}{12} = \frac{16 + 15}{36} = \frac{31}{36}.$$

Ans. $\frac{31}{36}$.

To be added, these fractions must be changed to like fractions, or to *fractions having a common denominator.*

$\frac{4}{9}$ may be changed to

18ths, 27ths, **36**ths, 45ths, 54ths, 63ds, **72**ds, etc.

$\frac{5}{12}$ may be changed to

24ths, **36**ths, 48ths, 60ths, **72**ds, etc.

36 is the least multiple (Art. 71) common to 9 and 12, and may be taken for the common denominator. Multiplying both terms of $\frac{4}{9}$ by 4, and both terms of $\frac{5}{12}$ by 3, the resulting fractions are $\frac{16}{36}$ and $\frac{15}{36}$, which added equal $\frac{31}{36}$. *Ans.* $\frac{31}{36}$.

202. Examples for Oral or Written Work.

62. $\frac{3}{4}+\frac{2}{5}=?$	**65.** $\frac{1}{6}+\frac{1}{8}=?$	**68.** $\frac{1}{5}+\frac{1}{7}=?$	**71.** $\frac{1}{11}+\frac{1}{12}=?$
63. $\frac{1}{5}+\frac{1}{6}=?$	**66.** $\frac{4}{5}+\frac{5}{8}=?$	**69.** $\frac{5}{7}+\frac{1}{4}=?$	**72.** $\frac{1}{8}+\frac{1}{12}=?$
64. $\frac{1}{4}+\frac{1}{6}=?$	**67.** $\frac{3}{4}+\frac{3}{8}=?$	**70.** $\frac{2}{3}+\frac{4}{7}=?$	**73.** $\frac{3}{8}+\frac{2}{9}=?$

203. In adding fractions, any common denominator may be used, but the *least common denominator* is to be preferred. This is always *the least multiple common to all the denominators* and is called the **least common multiple** (L. C. M.).

A common multiple of the denominators must contain all the denominators, and hence all their prime factors; the least common multiple must contain only these factors, each occurring as many times as it occurs in any one of the numbers. When these factors cannot be readily seen, the method at the left of the written work below may be used for finding them, and from them obtaining the least common denominator. For another explanation, see Supplement, Art. 8.

204. *Illustrative Example.* Add $\frac{5}{8}$, $\frac{8}{21}$, and $\frac{9}{14}$.

WRITTEN WORK.

$$\begin{array}{r|rrr} 2 & 8 & 21 & 14 \\ \hline 7 & 4 & 21 & 7 \\ \hline & 4 & 3 & 1 \end{array}$$

L. C. M. $=2\times7\times4\times3=168$

$$\frac{5}{8}=\frac{5\times7\times3}{168}=\frac{105}{168};$$

$$\frac{8}{21}=\frac{8\times2\times4}{168}=\frac{64}{168};$$

$$\frac{9}{14}=\frac{9\times4\times3}{168}=\frac{108}{168};$$

$$\frac{105+64+108}{168}=\frac{277}{168}$$

$$=1\tfrac{109}{168}.\ \textit{Ans.}$$

Here by repeated divisions the factors which are common to two or more of the denominators, and which, therefore, should enter into the common denominator but once, are taken out. The product of these factors with those that are not common must be the least common denominator, which is 168.

To change $\frac{5}{8}$ to 168ths, the denominator 8 is multiplied by 7×3, so the numerator 5 must be multiplied by 7×3.

In a similar way, $\frac{8}{21}$ is found to equal $\frac{64}{168}$, and $\frac{9}{14}$ to equal $\frac{108}{168}$. Adding these, the sum is $\frac{277}{168}=1\frac{109}{168}$ *Ans.*

205. From the preceding examples may be derived the following

Rules.

I. To change fractions to equivalent fractions having the least common denominator:

1. For the common denominator, *find the least common multiple of the given denominators.*

2. For the new numerators, *multiply the numerator of each fraction by the number by which its denominator must be multiplied to produce the common denominator.*

II. To add fractions:

1. If they have a common denominator, *add their numerators* for the numerator of the answer.

2. If they have not a common denominator, *change them to equivalent fractions that have a common denominator, and then add their numerators.*

Examples for Written Work.

206. Add the following in lines and in columns:

	78.		**79.**		**80.**		**81.**
74.	$\frac{23}{24}$	+	$\frac{1}{36}$	+	$\frac{2}{3}$	+	$\frac{1}{2}$.
75.	$\frac{1}{6}$	+	$\frac{2}{9}$	+	$\frac{13}{30}$	+	$\frac{4}{5}$.
76.	$\frac{17}{48}$	+	$\frac{7}{8}$	+	$\frac{3}{4}$	+	$\frac{5}{6}$.
77.	$\frac{5}{36}$	+	$\frac{9}{16}$	+	$\frac{2}{9}$	+	$\frac{13}{24}$.

	86.		**87.**		**88.**		**89.**		**90.**
82.	$\frac{3}{4}$	+	$\frac{5}{6}$	+	$\frac{7}{8}$	+	$\frac{8}{9}$	+	$\frac{11}{12}$.
83.	$\frac{2}{3}$	+	$\frac{7}{15}$	+	$\frac{7}{10}$	+	$\frac{7}{45}$	+	$\frac{5}{18}$.
84.	$\frac{7}{18}$	+	$\frac{7}{9}$	+	$\frac{11}{16}$	+	$\frac{5}{6}$	+	$\frac{5}{24}$.
85.	$\frac{75}{144}$	+	$\frac{53}{72}$	+	$\frac{17}{48}$	+	$\frac{25}{36}$	+	$\frac{102}{108}$.

Note. — In performing the following examples, add the integers and fractions separately. (See Supplement, Art. 9.)

	95.		**96.**		**97.**
91.	$25\frac{1}{3}$	+	$67\frac{3}{5}$	+	$14\frac{7}{25}$.
92.	$43\frac{3}{4}$	+	$16\frac{5}{9}$	+	$87\frac{13}{24}$.
93.	$75\frac{3}{5}$	+	$9\frac{7}{10}$	+	$6\frac{9}{40}$.
94.	$40\frac{5}{9}$	+	$77\frac{10}{27}$	+	$98\frac{13}{15}$.

	102.		**103.**		**104.**
98.	$25\frac{4}{7}$	+	$110\frac{1}{6}$	+	$84\frac{3}{4}$.
99.	$40\frac{25}{28}$	+	$17\frac{7}{12}$	+	$13\frac{7}{8}$.
100.	$12\frac{7}{11}$	+	$25\frac{41}{44}$	+	$91\frac{11}{12}$.
101.	$65\frac{21}{77}$	+	$84\frac{9}{14}$	+	$55\frac{5}{22}$.

105. Out of a barrel of vinegar were drawn $1\frac{1}{3}$ gallons, $3\frac{5}{8}$ gallons, and $2\frac{1}{4}$ gallons, after which $12\frac{1}{2}$ gallons remained. How many gallons did the barrel contain at first?

106. How much silk is there in 4 remnants measuring $12\frac{5}{8}$ yards, $15\frac{5}{16}$ yards, $11\frac{1}{8}$ yards, and $9\frac{1}{4}$ yards?

107. How much iron railing is needed to fence a court, whose sides are $4\frac{1}{11}$ rods, $3\frac{5}{22}$ rods, $2\frac{1}{2}$ rods, and $7\frac{3}{4}$ rods?

108. John weighs $92\frac{1}{2}$ pounds, James $65\frac{3}{16}$, Charles $85\frac{5}{8}$, and Richard $78\frac{3}{4}$ pounds. What is the sum of these weights?

109. From a piece of ribbon measuring 10 yards $3\frac{5}{8}$ yards, $2\frac{1}{2}$ yards, and $3\frac{3}{8}$ yards were cut. How much remained?

SUBTRACTION OF FRACTIONS.

Oral Exercises.

207. *Illustrative Example.* From $\frac{7}{8}$ of a yard of velvet $\frac{4}{8}$ of a yard was taken. What part remained? *Ans.* $\frac{3}{8}$.

Find the remainders in the following examples:

a. $\frac{4}{5}-\frac{3}{5}$.	*c.* $\frac{13}{14}-\frac{7}{14}$.	*e.* $3\frac{7}{8}-1\frac{3}{8}$.	*g.* $8-\frac{3}{4}$.
b. $\frac{9}{10}-\frac{3}{10}$.	*d.* $\frac{74}{100}-\frac{66}{100}$.	*f.* $1\frac{5}{8}-\frac{2}{8}$.	*h.* $2\frac{2}{4}-\frac{1}{4}$.

i. $5\frac{2}{5}-\frac{4}{5}$.	*m.* $3\frac{5}{7}-2\frac{6}{7}$.	*q.* $21\frac{3}{20}-9\frac{7}{20}$.
j. $2\frac{3}{7}-\frac{4}{7}$.	*n.* $2\frac{3}{10}-1\frac{7}{10}$.	*r.* $6\frac{8}{21}-5\frac{11}{21}$.
k. $10\frac{5}{9}-\frac{7}{9}$.	*o.* $7\frac{4}{15}-3\frac{8}{15}$.	*s.* $15\frac{7}{30}-2\frac{11}{30}$.
l. $8\frac{6}{8}-\frac{3}{8}$.	*p.* $5\frac{5}{16}-2\frac{4}{16}$.	*t.* $8\frac{4}{25}-3\frac{8}{25}$.

When the minuend and subtrahend are like fractions, how do you subtract? In mixed numbers when the fraction in the subtrahend is larger than the fraction in the minuend, how do you proceed?

208. *Illustrative Example.* If Mary had $\frac{3}{4}$ of a yard of satin and has used $\frac{2}{3}$ of a yard, how much has she left?

WRITTEN WORK.

$$\frac{3}{4}-\frac{2}{3}=\frac{9-8}{12}=\frac{1}{12}.$$

That the subtraction may be performed, these fractions must be changed to equivalent fractions having a common denominator. The least common denominator is 12. $\frac{3}{4}=\frac{9}{12}$, and $\frac{2}{3}=\frac{8}{12}$; $\frac{9}{12}-\frac{8}{12}=\frac{1}{12}$. *Ans.* $\frac{1}{12}$ yard.

Subtract the following in lines and in columns.

	c.	*d.*		*g.*	*h.*		*k.*	*l.*
a.	$\frac{1}{2}$ −	$\frac{1}{3}$.	*e.*	$\frac{1}{3}$ −	$\frac{1}{4}$.	*i.*	$\frac{1}{3}$ −	$\frac{2}{11}$.
b.	$\frac{1}{4}$ −	$\frac{1}{5}$.	*f.*	$\frac{1}{6}$ −	$\frac{1}{7}$.	*j.*	$\frac{2}{9}$ −	$\frac{1}{12}$.

m. To make 100, what must be added to each of the following numbers? $6\frac{1}{4}$; $12\frac{1}{2}$; $16\frac{2}{3}$; $18\frac{3}{4}$; $33\frac{1}{3}$; $37\frac{1}{2}$; $62\frac{1}{2}$; $66\frac{2}{3}$.

209. From the previous illustrations may be derived the following

Rule.

To subtract one fraction from another:

1. If they have a common denominator, *find the difference of their numerators for the numerator of the answer.*

2. If they have not a common denominator, *change them to equivalent fractions which have a common denominator, and then find the difference of their numerators.*

210. Examples for Written Work.

110. From $10\frac{1}{6}$ take $4\frac{1}{2}$; $3\frac{1}{4}$; $5\frac{5}{9}$; $6\frac{3}{7}$; $7\frac{7}{8}$; $9\frac{7}{12}$.

111.	$\frac{7}{8}-\frac{5}{12}$.	**115.**	$18\frac{7}{10}-11\frac{4}{15}$.	**119.**	$350\frac{5}{9}-2\frac{91}{126}$.
112.	$\frac{4}{5}-\frac{2}{15}$.	**116.**	$5\frac{5}{16}-3\frac{21}{40}$.	**120.**	$187\frac{111}{144}-1\frac{201}{240}$.
113.	$\frac{8}{9}-\frac{7}{12}$.	**117.**	$187\frac{1}{16}-8\frac{4}{9}$.	**121.**	$960\frac{34}{35}-3\frac{8}{63}$.
114.	$\frac{3}{4}-\frac{9}{15}$.	**118.**	$101\frac{15}{36}-7\frac{53}{72}$.	**122.**	$401\frac{15}{42}-1\frac{40}{54}$.

211. Examples in Addition and Subtraction.

123. Which is larger and how much, $\frac{5}{6}$ or $\frac{3}{4}$?

124. The thermometer this morning indicates $17\frac{1}{2}$ degrees above zero; yesterday morning it stood $8\frac{2}{3}$ degrees below. How much higher is the thermometer this morning?

125. Jane has $12\frac{5}{8}$ yards of silk. How much more must she get to complete a dress that requires $15\frac{1}{2}$ yards?

126. A pole is standing so that $\frac{2}{5}$ of it is in the water, $\frac{1}{7}$ in the mud, and the rest in air. What part is in the air?

127. The average age of the pupils in Arthur's school is $12\frac{97}{120}$ years; Arthur is $10\frac{7}{12}$ years old. How much is his age below the average?

128. If $69\frac{1}{8}$ yards of cloth shrank $1\frac{1}{7}$ yards in bleaching, what was the length of the cloth after bleaching?

129. If you lose $\$\frac{1}{8}$ a bushel by selling wheat at $\$1\frac{2}{5}$ a bushel, how much did your wheat cost you a bushel?

130. Mr. Gane bought flour at $\$5\frac{9}{10}$ a barrel and gained in selling it $\$\frac{5}{12}$ a barrel. For how much a barrel did he sell it?

131. Albert has $\$1\frac{7}{10}$. How much more must he get to buy a cap worth $\$2\frac{3}{8}$ and a book worth $\$\frac{3}{4}$?

132. A man willed $\frac{1}{12}$ of his property to his mother, $\frac{1}{3}$ to an orphan asylum, $\frac{1}{40}$ to the public library, and the remainder to a college. What part of his money did he will to the college?

133. From what number must you take $19\frac{4}{7}$ to leave $8\frac{5}{9}$?

134. What number must you take from $50\frac{2}{3}$ to leave $29\frac{6}{7}$?

135. What number must you add to $19\frac{5}{18}$ to equal $33\frac{1}{8}$? To equal $45\frac{2}{5}$?

136. From $35\frac{2}{3}$ take $5\frac{1}{8} + 2\frac{8}{16} + 4\frac{1}{4} + 9\frac{1}{6}$.

137. $5\frac{1}{6}$ pounds of powder were made by mixing saltpeter with $\frac{28}{32}$ of a pound of charcoal and an equal quantity of sulphur. How many pounds of saltpeter were used?

138. $\frac{3}{4}+\frac{2}{3}-\frac{5}{6}+\frac{6}{7}=?$	**143.** $16-4\frac{1}{2}+3\frac{5}{8}=?$
139. $\frac{4}{9}-\frac{2}{3}+\frac{7}{8}-\frac{3}{12}=?$	**144.** $16-4\frac{1}{2}-3\frac{5}{8}=?$
140. $\frac{4}{9}+\frac{2}{5}-(\frac{7}{8}-\frac{3}{12})=?$	**145.** $16-(4\frac{1}{2}-3\frac{5}{8})=?$
141. $\frac{8}{11}-\frac{3}{121}+\frac{4}{55}-\frac{1}{5}=?$	**146.** $16-(4\frac{1}{2}+3\frac{5}{8})=?$
142. $\frac{5}{6}-\frac{3}{16}-\frac{8}{15}+2\frac{1}{10}=?$	**147.** $54-(22\frac{1}{3}-18\frac{1}{2})=?$

What are *like fractions?* How do you prepare unlike fractions for adding and subtracting. Select three fractions of different denominators; add and explain. Give a rule for addition of fractions. How do you add mixed numbers? How do you subtract fractions?

MULTIPLICATION OF FRACTIONS.

212. To multiply a fraction by an integer.

Illustrative Examples. (1) How many are 5 times $\frac{7}{12}$? (2) 6 times $\frac{7}{12}$?

WRITTEN WORK.

(1) $\frac{7 \times 5}{12} = \frac{35}{12} = 2\frac{11}{12}$.

(2) $\frac{7 \times \cancel{6}}{\underset{2}{\cancel{12}}} = \frac{7}{2} = 3\frac{1}{2}$.

(1) 5 times $\frac{7}{12}$ are $\frac{35}{12} = 2\frac{11}{12}$. Ans. $2\frac{11}{12}$. Here the numerator, 7, is multiplied by 5.

(2) In multiplying $\frac{7}{12}$ by 6, the multiplication is expressed as in the previous case, and the 6 is canceled. This has the effect of dividing the denominator by 6. *Ans.* $3\frac{1}{2}$.

Hence to multiply a fraction by an integer: *Multiply the numerator or divide the denominator by the integer.*

In multiplying a fraction by an integer, when should you divide the denominator? Why?

Oral Exercises.

213. Expressing the results in their smallest terms,

a. Multiply $\frac{3}{8}$ by 2; 5; 8; 9.

b. Multiply $\frac{3}{4}$ by 3; 6; 7; 8; 11.

c. Multiply $\frac{5}{24}$ by 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12.

d. How many pounds of wool are there in 9 yards of cloth if there are $\frac{5}{8}$ of a pound to a yard?

Find the cost of:

e. 8 dozen eggs at \$ $\frac{1}{8}$ a dozen. At \$ $\frac{3}{8}$. At \$ $\frac{1}{4}$.

f. 6 bushels of potatoes at \$ $\frac{3}{10}$ a bushel. At \$ $\frac{5}{6}$.

g. 6 half-barrels of flour at \$ $2\frac{1}{4}$ apiece.

NOTE. — Multiply the integer and the fraction separately.

h. A tailor uses $5\frac{7}{8}$ yards of cloth for a suit of clothes. How much does he require for a dozen similar suits?

i. If a quart of kerosene will supply 5 lamps $2\frac{7}{12}$ hours, how long will it supply 1 lamp?

j. If \$ 1 will pay for $7\frac{3}{4}$ yards of ribbon, for how many yards will \$ 8 pay? For how many will \$ 11 pay?

Examples for Written Work.

214. *Illustrative Example.* What is the width of 7 house lots, each $93\frac{3}{4}$ feet wide?

WRITTEN WORK.

$$\begin{array}{r} 93\frac{3}{4} \\ 7 \\ \hline 5\frac{1}{4} \\ 651 \\ \hline 656\frac{1}{4} \end{array}$$

Multiplying the fraction and the integer separately, and adding the results, their sum is $656\frac{1}{4}$. *Ans.* $656\frac{1}{4}$ feet.

148. If 13 men can do a piece of work in $8\frac{7}{8}$ days, how long will it take 1 man to do it?

149. When an electric car goes at the rate of a mile in $7\frac{9}{20}$ minutes, in how many minutes will it go 9 miles? 20 miles?

150. By using 3 teams, a contractor can cart a lot of gravel in $14\frac{2}{3}$ days. How long will it take him if he uses but 1 team?

151. I own $21\frac{17}{33}$ acres of land which is 1 fifth of an undivided lot. How many acres are there in the lot?

152. There are $57\frac{3}{4}$ cubic inches in a quart, liquid measure. How many cubic inches are there in a gallon?

153. If 17 men can shear a lot of sheep in $9\frac{9}{20}$ days, in what time can 1 man shear the lot?

154. How many square inches are there in the surface of a desk $28\frac{3}{8}$ inches long and 15 inches wide? In a board $42\frac{5}{6}$ inches long and 17 inches wide?

155. What must be paid for 2 shares of Canadian Pacific stock at $\$63\frac{5}{8}$ a share, and 5 shares Fitchburg railroad stock at $\$69\frac{1}{2}$ a share?

156. Multiply $9\frac{7}{16}$ by 8.

157. Multiply $4\frac{9}{12}$ by 9.

158. Multiply $19\frac{2}{11}$ by 11.

159. Multiply $21\frac{18}{63}$ by 5.

160. $68\frac{9}{89} \times 13 = ?$

161. $95\frac{5}{42} \times 18 = ?$

162. $118\frac{5}{11} \times 15 = ?$

163. $324\frac{6}{15} \times 24 = ?$

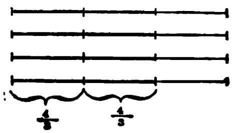
215. To multiply an integer or a fraction by a fraction.

Illustrative Examples. (1) Find $\frac{2}{3}$ of 4; (2) $\frac{2}{3}$ of $\frac{4}{5}$.

WRITTEN WORK. ILLUSTRATION.

(1)

$$\frac{2}{3}\text{ of }4=\frac{4\times 2}{3}=\frac{8}{3}=2\tfrac{2}{3}.$$

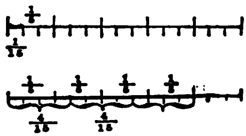


$\frac{2}{3}$ of 4 = $\frac{8}{3}$ = $2\frac{2}{3}$.

(2)

$$\frac{2}{3}\text{ of }\frac{4}{5}=\frac{4\times 2}{5\times 3}=\frac{8}{15}.$$

ILLUSTRATION.



$\frac{2}{3}$ of $\frac{4}{5}$ = $\frac{8}{15}$.

(1) $\frac{1}{3}$ of 4 = $\frac{4}{3}$ (Art. 180); hence $\frac{2}{3}$ of 4 = 2 times $\frac{4}{3}$, or $\frac{8}{3}$, = $2\frac{2}{3}$. *Ans.* $2\frac{2}{3}$.

(2) $\frac{1}{3}$ of $\frac{1}{5}$ is one of the three equal parts into which $\frac{1}{5}$ is divided. Since the entire unit, $\frac{5}{5}$, will contain 5 times or 15 such parts, one of the parts will be $\frac{1}{15}$ of the unit. Hence $\frac{1}{3}$ of $\frac{1}{5}$ = $\frac{1}{15}$, $\frac{1}{3}$ of $\frac{4}{5}$ = $\frac{4}{15}$, and $\frac{2}{3}$ of $\frac{4}{5}$ = $\frac{8}{15}$. *Ans.* $\frac{8}{15}$.

Finding the fractional part of a number is called **multiplying by a fraction.**

In finding $\frac{2}{3}$ of 4, by what was the integer 4 multiplied? By what was the result divided? In finding $\frac{2}{3}$ of $\frac{4}{5}$, how was the new numerator obtained? The new denominator?

216. Oral Exercises.

What is

a. $\frac{2}{3}$ of 11? *b.* $\frac{7}{9}$ of 30 years? *c.* $\frac{2}{9}$ of $\frac{3}{4}$ of a dozen?

Illustrative Example. Multiply 9 by $\frac{4}{5}$.

SOLUTION. — To multiply 9 by $\frac{4}{5}$ is to take $\frac{4}{5}$ of 9, etc.

d. Multiply 6 by $\frac{4}{5}$; 8 by $\frac{5}{6}$; 9 by $\frac{3}{4}$; 12 by $\frac{1}{3}$; 20 by $\frac{2}{13}$.

Name at sight the products of the following:

e. $\frac{1}{2}\times\frac{2}{3}$. *h.* $\frac{4}{5}\times\frac{1}{2}$. *k.* $\frac{2}{7}\times\frac{2}{7}$. *n.* $\frac{1}{10}\times\frac{4}{5}$. *q.* $\frac{7}{20}\times\frac{4}{5}$.

f. $\frac{1}{3}\times\frac{3}{4}$. *i.* $\frac{3}{5}\times\frac{3}{4}$. *l.* $\frac{4}{7}\times\frac{3}{5}$. *o.* $\frac{7}{10}\times\frac{1}{2}$. *r.* $\frac{3}{4}\times\frac{4}{15}$.

g. $\frac{2}{3}\times\frac{3}{4}$. *j.* $\frac{5}{6}\times\frac{4}{5}$. *m.* $\frac{2}{9}\times\frac{3}{4}$. *p.* $\frac{5}{11}\times\frac{1}{10}$. *s.* $\frac{3}{25}\times\frac{2}{3}$.

217. From the previous illustrations may be derived the following

Rules.

1. To multiply an integer by a fraction, or a fraction by an integer: *Multiply together the integer and the numerator of the fraction for the numerator of the product, and take the denominator of the fraction for the denominator of the product.*

2. To multiply a fraction by a fraction: *Multiply the numerators together for the numerator of the product, and multiply the denominators together for the denominator of the product.*

218. *Illustrative Example.* Multiply 87 by $2\frac{6}{7}$.

WRITTEN WORK.

$$\begin{array}{r} 87 \\ \underline{2\frac{6}{7}} \\ 7)\underline{522} \\ 74\frac{4}{7} \\ \underline{174} \\ 248\frac{4}{7} \end{array}$$

Multiplying by the fraction and integer separately, and adding the products as in the margin, the result is $248\frac{4}{7}$. *Ans.* $248\frac{4}{7}$.

219. Examples for Written Work.

164. $791 \times 11\frac{9}{10}$.	**166.** $487 \times 2\frac{11}{25}$.	**168.** $9012 \times 4\frac{8}{11}$.
165. $938 \times 1\frac{9}{20}$.	**167.** $804 \times 8\frac{7}{12}$.	**169.** $3456 \times 6\frac{10}{27}$.

Illustrative Example. Find $\frac{3}{4}$ of $2\frac{1}{3}$.

WRITTEN WORK.

$$\frac{7 \times \cancel{3}}{\cancel{3} \times 4} = \frac{7}{4} = 1\frac{3}{4}. \textit{ Ans.}$$

In multiplying together fractions and mixed numbers, *first change the mixed numbers to improper fractions.*

Multiply the following in lines and in columns as indicated:

	172.	**173.**		**176.**	**177.**		**180.**	**181.**		**184.**	**185.**
170.	$\frac{2}{3}$ of	$2\frac{1}{4}$.	**174.**	$\frac{5}{18}$ of	$8\frac{2}{3}$.	**178.**	$\frac{5}{9}$ of	$14\frac{2}{5}$.	**182.**	$2\frac{1}{4} \times$	$7\frac{1}{5}$.
171.	$\frac{2}{5}$ of	$11\frac{1}{4}$.	**175.**	$\frac{4}{9}$ of	$7\frac{5}{7}$.	**179.**	$\frac{5}{7}$ of	$4\frac{1}{5}$.	**183.**	$1\frac{5}{7} \times$	$12\frac{5}{6}$.

	188.	189.		192.	193.		196.	197.
186.	$\frac{3}{8}$ of	$62\frac{1}{2}$.	**190.**	$26\frac{6}{7}$ ×	$23\frac{5}{8}$.	**194.**	$87\frac{1}{2}$ ×	9.
187.	$15\frac{3}{8}$ ×	$\frac{2}{7}$.	**191.**	$34\frac{2}{3}$ ×	15.	**195.**	$\frac{14}{75}$ ×	$18\frac{3}{4}$.

220. *Illustrative Example.* What part of an estate is $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of it?

WRITTEN WORK.

$$\frac{\cancel{5} \times \cancel{4} \times \cancel{3}}{\underset{2}{\cancel{6}} \times \cancel{5} \times \cancel{4}} = \frac{1}{2}.$$

NOTE. — Such expressions as $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$, $\frac{2}{5}$ of $\frac{3}{4}$, etc., are called **compound fractions**, but they merely indicate the multiplication of fractions, or the finding of a part of a part.

198. $\frac{1}{2}$ of $\frac{1}{8}$ of \$ $2\frac{2}{5}$ = ?

199. $\frac{2}{5}$ of $\frac{2}{3}$ of $3\frac{1}{8}$ months = ?

200. $\frac{3}{2}$ of $\frac{1}{4}$ of $1\frac{8}{9}$ miles = ?

201. $\frac{1}{9}$ of $\frac{2}{3}$ of $1\frac{5}{7}$ yards = ?

Multiply the following in lines and in columns:

	205.	206.	207.		211.	212.	213.		217.	218.	219.
202.	$\frac{2}{3}$ ×	$\frac{3}{5}$ ×	$\frac{4}{5}$.	**208.**	$\frac{1}{2}$ ×	$\frac{2}{3}$ ×	$\frac{4}{5}$.	**214.**	$\frac{5}{18}$ ×	$\frac{6}{9}$ ×	$18\frac{2}{5}$.
203.	$\frac{3}{4}$ ×	$\frac{5}{6}$ ×	$\frac{6}{7}$.	**209.**	$\frac{3}{8}$ ×	$\frac{6}{7}$ ×	$\frac{14}{15}$.	**215.**	$\frac{3}{14}$ ×	$3\frac{1}{2}$ ×	8.
204.	$5\frac{1}{2}$ ×	$\frac{3}{11}$ ×	$\frac{5}{12}$.	**210.**	$\frac{7}{12}$ ×	$\frac{4}{9}$ ×	$2\frac{2}{5}$.	**216.**	$4\frac{3}{8}$ ×	$\frac{6}{7}$ ×	$18\frac{3}{10}$.

Oral Exercises.

221. *a.* If 14 yards of cloth will make a dress, and $\frac{5}{8}$ as much a wrapper, how much will make the wrapper?

b. How many acres remain of a 7-acre lot of land after $\frac{3}{8}$ of it is sold?

c. Of a journey of 56 miles, how many miles remain after $\frac{2}{9}$ of it is traveled.

d. A boy bought a flute for 80 cents, and sold it for $\frac{4}{5}$ of its cost. How much did he lose?

e. He bought a boat for \$24, and sold it for $\frac{9}{8}$ of its cost. How much did he gain?

f. Mining stock which cost \$100 a share has declined to $\frac{7}{12}$ of its cost. What is it now worth?

g. A book, the retail price of which was \$1, was sold at wholesale for $\frac{4}{5}$ of the retail price with $\frac{1}{10}$ off from that, for cash. For how much was it sold?

222. Examples for Written Work.

220. A house costing \$5260 was mortgaged for $\frac{7}{8}$ of its cost. For how much was it mortgaged?

221. If $\frac{7}{100}$ of \$475 is paid for its use for a year, how much is paid?

222. At a mark-down sale, a suit formerly marked \$37.50 was sold for $\frac{5}{6}$ of this price. How much did it bring?

223. Of 150 acres of land, $\frac{3}{4}$ was sold to one man, and $\frac{2}{9}$ to another. How many acres were sold?

224. Find the cost of $9\frac{1}{2}$ yards of cloth at 29 cents a yard.

225. A quotient is $15\frac{7}{18}$ and a divisor 117. What is the dividend?

226. Find the cost of $18\frac{3}{4}$ feet of boards at 23 cents a foot?

227. How many 1-ounce bags will be required to hold $32\frac{3}{4}$ pounds of flower seeds?

228. A store worth \$9550 was insured for $\frac{3}{4}$ of its value. How much was insured? What was the cost of insuring it at $\frac{2}{5}$ of a cent on every dollar insured?

229. A tax of $\frac{36}{100}$ of the value of a lot of glassware imported from France, and valued at \$250, was paid at the Custom House. How much was paid?

230. There are $272\frac{1}{4}$ square feet in a square rod. How many square feet are there in $9\frac{5}{14}$ square rods?

231. What is the value of $\frac{7}{8}$ of a square rod of land at $8\frac{1}{3}$ cents a square foot?

232. The wheel of a wagon, $12\frac{5}{12}$ feet around, revolves $28\frac{1}{2}$ times in going to the railroad station. How many feet is it to the station?

233. A piece of ground $16\frac{2}{3}$ yards long and $4\frac{1}{2}$ yards wide is to be sodded. What will the sodding cost at $1\frac{1}{2}$ cents a square foot?

234. How many square feet are there in a lot $87\frac{1}{4}$ feet long and $43\frac{1}{3}$ feet wide, and what is it worth at \$$\frac{3}{10}$ a square foot?

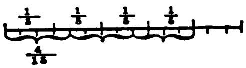
DIVISION OF FRACTIONS.

223. To divide a fraction by an integer.

Illustrative Examples. (1) Divide $\frac{4}{5}$ of a melon between 2 boys; (2) among 3 boys.

(1) Each of 2 boys will have 1 half of $\frac{4}{5}$, or $\frac{2}{5}$ of a melon. *Ans.* $\frac{2}{5}$ of a melon.

ILLUSTRATION.



$\frac{1}{3}$ of $\frac{4}{5} = \frac{4}{15}$.

(2) Each of 3 boys will have 1 third of $\frac{4}{5}$ of a melon. 1 third of $\frac{1}{5}$ is $\frac{1}{15}$; and 1 third of $\frac{4}{5}$ must be $\frac{4}{15}$.

***Ans.* $\frac{4}{15}$ of a melon.**

In dividing $\frac{4}{5}$ by 2, how was the division performed? In dividing $\frac{4}{5}$ by 3, how was the division performed? In what two ways, then, may a fraction be divided by an integer?

Comparing the examples above with those in Art. 215, it will be seen that they both teach the same thing, namely, finding one of the equal parts of a fraction. In Art. 215 the operation is called *multiplying by a fraction.* Here it is called *dividing by an integer.*

Dividing by 2 is equivalent to multiplying by what? Dividing by 5 is equivalent to multiplying by what? Multiplying by $\frac{1}{4}$ is equivalent to dividing by what? Multiplying by $\frac{1}{8}$ is equivalent to dividing by what?

Oral Exercises.

224. *a.* Divide $\frac{25}{72}$ by 5; $\frac{33}{40}$ by 11; $\frac{28}{29}$ by 7; $\frac{26}{51}$ by 13; $\frac{24}{25}$ by 8.

b. Divide $\frac{3}{7}$ by 5; $\frac{5}{8}$ by 3; $\frac{7}{11}$ by 9; $\frac{11}{12}$ by 7; $\frac{5}{9}$ by 8.

c. Divide $\frac{8}{9}$ of a cubic foot into 4 equal parts; into 5; 6; 7; 10.

d. Divide $1\frac{5}{7}$ $(=\frac{12}{7})$ into 2 equal parts; into 3; 5; 6; 7; 9; 10.

e. The charge for 8 hours' work was $\$2\frac{2}{5}$. How much was this for 1 hour?

f. A walk of $8\frac{4}{5}$ miles in 3 hours is an average of how many miles per hour?

WRITTEN WORK.

$$3 \overline{)8\frac{4}{5}}$$
$$2\frac{14}{15}.$$

When the dividend is a mixed number and less than the divisor, as in example *e.*, the dividend is changed to an improper fraction before dividing.

When the dividend is *greater* than the divisor, as in example *f.*, the integer is first divided, and the remainder only is changed to an improper fraction. *Ans.* $2\frac{14}{15}$.

g. A week's work for \$$10\frac{4}{5}$ is how much per day?

h. A tank can be emptied by 1 pipe in $10\frac{3}{5}$ hours; in what time can it be emptied by 3 such pipes?

i. When ice forms $11\frac{2}{3}$ inches thick in 4 successive nights, what is the average thickness formed per night?

j. If a boy working alone can pick a lot of pease in $6\frac{2}{5}$ days, how long will it take with 3 boys to help him?

k. Ralph travels on his bicycle $75\frac{3}{5}$ miles in a day of 9 hours; what is his average rate per hour?

225. Examples for Written Work.

235. If an engine goes $4\frac{39}{40}$ miles in five minutes, what is its average rate per minute?

236. At the rate of $51\frac{2}{3}$ miles per hour, how far does an express go in 1 minute?

237. When a product is $27\frac{5}{9}$ and the multiplier 8, what is the multiplicand?

238. When a product is $5\frac{4}{7}$ and the multiplicand 9, what is the multiplier?

239. A rope $148\frac{2}{3}$ feet long was cut into 12 equal lengths. How many feet were there in each length?

240. What is the price of 1 yard of fur trimming, the cost of 11 yards being \$$17\frac{7}{8}$?

241. A seamstress put $14\frac{3}{8}$ yards of ribbon into 46 equal loops. How much was put into each loop?

226. To divide an integer or a fraction by a fraction.

a. How many thirds are there in 2? how many fourths? eighths? tenths?

b. 3 are how many times $\frac{1}{2}$? $\frac{1}{4}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{12}$? $\frac{1}{20}$?

Illustrative Example. How many yards of cloth at $\frac{5}{8}$ of a dollar a yard can be bought for \$6?

WRITTEN WORK.

$$6 \div \frac{5}{8} = \frac{48}{8} \div \frac{5}{8} = \frac{48}{5} = 9\tfrac{3}{5};$$

$$\text{or, } \frac{6 \times 8}{8} \div \frac{5}{8} = \frac{6 \times 8}{5} = 9\tfrac{3}{5};$$

$$\text{or, simply, } \quad 6 \times \tfrac{8}{5} = 9\tfrac{3}{5}.$$

As many yards can be bought as there are times $\frac{5}{8}$ in 6. 6 changed to 8ths $= \frac{48}{8}$. There are as many times $\frac{5}{8}$ in $\frac{48}{8}$ as there are 5's in 48, which is $9\frac{3}{5}$. *Ans.* $9\frac{3}{5}$.

In the last form of written work, we have the dividend, 6, multiplied by $\frac{8}{5}$. But $\frac{8}{5}$ is the inverted expression of the divisor $\frac{5}{8}$.

Oral Exercises.

227. *c.* How many hours' work at $\frac{8}{10}$ of a dollar an hour can be obtained for \$3? For \$6? For \$9?

d. How many steps $\frac{2}{3}$ of a foot high will be required to ascend 8 feet? 12 feet?

e. How many pounds of tea at $\frac{7}{8}$ of a dollar a pound can be bought for \$5?

f. A heaped bushel is $\frac{5}{4}$ of an even bushel. How many bushels of apples, heaped measure, can be put into barrels that hold just 7 bushels of shelled beans, even measure?

g. A dealer bought \$9 worth of hats at $\frac{9}{10}$ of a dollar each. How many hats did he buy?

h. If the dealer had bought his hats at $\$1\frac{1}{8}$ each, how many could he have bought?

i. 6 are how many times $\frac{3}{4}$? $\frac{5}{8}$? $\frac{7}{9}$? $\frac{3}{10}$? $\frac{11}{12}$? $1\frac{1}{8}$?

j. How many are $8 \div \frac{3}{4}$? $6 \div \frac{5}{8}$? $4 \div \frac{9}{10}$? $9 \div 1\frac{1}{2}$?

Dividing by $\frac{1}{3}$ is equivalent to multiplying by what?

To divide an integer by a fraction, how must the integer be changed? What change must be made in a mixed number used as a divisor?

k. Divide $\frac{8}{11}$ by $\frac{2}{11}$; $\frac{9}{20}$ by $\frac{3}{20}$; $\frac{36}{50}$ by $\frac{9}{50}$; $\frac{28}{24}$ by $\frac{7}{24}$.

How is the division of a fraction by a fraction performed when both fractions have a common denominator?

m. How many pillow slips $\frac{3}{4}$ of a yard long can be made from 12 yards of cloth? From $8\frac{1}{4}$ yards?

n. How many paces, each $2\frac{1}{2}$ feet, are there in a rod?

o. When coffee cups hold $\frac{3}{4}$ of a pint each, how many cups can be filled with $5\frac{1}{4}$ pints of coffee? With a gallon?

228. *Illustrative Example.* How many yards of cloth at $\frac{5}{8}$ of a dollar a yard can be bought for $\frac{2}{3}$ of a dollar?

WRITTEN WORK.

$$\frac{2}{3} \div \frac{5}{8} = \frac{16}{24} \div \frac{15}{24} = \frac{16}{15} = 1\tfrac{1}{15};$$

or,

$$\frac{2 \times 8}{24} \div \frac{3 \times 5}{24} = \frac{2 \times 8}{3 \times 5} = 1\tfrac{1}{15};$$

or, simply, $$\frac{2 \times 8}{3 \times 5} = 1\tfrac{1}{15}.$$

The fractions $\frac{2}{3}$ and $\frac{5}{8}$ changed to fractions having a common denominator equal $\frac{16}{24}$ and $\frac{15}{24}$. Dividing the new numerator (16) of the dividend by the new numerator (15) of the divisor, the result is $1\frac{1}{15}$. *Ans.* $1\frac{1}{15}$ yards.

Or, using the factors by which the new numerators are obtained, in changing to a common denominator, we have the original dividend ($\frac{2}{3}$), multiplied by $\frac{8}{5}$. But $\frac{8}{5}$ is the inverted expression of the divisor $\frac{5}{8}$.*

229. From the above illustrations may be derived the following

Rules.

1. To divide a fraction by an integer: *Divide the numerator or multiply the denominator by the integer.*

2. To divide an integer or a fraction by a fraction: *Change the dividend and divisor to fractions having a common denominator, and then divide the numerator of the dividend by the numerator of the divisor.* Or,

Invert the expression of the divisor and proceed as in multiplication of fractions.

*** For other explanations of division of fractions, see Supplement, Art. 10.**

230. Examples for Written Work.

242. Divide 7 by $\frac{3}{5}$.

243. Divide 9 by $\frac{3}{2}$.

244. Divide 2 by $\frac{6}{5}$.

245. Divide 4 by $\frac{5}{6}$.

246. Divide $1\frac{1}{3}$ by $\frac{1}{2}$.

247. Divide $2\frac{1}{4}$ by $\frac{2}{3}$.

248. Divide $2\frac{2}{3}$ by $\frac{5}{6}$.

249. Divide $83\frac{1}{3}$ by $31\frac{1}{4}$.

250. $3 \div 1\frac{1}{2} = ?$

251. $5 \div 2\frac{2}{3} = ?$

252. $7\frac{1}{2} \div 1\frac{1}{4} = ?$

253. $10 \div 2\frac{1}{5} = ?$

254. $2\frac{1}{2} \div 1\frac{5}{6} = ?$

255. $1\frac{2}{3} \div 1\frac{1}{4} = ?$

256. $\frac{5}{8} \div 3\frac{1}{2} = ?$

257. $1\frac{7}{8} \div 5\frac{1}{4} = ?$

258. $10 \div \frac{5}{8} = ?$

259. $26 \div \frac{7}{10} = ?$

260. $39 \div \frac{11}{12} = ?$

261. $\frac{5}{6} \div \frac{7}{9} = ?$

262. $\frac{7}{12} \div \frac{4}{7} = ?$

263. $\frac{12}{13} \div \frac{8}{20} = ?$

264. $\frac{4}{5} \div \frac{5}{36} = ?$

265. $\frac{5}{24} \div \frac{4}{66} = ?$

266. $\frac{8}{33} \div \frac{4}{13} = ?$

267. $42 \div \frac{8}{24} = ?$

268. $\frac{7}{75} \div \frac{2}{25} = ?$

269. $1\frac{5}{8} \div \frac{5}{12} = ?$

270. $3\frac{7}{16} \div 2\frac{3}{8} = ?$

271. How many towels $\frac{5}{6}$ of a yard long can be cut from $24\frac{1}{2}$ yards, and what part of a yard will be left?

272. How many sheets $2\frac{5}{8}$ yards in length can be cut from $42\frac{1}{4}$ yards, and how many yards of cloth will be left?

273. In what time, paying \$ $1\frac{3}{4}$ per week, will a debt of \$ $12\frac{1}{2}$ be paid?

274. How many widths of cartridge paper $\frac{5}{6}$ of a yard wide will reach around a room 12 feet by 15 feet?

275. How many yards of sheeting at $16\frac{1}{2}$ cents a yard, will pay for $2\frac{3}{4}$ bushels of pease at 35 cents a peck?

276. $7\frac{1}{2}$ marks of Germany equal \$ $1.78\frac{1}{2}$. What is the value of 1 mark?

277. At \$ $23\frac{3}{10}$ per thousand feet for lumber, how many thousand feet can be obtained for \$ $186\frac{4}{10}$?

278. If \$ 13.51 were exchanged for francs at $19\frac{3}{10}$ cents each, how many francs were obtained?

279. If \$ $569\frac{1}{4}$ worth of flour was sold at \$ $4\frac{1}{8}$ a barrel, what was the number of barrels sold?

280. An agent sells flour, receiving $4\frac{1}{2}$ cents on a dollar of the amount of the sales. If he receives \$ 25.92, what is the amount of the sales?

281. If telegraph poles are set one to every $5\frac{1}{11}$ rods, how many are set in $3\frac{1}{2}$ miles?

282. How many railroad ties will be required for 300 feet of a single track railroad which is supported at the ends and throughout by ties $2\frac{1}{2}$ feet apart?

231. *Illustrative Example.* What is the quotient of $\frac{20}{5\frac{1}{2}}$?

WRITTEN WORK.

$$\frac{20}{5\frac{1}{2}} = \frac{20 \times 2}{11} = \frac{40}{11} = 3\tfrac{7}{11}.$$

$\frac{20}{5\frac{1}{2}}$ denotes that 20 is to be divided by $5\frac{1}{2}$. (Art. 125.) Multiplying both terms by 2, we have 40 divided by 11, which equals $3\frac{7}{11}$. *Ans.* $3\frac{7}{11}$.

Perform the operations indicated below:

283. $\dfrac{8}{8\frac{1}{3}}$ **286.** $\dfrac{2\frac{1}{3}}{4}$ **289.** $\dfrac{1}{6\frac{3}{4}}$ **292.** $\dfrac{7\frac{1}{2}}{10}$ **295.** $\dfrac{8\frac{1}{3}}{3\frac{1}{3}}$

284. $\dfrac{33\frac{1}{3}}{\frac{4}{5}}$ **287.** $\dfrac{1}{2\frac{1}{4}}$ **290.** $\dfrac{1\frac{2}{3}}{10}$ **293.** $\dfrac{3}{4\frac{2}{5}}$ **296.** $\dfrac{16\frac{2}{3}}{20}$

285. $\dfrac{\frac{2}{5}}{\frac{4}{7}}$ **288.** $\dfrac{5\frac{2}{5}}{\frac{1}{2}}$ **291.** $\dfrac{7\frac{3}{10}}{\frac{3}{4}}$ **294.** $\dfrac{6\frac{2}{3}}{\frac{4}{5}}$ **297.** $\dfrac{3\frac{1}{4}}{4\frac{1}{2}}$

Expressions like those above are called **complex fractions.**

How do you *multiply a fraction* by an integer? a mixed number by an integer? Explain, by an example, the method of multiplying an integer by a fraction. Multiply a fraction by a fraction; explain and give the rule. How do you multiply a mixed number by a mixed number or by a fraction? How can you simplify compound fractions?

How do you *divide a fraction* by an integer? a mixed number by an integer? an integer by a fraction? Dividing by $\frac{1}{8}$ is equivalent to multiplying by what? Multiplying by 4 is equivalent to dividing by what? Explain by an example the method of dividing a fraction by a fraction, and give the rule. How can you simplify the expressions called *complex fractions.*

SECTION IX.

MISCELLANEOUS AND REVIEWS.

232. To find what part one number is of another.

a. What part of 5 is 1? 2? 3? 4? Of 7 is 4? 5?

b. One is what part of 3? 5? 10? 20? 100?

In comparing 1 with any number to see what part it is of that number, what do you take as the numerator? as the denominator?

c. What part of 30 days is 1 day? of 32 quarts is 1 quart?

d. What part of 1 dozen is 6? 3? 4? 9? 8? 2? 10?

e. What part of 1 score is 10? 4? 5? 15? 16? 8? 12?

f. If a cistern can be drained in 12 minutes, what part of it can be drained in 5 minutes?

g. A firm failing can pay but 30 cents on a dollar of what they owe. What part of their debts can they pay?

h. Goods which cost 50 cents per yard sold for 75 cents. What part of the cost was the gain?

i. What part of the cost is the loss, when brooms which cost 30 cents sell for 24 cents?

j. Fred and Frank buy a quire of paper together; Fred uses 8 sheets, Frank uses 16. What part of the cost should each pay?

233. From the foregoing illustrations may be derived the following

Rule.

To find what part one number is of another: *Make the number which is the part the numerator of a fraction, and the number with which it is compared the denominator.*

Examples for Written Work.

234. What part of

1. $8\frac{3}{4}$ is 5?	**6.** 100 is $12\frac{1}{2}$?	**11.** $\frac{9}{20}$ is $\frac{7}{60}$?
2. $10\frac{7}{8}$ is 3?	**7.** 100 is $33\frac{1}{3}$?	**12.** $\frac{5}{8}$ is $\frac{5}{12}$?
3. $87\frac{1}{2}$ is 7?	**8.** 100 is $87\frac{1}{2}$?	**13.** $4\frac{1}{6}$ is $3\frac{1}{3}$?
4. $56\frac{1}{4}$ is 9?	**9.** $\frac{2}{3}$ is $\frac{7}{15}$?	**14.** $16\frac{2}{3}$ is $6\frac{2}{3}$?
5. 25 is $1\frac{7}{9}$?	**10.** $\frac{5}{12}$ is $\frac{3}{16}$?	**15.** $83\frac{1}{3}$ is 100?

16. What part of $272\frac{1}{4}$ square feet is 9 square feet?

17. The material for a cloak cost $\$12\frac{1}{2}$, the making $\$11\frac{1}{2}$. What part of the entire cost was the cost of making?

18. If a man can do a piece of work in 25 days, what part of it can he do in $6\frac{1}{4}$ days?

19. A can lay a hall floor in 6 days, B can do it in 8 days. What part of the work can each do in one day? What part can both together do in one day?

20. Three men working at the same rate per day did a piece of work for \$90. A worked 5 days, B $6\frac{1}{2}$ days, and C $8\frac{1}{2}$ days. How much did each receive per day? How much did each receive for his work?

21. In a rectangular lot 100 ft. long and 80 ft. wide there is an asparagus bed, also rectangular, 50 ft. long and 12 ft. wide. What part of the lot does the bed occupy?

22. What part of his earnings does a man spend who works 6 days a week, receiving \$1.75 per day, and paying for his board \$4 a week, and for all other expenses \$32.50 a quarter (13 weeks)?

235. To find the whole when a part is given.

a. In $\frac{2}{5}$ of a ream of paper there are 8 quires. How many quires are there in $\frac{1}{5}$ of a ream? In a whole ream?

b. 8 is $\frac{2}{5}$ of what number?

SOLUTION. — Since 8 is $\frac{2}{5}$ of a number, 1 half of 8, or 4, is $\frac{1}{5}$ of the number, and $\frac{5}{5}$ of the number is 5 times 4 or 20.

c. If the weight of $\frac{3}{4}$ of a bushel of potatoes is 45 pounds, what is the weight of 1 bushel?

d. 6 hours equal $\frac{3}{8}$ of what number of hours?

e. 12 days equal $\frac{3}{4}$ of what number of days?

f. $\frac{4}{11}$ of a mile equals $\frac{2}{5}$ of what part of a mile?

g. Mrs. Pratt has been 6 hours tacking $\frac{3}{7}$ of a comforter. At the same rate, how long will it take her to finish it?

h. If I sell grapes for 60 cents and thereby lose a sum equal to $\frac{1}{6}$ of the cost, what was the cost?

NOTE. — If $\frac{1}{6}$ was lost, $\frac{5}{6}$ remained.

i. A vessel having lost $\frac{1}{5}$ of her cable, has 200 feet remaining. How many feet had she at first?

j. Carl is 14 years old, which is $1\frac{1}{6}$ times the age of Alfred. How old is Alfred?

NOTE. — Carl's age is $\frac{7}{6}$ of the age of Alfred.

k. What was the cost of mutton per pound which sells for 8¢, at a gain of $\frac{1}{3}$ of the cost? Of yearlings which sell for 10¢, at a gain of $\frac{1}{4}$ of the cost? Of lamb which sells for 12¢, at a gain of $\frac{1}{5}$ of the cost?

l. By reason of large importations from Vera Cruz, hard asphalt sold at $ 9 per ton. If this was a falling off of $\frac{1}{4}$ from the former price, what was the former price?

m. A and B are to share $ 100. B is to have $\frac{1}{4}$ as much as A. What is each one's part?

NOTE. — A's part must be $\frac{4}{4}$ of itself, and B's part with A's part will equal $\frac{5}{4}$ of A's part. But the two together have $100; then $100 is $\frac{5}{4}$ of A's part.

n. A mother and her son together have $ 45; the son's part is $\frac{2}{3}$ as great as the mother's. What is each one's part?

o. When $ 6 received for the use of money is $\frac{2}{100}$ of the sum loaned, what is the sum loaned?

p. Of what sum of money is 4 cents $\frac{2}{100}$? is 6 cents $\frac{3}{100}$? is 18 cents $\frac{6}{100}$?

236. Examples for Written Work.

23. $\frac{7}{8}$ lb. = $\frac{2}{25}$ of what?

24. $\frac{5}{12}$ ft. = $\frac{10}{7}$ of what?

25. $\frac{16}{25}$ A. = $\frac{4}{11}$ of what?

26. $6\frac{1}{4}$¢ = $\frac{2}{3}$ of what?

27. $3\frac{3}{8}$ gal. = $\frac{9}{10}$ of what?

28. $16\frac{2}{3}$¢ = $\frac{5}{8}$ of what?

29. If $\frac{3}{8}$ of a barrel of flour weighs $73\frac{1}{2}$ pounds, what is the weight of a barrel?

30. A farmer received in one year $3575 as a royalty for oil produced on his farm. If this was $\frac{2}{3}$ of the value produced, what is the value produced?

What is the cost per pound

31. Of veal when $5\frac{3}{4}$ pounds can be bought for 92 cents?

32. Of butter when $4\frac{1}{8}$ pounds can be bought for $1.65?

33. Of sugar when $12\frac{1}{2}$ pounds can be bought for a dollar?

34. Of tea when a box of $13\frac{1}{3}$ pounds costs $10?

35. What is the cost per gross of pencils when $5\frac{1}{4}$ gross can be bought for $$7\frac{7}{8}$?

36. When $7\frac{1}{2}$ gallons of molasses costs $3.15, what is the price per gallon?

37. If $4\frac{1}{4}$ pounds of halibut costs $1.19, what is the cost per pound?

38. After losing $\frac{1}{3}$ of his property a man had $2400 left. How much money had he at first?

39. A sold $\frac{5}{9}$ of his farm and had 412 acres left. How many acres had he at first?

40. Mr. White owning $\frac{1}{3}$ of a yacht sold $\frac{1}{2}$ of his share for $550. What was the value of the yacht at the same rate?

41. What number is that which increased by $\frac{1}{4}$ of itself equals $291\frac{1}{9}$?

42. What number is that which diminished by $\frac{1}{4}$ of itself equals $291\frac{1}{9}$?

43. Water in changing from the temperature of ice to that of boiling, increases its bulk about 1 cubic foot in every 24. At this rate of increase, how many cubic feet of boiling water will equal the weight of 1000 cubic feet of ice water?

44. From a hogshead of molasses $31\frac{1}{4}$ gallons were taken, after which $\frac{4}{9}$ of the original quantity remained. How many gallons did the hogshead contain at first?

45. When $\frac{2}{3}$ of $\frac{7}{8}$ of a vessel is worth $980, how much is the whole vessel worth?

46. In a college of 342 students there were $\frac{1}{8}$ as many women as men. What was the number of each?

47. A dealer sold an umbrella for $5.20, which was $1\frac{5}{8}$ times what he paid for it. How much did he pay for it, and what was his gain?

48. A man failing in business paid only $\frac{2}{5}$ of the value of his bills. If Mr. Taft lost $2050 by him, what was the amount of Mr. Taft's bill?

49. Four children picked cranberries which they sold for $2.60. One picked 5 quarts, another 7 quarts, another 6 quarts, another 8 quarts. How much did they receive per quart? How much should each receive for all he had?

ALIQUOT PARTS OF NUMBERS.

237. What is one of the four equal parts of 8? of 9?

One of the equal parts of a number is an **aliquot part** of the number. Thus, $2\frac{1}{4}$ is an aliquot part of 9.

Oral Exercises.

a. Of the number 10 find $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{2}{3}$; $\frac{3}{4}$.
b. Of the number 30 find $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{10}$; $\frac{2}{3}$; $\frac{3}{4}$.
c. Of the number 50 find $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{2}{3}$; $\frac{3}{4}$.
d. Of the number 60 find $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{10}$; $\frac{1}{12}$; $\frac{1}{15}$; $\frac{1}{20}$.
e. Of the number 100 find $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{10}$; $\frac{1}{12}$; $\frac{1}{16}$.
f. Of the number 100 find $\frac{2}{3}$; $\frac{3}{4}$; $\frac{3}{5}$; $\frac{4}{5}$; $\frac{5}{6}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{3}{10}$.
g. Of the number 144 find $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{9}$; $\frac{1}{12}$; $\frac{2}{3}$; $\frac{3}{4}$.
h. Of the number 200 find $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{10}$; $\frac{1}{12}$; $\frac{2}{3}$.

By using the aliquot parts of numbers, especially of 10, 100, and 1000, the work of multiplying and dividing can often be materially shortened.

238. *Illustrative Example.* What is the cost of 26,500 shingles at $2½ per thousand?

WRITTEN WORK.

$$\begin{array}{r} 4\underline{)265.00} \\ 66.25 \end{array}$$

26,500 shingles at $1 a thousand would cost $26.50; 26,500 shingles at $10 per thousand would cost ten times as much, or $265.00; at $2½ per thousand, they would cost ¼ of $265.00, or $66.25. *Ans.* $66.25.

i. Give the shortest way of multiplying by 25; 12½; 37½; 62½; 87½; 3⅓; 33⅓; 66⅔.

j. Give the shortest way of dividing by 25; 12½; 3⅓; 33⅓, etc.

Examples for Written Work.

239. What is the amount of the following purchases:

50. 2 tubs of butter, each containing 35 pounds at 25¢, and 1 tub of 45 pounds at 37½¢?

51. 2 boxes of cheese, each containing 75 pounds at 12½¢, and 1 box of 82 pounds at 16⅔¢?

52. 1 quarter of beef weighing 190 pounds at 6¼¢, and 1 quarter weighing 210 pounds at 8⅓¢?

53. 60 yards of cashmere at 62½¢; 54 yards at 75¢; 36 yards at 87½¢?

54. 19 yards of velvet at $1.33⅓; 25 yards at $1.50; 22 yards at $1.66⅔?

55. 3⅓ yards of crash at 13¢; 7½ yards at 11¢; 5 yards of broadcloth at $4.50.

NOTE. — First find the cost of 10 yards.

56. 12½ yards of silk at 79¢; 33⅓ yards at 85¢; 37½ yards at $2.66.

57. How many pounds of cocoa at 33⅓¢ per pound, can be bought for $1? For $25?

58. $428 \times 16\frac{2}{3} = ?$ **60.** $521 \div 12\frac{1}{2} = ?$ **62.** $186 \div 6\frac{1}{4} = ?$

59. $262 \times 62\frac{1}{2} = ?$ **61.** $482 \times 37\frac{1}{2} = ?$ **63.** $216 \div 33\frac{1}{3} = ?$

Miscellaneous Oral Examples.

240. *a.* At $37\frac{1}{2}$¢ per dozen, what is the price of 1 spool of cotton?

b. How many potatoes at $\$\frac{4}{5}$ per bushel must be given for 4 bushels of walnuts at $\$1\frac{3}{4}$ a bushel?

c. If 8 men can mow a field in $\frac{2}{3}$ of a day, in how many days can 3 men mow the same field?

d. What part of $37\frac{1}{2}$¢ is $\frac{1}{8}$ of $\$1$?

e. A drover put $\frac{1}{2}$ of his cattle in the barn, $\frac{2}{5}$ of them in the yard, and the remainder, which were 10, were left in the field. How many cattle had he?

f. When gloves at wholesale can be bought for $\$9$ per dozen pairs, and thus $\frac{1}{4}$ of the retail price be saved, what is the retail price per pair?

g. If my house is 10 years old, and $\frac{3}{5}$ of its age is $\frac{3}{4}$ of the age of my son, what is my son's age?

h. X paid $\$4\frac{1}{2}$ a week for board, which was $\frac{3}{5}$ of what his brother paid, and what his brother paid was $\frac{5}{6}$ of what his father paid. How much did his father pay?

i. $\frac{4}{7}$ of 35 is $\frac{2}{9}$ of what number?

j. A can paint a house in 5 days, and B can do the same work in 4 days. What part of the work can each do in one day? What part can both do in one day? In how many days can both do it working together?

k. One seamstress can make a dress in 6 days and another in 5 days. How long will it take both to make it working together?

l. If you buy wool at 24 cents per pound, and it loses $\frac{1}{7}$ of its weight in cleansing, for how much must you sell it to make 10 cents per lb.?

NOTE.—If it loses $\frac{1}{7}$ of its weight, the $\frac{6}{7}$ which remains of each pound must have cost 24 cents.

m. The cars were due at 45 minutes past 3 A.M., but arrived $2\frac{3}{4}$ hours late. At what time did they arrive?

n. A can walk $\frac{5}{8}$ of a mile in 15 minutes. If he starts at 2 o'clock to walk $2\frac{1}{2}$ miles, at what time can he finish his walk?

241. Miscellaneous Written Examples.

64. What is the cost of $13\frac{3}{8}$ pounds of Formosa tea at 75¢ per pound and $18\frac{1}{4}$ pounds of bacon at 10¢ per pound?

65. A woman bought remnants of silk as follows: $\frac{1}{8}$ yd., $\frac{5}{16}$ yd., $1\frac{1}{4}$ yd., and $\frac{7}{8}$ yd. How much did she buy in all, and how much did she pay, at 80¢ a yard?

66. From a piece of duck containing $42\frac{1}{4}$ yd. were cut $3\frac{1}{8}$ yd., $2\frac{1}{2}$ yd., $5\frac{3}{4}$ yd., and $3\frac{1}{8}$ yd. How much remained?

67. From a piece of cotton containing 38 yd., worth 18¢ per yard, were made as many sheets, $2\frac{5}{8}$ yd. long, as possible. How many sheets were made, and what was the cost of cloth for each?

68. At 50¢ a cord for each time the sticks are sawed, how much must be paid for sawing $8\frac{1}{2}$ cords of wood, each stick to be sawed into three pieces?

69. Suppose each stick of the above to be sawed into four pieces, what should be paid?

70. How many bushels of wheat at $56\frac{1}{4}$¢ per bushel can be bought for $20?

71. If land is worth 4.62\frac{1}{2}$ per square foot, what is the value of a lot which measures 100 feet by $26\frac{1}{2}$ feet?

72. When $\frac{3}{7}$ of a mill is worth $2700, what is $\frac{5}{8}$ of the mill worth?

73. Having traveled 564 miles, I find I have gone just $\frac{5}{8}$ of my journey. How many miles have I still to go?

74. A's share in the profits of a speculation was $74.10. If this was $\frac{15}{100}$ of the whole profit, what was the whole profit?

75. My strawberry patch yielded 4140 boxes of strawberries in two years, yielding $\frac{4}{5}$ as much the second year as the first. What was the yield each year?

76. A mixed school of 800 pupils had $\frac{1}{3}$ as many boys as girls. What was the number of each?

77. A farm was sold for \$6580, which was $3\frac{1}{2}$ times what it cost. How much did it cost?

78. 9 men can do a piece of work in a certain time, working $8\frac{1}{3}$ hours a day; how many men would be required to do the work in the same time, working $6\frac{1}{4}$ hours a day?

79. When wood was \$$7\frac{1}{2}$ a cord, I gave $1\frac{1}{2}$ cords for $1\frac{7}{8}$ tons of coal. What was the coal worth a ton?

80. Owning $\frac{2}{3}$ of a yacht, I sold $\frac{3}{4}$ of my share for \$1520. What was the value of the whole yacht at the same rate?

81. How long will a barrel of flour (196 lb.) last 12 persons at the rate of $\frac{7}{8}$ of a pound a day to each person?

82. What will $42\frac{7}{8}$ quires of paper weigh, at 12 pounds to the ream?

83. I shipped to a man in Paris \$318 worth of cotton, and in return received 1200 francs. If the exchange value of a franc is $19\frac{3}{10}$ cents, how many dollars are still due me?

84. Two ships, $175\frac{1}{6}$ miles apart, sailed towards each other. After one had sailed $26\frac{4}{5}$ miles and the other $37\frac{2}{3}$ miles, how far apart were they?

85. Two persons start at the same place at 6 o'clock 5 min. A.M., and travel in opposite directions, one at the rate of $7\frac{3}{4}$ miles an hour, the other at the rate of $4\frac{1}{8}$ miles an hour. How far apart will they be at 2 o'clock 15 min. P.M.?

86. If a body falls $16\frac{1}{12}$ feet the first second, 3 times as far the next second, and 5 times as far the third second, how far does it fall in 3 seconds?

87. If the circumference of a wheel is $9\frac{11}{12}$ feet, how many times will it turn in going a mile?

88. A shilling of the old New England currency was worth $16\frac{2}{3}$ cents, and 12 pence made a shilling. What was the value in cents of the coin called "ninepence"? How much was "fourpence ha'penny" worth?

89. A New York shilling was worth $12\frac{1}{2}$ cents. How many New York shillings were equal to 6 shillings N. E. currency?

90. What part of a shilling N. E. currency was a New York shilling? What was the difference of their values?

91. When the exchange value of a franc is $19\frac{1}{4}$ ¢, what is the value of a 5-franc piece? How many francs can be bought for $8.50, and how many cents will remain?

92. Find the cost of storing, at $4\frac{1}{2}$ cents a month per barrel, the following goods:

2000 barrels apples,	$1\frac{3}{8}$ mo.	124 barrels squashes,	$\frac{5}{6}$ mo.
78 " parsnips,	$1\frac{4}{5}$ "	66 " onions,	$1\frac{3}{10}$ "

93. A dealer purchased 16 acres of woodland which averaged 40 cords to the acre; he sold $175\frac{3}{4}$ cords of oak, $49\frac{1}{2}$ cords of beech, and $362\frac{7}{8}$ cords of pine. What would the balance come to at the rate of $6.40 a cord?

94. If the average yield of sap per maple tree is $16\frac{2}{3}$ gallons, and 30 gallons of sap make 7 pounds of sugar, what weight of sugar will be made from 500 trees?

95. In a certain school $\frac{3}{8}$ of the pupils belong to the fourth grade, $\frac{1}{4}$ to the third, $\frac{5}{24}$ to the second, and the remainder, which is 96, belong to the upper grade. How many pupils are there in the whole school?

96. A man lost $\frac{1}{3}$, $\frac{1}{7}$, and $\frac{2}{5}$ of his money, and then had $2613 left. What sum had he originally?

97. A farmer had 2364 sheep, and sold half of them to one man, and $\frac{1}{3}$ to another. How many had he left?

98. A man owning $261\frac{4}{5}$ acres of land, sold $\frac{5}{11}$ of it to one man and $\frac{1}{2}$ of the remainder to another. How much did he sell to both men?

99. A man owning a meadow of 182 acres, sold $27\frac{1}{8}$ acres to one man and $\frac{2}{3}$ of the remainder to another. At $32 an acre, what was the value of the land he kept, and how much did he receive for the land he sold?

100. James spent $\frac{1}{2}$ of his money, then $\frac{2}{3}$ of what remained, then $\frac{3}{4}$ of what still remained, after which he had \$15. How much had he at first?

101. If it requires 1000 shingles laid 4 inches to the weather to cover a certain surface, how many will be required if they are laid $5\frac{1}{3}$ inches to the weather?

102. Two wagons were sold at \$120 apiece. On one wagon there was a gain of $\frac{1}{5}$ of the cost, and on the other a loss of $\frac{1}{5}$. What was the cost of each wagon, and what was the net gain or loss on the two?

103. A Jersey cow, which cost \$125, was sold three times at a profit of $\frac{1}{5}$ at each sale. How much did the last buyer pay for her?

104. A fruit dealer paid \$7 for $2\frac{3}{4}$ bushels of peaches, and sold them at 75¢ a peck. How much did he gain?

105. Mr. Snow left by will, $\frac{5}{8}$ of his estate to his wife, $\frac{1}{3}$ of the remainder to each of his two sons, and the remainder, which was \$20,168, to an Industrial School. How much did he leave to his wife?

106. If Mary can sweep a room in 30 minutes, and Edna in 40 minutes, how long will it take for both to sweep it?

107 A certain piece of work can be performed by a farmer in 8 days, by his hired man in 9 days, and by his son in 12 days. In what time can the farmer and his hired man do the work?

108. In what time can the farmer and son do it?

109. In what time can the hired man and son do it?

110. In what time can all do it, working together?

111. A merchant sold goods for a farmer, and was paid as follows:

For selling 50 barrels of flour at \$5.37$\frac{1}{2}$ per bbl., $2\frac{1}{2}$¢ per \$1.
For selling 750 pounds of cheese at $8\frac{1}{3}$¢ per lb., $2\frac{1}{4}$¢ per \$1.
For selling 640 acres of land at \$7.50 per acre, $\frac{7}{8}$¢ per \$1.
For selling 220 bushels of grain at $62\frac{1}{2}$¢ per bush., $1\frac{7}{8}$¢ per \$1.

What was the total paid the merchant? (Ans. to cents.)

DRILL TABLE No. 5. **(See Supplement, Art. 1.)**

242. For supplementary practice in fractions.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
A	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{10}{21}$	$\frac{3}{5}$	$\frac{7}{12}$	$\frac{5}{6}$	$\frac{3}{4}$	$\frac{1}{9}$	$\frac{4}{7}$	$\frac{2}{3}$	$\frac{1}{15}$	$\frac{1}{2}$	$\frac{5}{7}$	$\frac{2}{3}$	**A**
B	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{7}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{1}{9}$	$\frac{2}{5}$	$\frac{3}{7}$	$\frac{8}{9}$	$\frac{5}{8}$	**B**
C	$\frac{21}{24}$	$\frac{50}{60}$	$\frac{32}{36}$	$\frac{18}{21}$	$\frac{15}{20}$	$\frac{22}{24}$	$\frac{21}{24}$	$\frac{24}{32}$	$\frac{28}{36}$	$\frac{18}{21}$	$\frac{36}{45}$	$\frac{18}{45}$	$\frac{24}{28}$	$\frac{38}{42}$	$\frac{40}{48}$	**C**
D	$5\frac{5}{6}$	$4\frac{3}{10}$	$7\frac{1}{3}$	$4\frac{1}{6}$	$6\frac{3}{10}$	$3\frac{2}{3}$	$2\frac{1}{3}$	$1\frac{5}{8}$	$3\frac{1}{2}$	$4\frac{2}{3}$	$6\frac{1}{5}$	$1\frac{1}{3}$	$4\frac{1}{8}$	$4\frac{1}{7}$	$3\frac{3}{4}$	**D**

Exercises.

In each of the columns, 1, 2, 3, etc.,

112–126. Add A and B.
127–141. Add B and C.
142–156. Add C and D.
157–171. Add A, B, and C.
172–186. Add B, C, and D.
187–201. Add A, B, C, and D.
202–216. Take C from D.
217–231. Take B from D.
232–246. Take B + C from D.
247–261. Multiply A by B.
262–276. Multiply B by C.
277–291. Multiply D by C.
292–306. Multiply D by $2\frac{1}{4}$.
307–321. Divide C by B.
322–336. Divide D by 3.
337–351. Divide A by B.
352–366. Divide B by C.
367–381. Divide D by A.
382–396. Multiply C by A by B.
397–411. Find A of B × C of D.
412–426. Divide C + D by B.
427–441. Divide (D − A) by (A + C).
442–456. $C = \frac{3}{8}$ of what?
457–471. $D = \frac{8}{9}$ of what?
472–486. $A \times C = \frac{5}{9}$ of what?
487–501. $4\frac{1}{2} \times D$ is $\frac{7}{12}$ of what?

How do you find what part one number is of another?

What effect does multiplying the numerator of a fraction have upon the fraction? Why? In what other way could you produce the same effect, and why? What effect does dividing the numerator have upon a fraction? Why? In what other way could you produce the same effect, and why?

SECTION X.

DECIMAL FRACTIONS.

243. A decimal fraction is one or more of the decimal parts of a unit, as 1 tenth, 2 hundredths, 25 thousandths, etc.

NOTE.—Some elementary applications of Decimals were treated with integral numbers. (Art. 27–30, 44, 60, 89, 135.) In this section are given applications which require a knowledge of principles taught in common fractions.

244. To read and write decimals.

The manner of reading and writing decimals is taught in Art. 29 and 30. The following table shows the correspondence between the names of decimals and integers:

TABLE.

PLACE NAMES.	ETC.	MILLIONS.	HUNDRED-THOUSANDS.	TEN-THOUSANDS.	THOUSANDS.	HUNDREDS.	TENS.	UNITS.		TENTHS.	HUNDREDTHS.	THOUSANDTHS.	TEN-THOUSANDTHS.	HUNDRED-THOUSANDTHS.	MILLIONTHS.	ETC.
PLACES.		7TH.	6TH.	5TH.	4TH.	3D.	2D.	1ST.		1ST.	2D.	3D.	4TH.	5TH.	6TH.	
FIGURES.		4	8	6	9	3	7	8	.	7	4	8	9	3	5	
		Integers.								Decimals.						

Name the orders of decimals from tenths to millionths; from millionths to tenths.

Illustrate the manner of reading and writing decimals.

How are integral numbers and decimals read when written together?

Exercises in Reading and Writing Decimals.

245. Read or write in words the following:

1. 202.9	**7.** 0.4721	**13.** 185.0007	**19.** 0.205
2. 0.325	**8.** 30.492	**14.** 16.4076	**20.** 200.005
3. 0.016	**9.** 0.0039	**15.** 0.003072	**21.** 500.013
4. 71.83	**10.** 6.1014	**16.** 100.1005	**22.** 0.215
5. 4.324	**11.** 3.7818	**17.** 0.78091	**23.** 0.0506
6. 63.07	**12.** 13.002	**18.** 0.060071	**24.** 500.0006

Write in figures the following:

25. 208 ten-thousandths.
26. 1, and 45 thousandths.
27. 18 ten-thousandths.
28. 582 hundred-thousandths.
29. 648 millionths.
30. 6 thousand 48 millionths.
31. 41 hundred-thousandths.
32. 8, and 4003 millionths.
33. 45, and 673 millionths.
34. 5 thousand, and 51 thousandths.
35. 6 hundred, and 48 millionths.
36. 10 million, and 75 ten-millionths.

Write in decimal form

37. $5\frac{2}{10}$; $\frac{54}{100}$; $2\frac{816}{1000}$; $\frac{5}{10000}$; $4\frac{2}{100}$; $3\frac{5}{1000}$; $\frac{18}{10000}$; $\frac{847}{100000}$.

38. Write $\frac{45}{10}$, and $\frac{33\frac{1}{3}}{100}$. *Ans.* 4.5; $0.33\frac{1}{3}$.

39. Write $\frac{18}{10}$; $\frac{33\frac{1}{3}}{10}$; $\frac{185}{100}$; $\frac{4868}{1000}$; $\frac{333\frac{1}{3}}{100}$.

Write the following in decimal form, each example as one number:

40. $\frac{3}{10}$, $\frac{8}{100}$, and $\frac{6}{1000}$.
41. $\frac{7}{10}$, $\frac{4}{1000}$, and $\frac{8}{100}$.
42. $\frac{5}{100}$, $\frac{6}{1000}$, and $\frac{9}{10}$.
43. $\frac{6}{10}$, $\frac{7}{1000}$, $\frac{8}{10000}$, and $\frac{5}{100}$.
44. $\frac{46}{10}$, $\frac{5}{100}$, $\frac{6}{10000}$, and $\frac{8}{1000}$.
45. 50, $\frac{4}{100}$, $\frac{6}{1000000}$, and $\frac{14}{10000}$.

a. What is the denominator of the decimal 0.1? 0.25? 0.41? 4.0075?

b. What are the numerators of the above decimals?

REDUCTION OF DECIMALS.

246. To change decimals to lower denominations.

Illustrative Example. Change 0.26 to thousandths; to millionths.

WRITTEN WORK.

$0.26 = 0.260$
$0.26 = 0.260000$

To express a decimal in a lower denomination, *annex zeros to the given expression until the place of the required denomination is filled.* *Ans.* 0.260; 0.260000.

46. Change 4 to tenths; to hundredths; to thousandths.
47. Change 0.7 to thousandths; to millionths.
48. Change 4.5 to hundredths; 9.27 to ten-thousandths.
49. Express 5, 32, and 465 each as tenths; as thousandths.

247. To change decimals to common fractions.

Illustrative Example. Change 0.75, 0.875, and $0.37\frac{1}{2}$ each to common fractions in their simplest forms.

WRITTEN WORK.

$0.75 = \frac{75}{100} = \frac{3}{4}$.
$0.875 = \frac{875}{1000} = \frac{7}{8}$.
$0.37\frac{1}{2} = \frac{37\frac{1}{2}}{100} = \frac{75}{200} = \frac{3}{8}$.

To change a decimal fraction to a common fraction, *write the decimal with its denominator and then change the common fraction, if necessary, to its smallest terms.* *Ans.* $\frac{3}{4}$; $\frac{7}{8}$; $\frac{3}{8}$.

Change to common fractions in their smallest terms:

50. 0.25	**53.** $0.06\frac{1}{4}$	**56.** $0.31\frac{1}{4}$	**59.** $0.056\frac{1}{4}$
51. 0.125	**54.** $0.16\frac{2}{3}$	**57.** $0.71\frac{3}{7}$	**60.** $0.015\frac{5}{18}$
52. 0.875	**55.** $0.37\frac{1}{2}$	**58.** $0.83\frac{1}{3}$	**61.** $0.142\frac{6}{7}$

248. To change common fractions to decimal fractions.

Illustrative Example. Change $\frac{5}{8}$ to a decimal.

WRITTEN WORK.

$$\begin{array}{r} 8\underline{)5.000} \\ 0.625 \end{array}$$

$\frac{5}{8}$ of 1 equals $\frac{1}{8}$ of 5 (Art. 180).

5 equals 50 tenths, 500 hundredths, or 5000 thousandths. $\frac{1}{8}$ of 5000 thousandths is 625 thousandths. *Ans.* 0.625.

249. From the preceding example may be derived the following

Rule.

To change a common fraction to a decimal fraction: *Annex zeros to the numerator and divide it by the denominator.*

ADDITION AND SUBTRACTION OF DECIMALS.

For addition and subtraction of decimals, see Arts. 44 and 60.

250. *Illustrative Example.* Change $\frac{3}{8}$ and $\frac{9}{20}$ to decimals and add them.

WRITTEN WORK.

$$\begin{array}{rl} \frac{3}{8} = & 0.375 \\ \frac{9}{20} = & \underline{.45} \\ & 0.825 \end{array}$$

Changed to decimals, $\frac{3}{8} = 0.375$ and $\frac{9}{20} = 0.45$, the sum of which is 0.825. *Ans.* 0.825.

Change the following to decimals, and add in lines and in columns:

	65. 66. 67.		71. 72. 73.		77. 78. 79.
62.	$\frac{1}{2}+\frac{1}{4}+\frac{3}{8}=?$	68.	$\frac{4}{25}+\frac{3}{5}+\frac{7}{40}=?$	74.	$\frac{1}{32}+\frac{3}{64}+\frac{4}{5}=?$
63.	$\frac{3}{4}+\frac{2}{5}+\frac{3}{20}=?$	69.	$\frac{1}{2}+\frac{4}{50}+\frac{9}{25}=?$	75.	$\frac{9}{16}+\frac{4}{125}+5\frac{1}{8}=?$
64.	$\frac{4}{5}+\frac{3}{8}+\frac{5}{16}=?$	70.	$\frac{3}{4}+\frac{641}{25}+\frac{12}{5}=?$	76.	$\frac{7}{8}+\frac{7}{10}+\frac{9}{40}=?$
	$?+?+?=?$		$?+?+?=?$		$?+?+?=?$

251. *Illustrative Example.* Change $6\frac{1}{3}$ and $4\frac{3}{7}$ to decimals and add them.

WRITTEN WORK.

$$\begin{array}{rl} 6\frac{1}{3} = & 6.333\frac{1}{3} \\ 4\frac{3}{7} = & \underline{4.428\frac{4}{7}} \\ \textit{Exact Ans.} & 10.761\frac{19}{21} \end{array} \qquad \text{or} \qquad \begin{array}{rl} 6\frac{1}{3} = & 6.3333\ldots \\ 4\frac{3}{7} = & \underline{4.4285}\ldots \\ \textit{Approximate Ans.} & 10.762 \end{array}$$

Neither $\frac{1}{3}$ nor $\frac{3}{7}$ can be completely expressed in decimal form; however far the division is carried, there will still be a remainder. Whenever the denominator contains factors other than 2 and 5, the division is *interminable.* Further explanations of such fractions will be found in the Supplement, Arts. 11–14.

The degree of exactness required in any particular example will determine the extent to which the division shall go. It is customary to express the answer to the nearest tenth, hundredth, thousandth, etc. This is done by carrying the quotient one place farther than is required in the answer, and adding one to the tenths, hundredths, thousandths, etc., whenever the next term of the quotient equals or exceeds 5.

In the examples which follow, the pupil should give the decimal values to the nearest thousandth, unless otherwise directed.

Change to decimals and add in lines and in columns:

	83.		**84.**		**85.**
80.	$\frac{4}{5}$	+	$\frac{1}{3}$	+	$\frac{6}{9}$
81.	$\frac{5}{8}$	+	$\frac{5}{6}$	+	$\frac{5}{9}$
82.	$\frac{6}{7}$	+	$\frac{6}{11}$	+	$\frac{4}{13}$

	89.		**90.**		**91.**
86.	$\frac{7}{12}$	+	$\frac{8}{9}$	+	$\frac{1}{6}$
87.	$\frac{4}{15}$	+	$\frac{5}{16}$	+	$\frac{1}{2}$
88.	$\frac{2}{33}$	+	$\frac{4}{15}$	+	$\frac{8}{27}$

	95.		**96.**
92.	$5.0\frac{2}{3}$	+	$6.00\frac{1}{7}$
93.	$0.0\frac{1}{25}$	+	$5.93\frac{1}{4}$
94.	$18.3\frac{2}{13}$	+	$10.5\frac{4}{11}$

Change to decimals and subtract in lines and in columns:

	99.		**100.**
97.	$3\frac{1}{3}$	−	$2\frac{3}{4}$
98.	$1\frac{1}{12}$	−	$\frac{7}{8}$

	103.		**104.**
101.	$0.24\frac{1}{3}$	−	$0.17\frac{1}{2}$
102.	$0.0\frac{3}{4}$	−	$0.0\frac{2}{7}$

	107.		**108.**
105.	$8.33\frac{1}{3}$	−	$2\frac{6}{11}$
106.	$5.00\frac{1}{4}$	−	$1\frac{7}{15}$

	111.		**112.**
109.	$1\frac{3}{4}$	−	$\frac{17}{27}$
110.	$\frac{9}{13}$	−	$\frac{5}{11}$

	115.		**116.**
113.	$1\frac{18}{25}$	−	$\frac{3}{8}$
114.	$\frac{17}{200}$	−	$\frac{1}{4000}$

	119.		**120.**
117.	8.075	−	$1\frac{5}{9}$
118.	$4.00\frac{1}{7}$	−	$\frac{9}{125}$

121. From my home to church is $0.12\frac{1}{2}$ of a mile; thence to my neighbor Crook's is 1.8 miles; and to Mr. Knight's is 3.89 miles still further. What is the distance from my home to Mr. Knight's?

122. If water covers 0.734 of the surface of the earth, what part of the surface is land?

123. When $0.087\frac{1}{2}$ of a journey is completed, what part of the journey remains?

124. If from a meter, which is 39.375 inches, are cut 6.16 inches, $7\frac{1}{4}$ inches, $5\frac{1}{8}$ inches, and 4.2 inches, how much will be left?

125. If a balloon rises $1\frac{1}{5}$ miles, then sinks 0.875 of a mile, and again rises $\frac{3}{4}$ of a mile, how high is it?

126. If the entire surface of the earth is taken at 1000 parts, the Torrid Zone contains 398.7491 parts, and the Temperate Zones, 259.1555 parts. How many parts are in the Frigid Zones?

127. The four sides of a park measure, respectively, $140\frac{3}{5}$ rods, $128\frac{5}{8}$ rods, $98\frac{3}{40}$ rods, and 86.39 rods. After riding a mile, how much farther has one to go to ride around the park?

MULTIPLICATION OF DECIMALS.

For multiplication of decimals by integers, see Art. 89.

In multiplying a decimal by an integer, how many decimal places must there be in the product?

252. *Illustrative Examples.* (1) Multiply 28 by 0.1; (2) Multiply 28 by 0.7; (3) Multiply 2.8 by 0.7.

WRITTEN WORK.

(1) $28 \times 0.1 = 2.8$

$$\begin{array}{rr} (2) & (3) \\ 28 & 2.8 \\ \underline{0.7} & \underline{0.7} \\ 19.6 & 1.96 \end{array}$$

(1) To multiply 28 by 0.1 is to take 1 tenth of 28, which is expressed by placing the decimal point one place from the right. *Ans.* 2.8.

(2) To multiply 28 by 0.7 is to take 7 tenths of 28. One tenth of 28 is 2.8 (28 tenths), and 7 tenths is 7 times 2.8, or 19.6. *Ans.* 19.6.

(3) To multiply 2.8 by 0.7 is to take 7 tenths of 2.8. One tenth of 2.8 is 0.28 and 7 tenths of 2.8 is 7 times 0.28, or 1.96. *Ans.* 1.96.

253. From the written work above may be derived the following

Rule.

To multiply decimals: *Multiply as in integers, and point off as many places for decimals in the product as there are decimal places in the multiplicand and the multiplier counted together.*

NOTE.—If there are not figures enough in the product, prefix zeros.

254. Examples for Written Work.

128.	$0.14 \times 9 = ?$	**134.**	$0.69 \times 0.4 = ?$
129.	$0.014 \times 0.9 = ?$	**135.**	$43.7 \times 0.045 = ?$
130.	$1.12 \times 0.8 = ?$	**136.**	$66.66 \times 0.003 = ?$
131.	$18.7 \times 0.07 = ?$	**137.**	$6.002 \times 2.08 = ?$
132.	$60.37 \times 2.6 = ?$	**138.**	$77.034 \times 0.002 = ?$
133.	$8.75 \times 2.04 = ?$	**139.**	$10.00 \times 0.375 = ?$

Multiply the following in lines and in columns:

		142.	**143.**		**146.**	**147.**		**150.**	**151.**
Multiply	**140.**	0.75 ×	25	**144.**	0.09 ×	48	**148.**	0.009 ×	0.62
by	**141.**	46 ×	0.95	**145.**	16 ×	1.07	**149.**	1.463 ×	1.01

152. What is the cost of making 1.75 square yards of concrete walk at $0.90 a square yard?

153. What must be paid for skim coating 7.31 square yards of concrete walk at $0.35 a square yard?

154. George IV. spent 1,000,000 pounds English money on Windsor Castle. Reckoning a pound at $4.866½, what was the amount in dollars?

155. In Calcutta, in 1887, there were contributed 100,400 rupees for a hospital for sick animals. To how many dollars does this amount, at $0.292 to a rupee?

Change to dollars:

156. 100 marks of Germany at $0.238 each.

157. 50 drachmas of Greece at $0.193 each.

158. 75 florins of Netherlands at $0.402 each.

159. 750 francs of France at $0.193 each.

160. A man borrowed $540, paying for its use a sum equal to 0.07 of its value. How much did he pay?

161. A banker lent money at 0.06 of its value for each year. What should he receive for the use of $1500 for 2 years?

162. When a yen of Japan equals $0.997, what is the value of 250 yens?

At a profit of $0.25 on the dollar how much money would I make

163. On a carriage costing $368?

164. On a lot of apples costing $246?

165. On a house costing $10,500?

166. On a piece of property costing $635?

167. On a gold watch costing $375?

168. On a quantity of flour costing $3525?

169. At the rate of $14.50 on $1000 for taxes, what tax should be paid on $5000? on $4800? on $986.54?

170. At the rate of $0.06 on one dollar, how much should be paid for the use of $3475 for one year?

DIVISION OF DECIMALS.

For division of decimals by integers, see Art. 135.

In dividing a decimal by an integer, how many decimal places must be allowed in the quotient?

255. *Illustrative Examples.* (1) Divide 99.552 by 12. (2) Divide 0.9552 by 12. (3) Divide 99.552 by 0.12.

WRITTEN WORK.

(1)	(2)	(3)
12)99.552	12)0.9552	0.12)99.55|2
8.296	0.0796	829.6

In Art. 135, we learned that in dividing a decimal by an *integer, the decimal point must be inserted in the quotient when the point in the dividend is reached.* (1) *Ans.* 8.296. (2) *Ans.* 0.0796.

(3) This example differs from the other two inasmuch as the divisor contains a decimal as well as the dividend. Since multiplying the dividend and divisor by the same number does not alter the quotient (Art. 171), we may multiply the divisor 0.12 by 100, thus making it an integer, 12, provided we also multiply the dividend by 100, making it 9955.2. This increase in the dividend may be expressed by a mark, as a vertical line, put as many places at the right of the decimal point as the divisor has decimal places. The example will then be read, divide 9955.2 by 12, and the work will be done as in example 1. *Ans.* 829.6.

256. From the preceding illustrations may be derived the following

Rule.

To divide decimals: *Divide as in integers. If the divisor is an integer, point off as many places for decimals in the quotient as there are decimal places in the dividend.*

If the divisor is not an integer, place a mark in the dividend as many places to the right of the decimal point as there are decimal places in the divisor, and place a decimal point in the quotient when the terms of the dividend are used as far as the mark.

Note. — When there is a remainder after all the terms of the dividend have been used, the division may be continued as in Art. 135.

Examples for Written Work.

257. Divide the following in lines and in columns:

		173.			176.	177.		180.	181.
Divide	**171.**	23.42	÷ 7	**174.**	3.628	÷ .5	**178.**	5.681	÷ 0.06
by	**172.**	17.31	÷ 5	**175.**	4.095	÷ 81	**179.**	56.84	÷ 0.7

		184.	185.		188.	189.		192.	193.
Divide	**182.**	42	÷ 1.765	**186.**	18.76	÷ 0.004	**190.**	8.7	÷ 0.0009
by	**183.**	1.8	÷ 0.007	**187.**	4.357	÷ 5.1	**191.**	600	÷ $1.008\frac{1}{3}$

Note. — In Ex. 191 first multiply both dividend and divisor by 3.

	196.	197.		200.	201.		204.	205.
194.	1.36	÷ $5.7\frac{2}{3}$	**198.**	400.5	÷ $8.09\frac{6}{7}$	**202.**	\$185	÷ $0.37\frac{1}{2}$
195.	$3.33\frac{1}{3}$	÷ $9.3\frac{1}{3}$	**199.**	56.87	÷ $0.03\frac{1}{7}$	**203.**	\$982	÷ $0.16\frac{2}{3}$

206. How many pounds English money, at \$4.86 each, can be paid for with \$50, and how many cents remain?

207. How many rubles of Russia, at \$0.772 each, can be bought for \$100, and how many cents remain?

258. Miscellaneous Examples.

208. I have 3.75 yards of binding for a square bed spread 2.25 yards on each side. How much more binding do I need?

209. A cubic foot of water weighs 62.5 pounds; granite is 2.72 times as heavy as wáter. What is the weight of a cubic foot of granite? of 3.2 cubic feet?

210. How many meters of silk, 1.0936 yards to a meter, should I buy to make a dress requiring 18 yards?

211. How many yards are there in a piece of dress goods measuring 15 meters, 39.375 inches to a meter?

212. How many inches are there in 4.8 meters? In $6.66\frac{2}{3}$ meters?

213. What would be received for the use of $ 480 for a certain time, at $ 0.076 on each dollar?

214. An architect received $ 314.50 for planning a house. If this sum was 0.075 of the cost of the house, what was the cost?

215. Edinburgh is 393.6 miles from London. With close connections a traveler can go from London to Edinburgh and return in 18 hours, and make stops equal to 24 minutes. What is the average speed of the cars per hour while in motion? (Answer to the nearest hundredth.)

216. If 3.5 pounds of grain loses 1.5 ounces of its weight in thoroughly drying, what part of the original weight of the grain was liquid? (Answer to the nearest thousandth.)

217. If, by exposure to dampness, a certain article weighing 3.5 pounds absorbs 1.5 ounces of water, what part of the full weight is the weight of the water which is absorbed?

218. What would be received on $ 120 of an account that is worth $62\frac{1}{2}$ cents on a dollar?

219. A man owning $0.33\frac{1}{3}$ of a store bought half of the remainder. What part of the store did he then own? If his share was then worth $ 1700, what was the value of the whole store?

220. A man owning 0.15 of a ship sold $\frac{1}{3}$ of his share for \$ 800. What part of the ship did he sell, and what was the whole ship worth at the same rate?

221. Mr. James loaned \$ 1268 for 3 years, receiving for its use for the first year 0.07 of its value, for the second year 0.06 of its value, and for the third year 0.05 of its value. How much in all did he receive for its use?

222. How many days, at \$ 1.75 a day, must David work to earn enough to buy a coat worth \$ 8, a vest worth \$ 1.12½, and a hat worth \$ 3.87½?

223. How many miles an hour must a person travel to go 153 miles in 3 days, traveling 8.5 hours a day?

224. If a road rises 4.3 feet in every 50 feet, how much does it rise in a mile?

225. A wagon wheel is 3.5 feet across and 3.1416 times as far around; what is the distance around the wheel? How many times does it turn in going a mile?

226. A man sold a house for \$ 7250, which was 0.04 more than he paid for it. How much did he pay for it?

SUGGESTION. — \$ 7250 = 1.04 of what he paid.

227. When mining stock sells for 0.23 more than its original value, and its value was \$ 100 a share, what must I pay for 1 share? for 200 shares?

228. By giving a bank his written promise to pay the sum of \$ 300 in 2 months, Mr. A. obtained 0.986 of \$ 300 for present use. What sum did he obtain?

229. A bushel even measure equals 2150.42 cubic inches. If this is 0.783 of a heaped bushel, how many cubic inches are there in a heaped bushel?

230. 50 shares of railroad stock, which cost \$ 114 a share, were sold at a loss of \$ 432. At what price per share was the stock sold?

231. If an ore loses 0.4 of its weight in roasting and 0.4 of the remainder in smelting, how much ore will be required to make 1 ton of metal?

232. What is the cost of 475 melons at $12 per hundred?

NOTE. — 475 equals 4.75 ($4\frac{75}{100}$) hundreds. Since 1 hundred melons cost $12, 4.75 hundreds will cost 4.75 times $12.

Find the cost of:

233. 6 casks of lime, each 240 pounds, at $1.75 per C.*

234. 1260 posts at $20 per C.

235. 2842 pounds bituminous coal at 43¢ per C.

236. 220 bunches of onions at $8.50 per C.

237. 397 heads of lettuce at $5.20 per C.

238. What will be the cost of printing 6760 ems at 30¢ per thousand ems?

239. Make out a lumberman's bill for the following items:

1230 ft. White Wood	at	$18 per M.*
284 ft. Spruce	at	$17 per M.
93 ft. $\frac{7}{8}$ Pine	at	$48 per M.

240. My water meters registered 33,660 gallons for my house and 32,164 gallons for my store. Find the amount of the bill for water at 35¢ per thousand gallons.

241. My gas meter registered April 1st 207,300 feet; the bill for January 1st showed 202,800 feet. What is the bill for gas for the 3 months at $1.80 per thousand feet, if $0.16\frac{2}{3}$ of the bill is deducted for prompt payment?

What are *Decimal Fractions?* How are their written expressions distinguished from those of integral numbers? What indicates the denomination of the decimal? How do you read a decimal expression? How do you write decimals? What is the effect of annexing ciphers to a decimal expression?

How do you change decimals to common fractions? Common fractions to decimals? How do you add and subtract decimals? Perform an example in multiplication by a decimal, explain and give the rule. Perform an example in division by a decimal, explain and give the rule. How do you express the multiplication of a decimal by 10; 100; 1000; 0.1; 0.01; 0.001? How do you express the division of a decimal by 10; 100; 1000; 0.1; 0.01; 0.001?

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* C is used for "hundred" and M for "thousand."

DRILL TABLE NO. 6. **(See Supplement, Art. 1.)**

259. For supplementary practice in decimals.

Examples.	E.	F.	G.	H.
1.	0.0516	Two, and 206 *thousandths.*	9.16	$\frac{17}{16}$
2.	1.732	936 *ten-thousandths.*	800.1	$\frac{8}{21}$
3.	8016.	54, and 54 *thousandths.*	0.0052	$\frac{11}{12}$
4.	4.95	806, and 1047 *millionths.*	2.0763	$\frac{5}{9}$
5.	0.012	5 hundred, and 26 *hundredths.*	$18.26\frac{1}{2}$	$\frac{7}{15}$
6.	12.007	One thousand *millionths.*	0.0101	$\frac{5}{182}$
7.	45.9	Twenty-nine *millionths.*	3.0712	$\frac{181}{22}$
8.	8.621	846291 *hundred-thousandths.*	80.06	$\frac{19}{28}$
9.	0.00562	Five hundred eleven *thousandths.*	5.4	$\frac{8}{39}$
10.	1002.	4271, and 4271 *ten-millionths.*	0.6805	$\frac{8}{42}$
11.	$1.87\frac{1}{2}$	68 thousand, and $4\frac{1}{2}$ *tenths.*	4.071	$\frac{5}{17}$
12.	$0.12\frac{1}{5}$	One hundred 22 *thousandths.*	24.40	$\frac{18}{63}$
13.	1.015	Eight, and $4\frac{2}{7}$ *hundredths.*	0.0002	$\frac{168}{57}$
14.	$8.33\frac{4}{9}$	Five hundred, and $\frac{2}{8}$ *tenths.*	1.071	$\frac{49}{560}$

Exercises upon the Table.

242–255. Read the numbers expressed in E and in G.

256–269. Write in figures the numbers expressed in F.

270–283. Change E to equivalent common fractions in lowest terms.

284–297. In the same way change G.

298–311. Change H to equivalent decimals (4 places).

312–325. Add E, F, and G.

Find the difference between:

326–339. E and F. **340–353.** E and G. **354–367.** F and G.

368–381. Multiply E by F. **410–423.** Divide E by F.

382–395. Multiply E by G. **424–437.** Divide G by E.

396–409. Multiply F by G. **438–451.** Divide G by F.

452–465. Add $E \times 10$, $F \div 100$ and G.

SECTION XI.

COMPOUND DENOMINATE NUMBERS.

260. *Illustrative Example.* How many pints are there in 2 quarts 1 pint? *Ans.* 5 pints.

The number, 5 pints, is made up of units of one denomination only. Such a number is a **simple number,** and because the kind of unit is named, it is a **denominate number;** hence, it may be called a **simple denominate number.**

261. The number, 2 quarts 1 pint, is made up of units of two different denominations. A number made up of units of two or more different denominations is a **compound denominate number.**

262. Denominate numbers are also called **concrete numbers,** and numbers considered apart from any object are called **abstract numbers.**

Name a simple denominate number; a compound denominate number; a concrete number; an abstract number.

NOTE. — The tables of measures in common use required to solve the examples in this section will be found in the Supplement, Arts. 20-46.

MEASURES OF EXTENSION.

Examples for Oral and Written Work.

263. *a.* Repeat or write from memory the tables of Long Measure; of Square Measure; of Cubic Measure.

b. What is the standard unit of length?

c. What divisions are there in the yard besides those given in the table?

1. Copy and fill out the following:

1 rod	=	yards	=	feet	=	inches.
1 mile	=	rods	=	yards	=	feet.
1 square mile	=	acres	=	square rods.		
1 acre	=	square rods	=	square yards	=	square feet.
1 cubic yard	=	cubic feet	=	cubic inches.		
1 cord	=	cord feet	=	cubic feet		

d. What is a fathom, and how used?

MEASURES OF CAPACITY.

264. *e.* Repeat or write from memory the tables of Liquid Measure. Of Dry Measure.

f. What is the standard unit of Liquid Measure? Of Dry Measure?

2. Copy and fill out the following:

1 gallon	=	quarts	=	pints	=	gills.
1 bushel	=	pecks	=	quarts	=	pints.

g. What is the approximate weight of a pint of water?

265. Comparison of Liquid and Dry Measures.

Liquid Measure.	*Cu. in.*	*Dry Measure.*	*Cu. in.*
1 gallon =	231	1 bushel =	2150.42
1 quart =	?	1 quart =	?

3. How many more cubic inches are in a quart dry measure than in a quart liquid measure? (Answer to hundredths.)

4. How many gallons of water will a tank contain which measures on the inside 3 feet square at the bottom and which is 3 feet deep?

5. How many gallons will a tank contain that is 3 ft. by 2 ft. 6 in. by 1 ft. 6 in. inside measurement?

6. How many bushels will a bin contain that measures on the inside 5 ft. by 4 ft. 3 in. by 2 ft. 10 in.?

MEASURES OF WEIGHT.

266. *h.* Repeat or write from memory the table of Avoirdupois Weight; of Troy Weight.

7. Copy and fill out the following:

1 ton	=	pounds =		ounces.		
1 long ton =		cwt.	=	pounds.		
1 pound Troy =		oz.	=	pwt. =		gr.

NOTE. — Apothecaries use the Troy pound and ounce, but for mixing medicines they divide the ounce into 8 drams of 3 scruples of 24 grains each.

i. What weight is used in weighing silver, gold, precious stones, etc.?

j. What is the standard unit of weight?

k. How many ounces are there in 3 pounds of silver? in 3 pounds of coal?

l. At 80 cents a Troy pound for camphor, what is the cost of an ounce?

m. How many more pounds are there in a long ton than in a common ton?

8. What is the value of a gold chain weighing $2\frac{1}{4}$ ounces at 90 cents a pwt.?

267. Comparison of Weights.

175 lb.	Troy	= 144 lb.	Avoirdupois.
175 oz.	"	= 192 oz.	"
7000 gr.	"	= 1 lb.	"

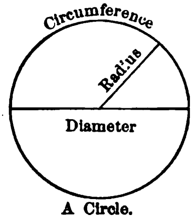
n. Which is heavier, a pound of gold or a pound of lead? an ounce of gold or an ounce of lead?

9. From two Australian mines, 16,000 ounces of gold were produced during one week in 1894. What was the product in pounds Troy? in pounds Avoirdupois?

10. Change 12 pounds Avoirdupois to pounds Troy.

CIRCULAR AND ANGULAR MEASURES.

268. A **circle** is a plane surface bounded by a line, every point of which is equally distant from a point within called the center.

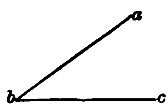


A Circle.

269. The bounding line of a circle is the **circumference.** Any part of the circumference is an **arc.**

270. A straight line drawn from the center to the circumference of a circle is its **radius.** A straight line drawn through the center, having its opposite ends in the circumference of the circle, is its **diameter.**

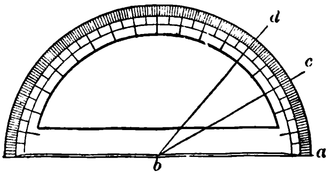
271. The circumference of a circle is divided into 360 equal arcs, called *degrees* (°), each degree into 60 *minutes* ('), and each minute into 60 *seconds* (").



An Angle.

272. When two lines, as *ab* and *bc*, meet each other, they form an **angle.** The lines are the **sides** of the angle, and the point where they meet is the **vertex.** The size of the angle is the amount by which one side is turned away from the other.

273. If the center of a circle is placed at the vertex of an angle, the arc included between the sides is the measure of the angle. Thus, if the arc *ac* contains 30 degrees, the angle *abc* is an angle of 30 degrees. Again, if the arc *ad* contains 50 degrees, the angle *abd* is an angle of 50 degrees.



274. An angle of 90 degrees is a **right** angle; an angle of less than 90 degrees is an **acute** angle; an angle of more than 90 degrees is an **obtuse** angle.

275. Lines which together form a right angle are **perpendicular** to each other.

NOTE. — As the circumference of every circle has 360 degrees, the length of 1 degree of a circumference differs in different circles.

276. The length of a degree of the circumference of the earth at the equator is about 69.16 common miles.

277. The length of a minute of the circumference of the earth at the equator is a **geographical** or **nautical mile,** and is about 1.15 common miles. When measuring the speed of vessels this is called a **knot.**

Exercises.

278. *a.* Repeat or write from memory the table of Circular and Angular Measure.

11. Copy and fill out the following:

1 circumference = $\quad ^\circ = \quad ' = \quad ''$.

b. How many degrees are there in a semi-circumference? in a quadrant, or $\frac{1}{4}$ of a circumference? in a sextant, or $\frac{1}{6}$ of a circumference?

c. How many degrees are there in a right angle?

d. Through how many degrees does the hour hand of a clock move in 3 hours? in 1 hour? in 2 hours?

e. Through how many degrees does the minute hand of a clock move in half an hour? in a quarter of an hour? in 5 minutes? in 1 minute?

f. The hour and minute hands of a clock form an angle of how many degrees at 2 o'clock? at 4 o'clock? at 5 o'clock? at 7 o'clock? at 10 o'clock?

g. The torrid zone is 47 degrees wide. How could you find its width in nautical miles?

MEASURES OF TIME.

279. Repeat or write the table of Time Measure.

12. Copy and fill out the following:

1 common year =	mo. =	wk. =	d.
1 day =	h. =	min. =	sec.

h. Name the months which contain 30 days each; name the months which contain 31 days each.

i. How do you know what years are leap years?

MISCELLANEOUS MEASURE.

280. Repeat or write the Miscellaneous Table.

13. Copy and fill out the following:

1 great gross =	gross =	dozen =	units.
1 bale =	bundles =	reams.	
1 ream =	quires =	sheets.	

MEASURES OF VALUE.

281. *j.* Repeat or write the table of U. S. and Canada Money; of English Money. What is the standard unit of U. S. money? Of Canada money? Of English money?

14. Copy and fill out the following:

U.S. money:	1 dollar =	dimes =	cents =	mills.	
English money:	1 pound =	shillings =	d. =	far. = $4.866½	
	1 guinea =	shillings.			
	1 crown =	shillings.			

			U. S. Money.
French money:	1 franc =	centimes =	$0.193
German money:	1 mark =	pfennigs =	$0.238
Russian money:	1 rouble =	kopecks =	$0.772

NOTE.—The values of the monetary units of foreign countries in United States money vary somewhat from year to year. For ordinary purposes, the above values which are those of Jan. 1, 1895, will be found sufficiently accurate; for exact values, the report of the United States Treasurer should be consulted each year.

In the next article give answers to thousandths.

Examples for Written Work.

282. Change to dollars: $100 equals

15. £100 (pounds).	**19.** How many pounds?
16. 100 francs.	**20.** How many marks?
17. 100 marks.	**21.** How many roubles?
18. 100 roubles.	**22.** How many francs?

23. The gold coin of Great Britain of the value of a pound is called a sovereign. How many shillings are there in 1 guinea, 2 crowns, 3 sovereigns, 1 half-sovereign, and 3 half-crowns?

REDUCTION.

283. To change denominate numbers to numbers of lower denominations.

Illustrative Example. Change 3 yd. 2 ft. 6 in. to inches.

WRITTEN WORK.

$$\begin{array}{l} 3 \text{ yd. } 2 \text{ ft. } 6 \text{ in.} \\ \underline{3} \\ 11 \text{ ft.} \\ \underline{12} \\ 138 \text{ in.} \end{array}$$

Since there are 3 ft. in 1 yd., in 3 yd. there are 3 times 3 ft. or 9 ft.; 9 ft. and 2 ft. = 11 ft. Since there are 12 in. in 1 ft., in 11 ft. there are 11 times 12 in. or 132 in.; 132 in. and 6 in. = 138 in. *Ans.* 138 in.

284. From the foregoing illustration may be derived the following

Rule.

To change a compound denominate number to units of a lower denomination: *Multiply the number of the highest denomination by the number of units it takes of the next lower denomination to make one of that higher, and to the product add the given number of the next lower denomination. Multiply this sum in like manner, and so proceed till the given number is changed to units of the required denomination.*

Examples for Written Work.

285. How many

24. Inches are 3 yd. 1 ft. 4 in.?
25. Minutes are 5 h. 40 min.?
26. Pounds are 2 T. 180 lb.?
27. Ounces are 6 lb. 9 oz. Av.?
28. Pence are £5 6*s.*?
29. Feet are 3 rd. 4 yd.?
30. Feet are 2 mi. 3 rd.?
31. Sq. ft. are 4 A. 9 sq. yd.?
32. Cd. ft. are 2 cd. 5 cd. ft.?
33. Minutes are 4° 13′.

34. What is the value of 2 lb. 8 oz. of gold at $20 an ounce?

35. At 11¢ a quart for nuts and 45¢ a peck for apples, find the value of $4\frac{1}{2}$ bushels of nuts and 12 bushels of apples.

36. At 7¢ per quart for milk, and 30¢ per pint for cream, how much will a dealer receive for 24 gal. 2 qt. of milk and 3 qt. 1 pt. of cream?

37. What will it cost to fence both sides of a road 36 rd. 6 ft. long, at 28¢ a foot?

38. The tropic of Cancer is 23° 30′ north of the equator. What is the distance in geographical miles? What is the distance in common miles? (See Art. 276.)

286. *Illustrative Examples.* (1) Change $\frac{2}{3}$ of a rod to yards and feet. (2) Change 0.66 of a rod to yards and feet.

WRITTEN WORK.

(1)

$\frac{2}{3}$ rd. $= \frac{2}{3}$ of $\frac{11}{2}$ yd. $= 3\frac{2}{3}$ yd.
$\frac{2}{3}$ yd. $= \frac{2}{3}$ of 3 ft. $= 2$ ft.
Ans. 3 yd. 2 ft.

(2)

$$\begin{array}{r} 0.66 \text{ rd.} \\ \underline{5\frac{1}{2}} \\ 33 \\ \underline{330} \\ 3.63 \text{ yd.} \\ \underline{3} \\ 1.89 \text{ ft.} \end{array}$$
Ans. 3 yd. 1.89 ft.

(1) Since $5\frac{1}{2}$ yd. = 1 rd., $\frac{2}{3}$ of a rd. = $\frac{2}{3}$ of $\frac{11}{2}$ yd. or $3\frac{2}{3}$ yd. Since 3 ft. = 1 yd., $\frac{2}{3}$ of a yd. = $\frac{2}{3}$ of 3 ft. = 2 ft. *Ans.* 3 yd. 2 ft.

(2) 0.66 rd. = 0.66 of $5\frac{1}{2}$ yd. = 3.63 yd.; 0.63 yd. = 0.63 of 3 ft. = 1.89 ft. *Ans.* 3 yd. 1.89 ft.

287. From the foregoing illustration may be derived the following

Rule.

To change a fraction of one denomination to integers of lower denominations: *Change the fraction, as far as possible, to an integer of the next lower denomination. If a fraction occurs in the result, proceed with it as with the first fraction, and so continue as far as required.*

Change

39. $\frac{5}{7}$ rd. to ft.	**43.** 0.48 lb. Troy to pwt.
40. $\frac{4}{5}$ A. to sq. ft.	**44.** 0.39° to ′.
41. $\frac{8}{9}$ mi. to rd.	**45.** 0.45 y. to days.
42. $\frac{2}{3}$ oz. to gr.	**46.** $\frac{7}{8}$ mi. to in.

47. At 20¢ a foot, what is the cost of $\frac{3}{4}$ of an acre of land?

Change to units of lower denominations:

48. $\frac{7}{8}$ T.	**53.** $\frac{7}{11}$ mi.	**58.** $0.56\frac{1}{4}$ gal.
49. $\frac{5}{9}$°.	**54.** $\frac{5}{8}$ h.	**59.** 0.73 sq. rd.
50. $\frac{7}{18}$ cu. yd.	**55.** $\frac{1}{3}$ of $\frac{4}{5}$ A.	**60.** 0.316 rd.
51. £ $\frac{7}{8}$.	**56.** 0.7 bu.	**61.** 0.875 lb. Avoir.
52. £ 5.624.	**57.** $\frac{9}{16}$ lb. Troy.	**62.** 0.0756°.

288. To change a simple denominate number to a compound number of higher denominations.

Illustrative Example. Change 424 feet to rods, yards, etc.

WRITTEN WORK.

3)424
$5\frac{1}{2}$)141 yd. + 1 ft. rem.
 2   2
11)282
 25 rd. + $\frac{7}{2}$ yd. rem.

Ans. 25 rd. $3\frac{1}{2}$ yd. 1 ft.,
or 25 rd. 3 yd. 2 ft. 6 in.

Since there are 3 ft. in 1 yd., in 424 ft. there are $424 \div 3 = 141$ yd. and 1 ft. remaining. Since there are $5\frac{1}{2}$ yd. in 1 rd., in 141 yd. there are $141 \div \frac{11}{2} = 25$ rd. and $3\frac{1}{2}$ yd. remaining; $\frac{1}{2}$ yd. = 1 ft. 6 in.; 1 ft. + 1 ft. = 2 ft. *Ans.* 25 rd. 3 yd. 2 ft. 6 in.

289. From the foregoing illustration may be derived the following

Rule.

To change a simple denominate number to a compound number of higher denominations: *Divide the given number by the number of units it takes of its denomination to make one of the next higher. Set aside the remainder, and divide, as before, the quotient thus obtained. Proceed in the same manner till the required denomination is reached. The last quotient with the several remainders is the compound number sought.*

Examples for Written Work.

290. Change to units of higher denominations:

63. 321 pints of berries.

64. 562 pints of water.

65. 428 ounces of salt.

66. 1418 grains of gold.

67. 3286 sheets of paper.

68. 18,642 minutes.

69. 38,741 seconds.

70. 2320 feet.

71. 5841 square feet.

72. 31,728 cubic inches.

73. What will 20 old silver dollars weigh in ounces, pennyweights, and grains, each dollar weighing $412\frac{1}{2}$ grains?

74. How many miles is it through the earth from pole to pole, the distance being 41,707,308 feet?

75. Change 58,642 farthings English money to pounds, shillings, etc.

76. Change 9862 pence to pounds, shillings, etc.

291. To change a compound denominate number to a fraction of a higher denomination.

Illustrative Examples. (1) Change 3 qt. 1 pt. 3 gi. to the fraction of a gallon. (2) Change 4 yd. 2 ft. 3 in. to the decimal of a rod.

Solutions. — (1) 3 qt. 1 pt. 3 gi. changed to gills equals 31 gills; 1 gallon equals 32 gills. 31 gills is $\frac{31}{32}$ of a gallon. *Ans.* $\frac{31}{32}$ gal.

(2) 4 yd. 2 ft. 3 in. changed to inches equals 171 inches; 1 rod equals 198 inches. 171 in. is $\frac{171}{198}$ rd.; $\frac{171}{198}$ equals 0.8636. *Ans.* 0.8636 rd.

292. From the preceding illustration may be derived the following

Rule.

1. To change a compound number to a fraction of a higher denomination: *Change the given compound number to its lowest denomination, and make it the numerator of a fraction; also change to the same denomination a unit of the required denomination and make it the denominator of the fraction.*

2. To change a compound number to a decimal of a higher denomination: *Proceed as above and then change the common fraction to a decimal.*

77. 1 pk. 2 qt. is what part of a bushel?

78. 4 yd. 0 ft. 9 in. is what part of a rod?

79. 75 sq. rd. 90$\frac{3}{4}$ sq. ft. is what part of an acre?

80. Change 6 oz. 5 pwt. to the decimal of a pound.

81. Change 87 rd. 10 ft. to the decimal of a mile.

82. Change 4s. 6d. to the decimal of a £.

83. Change 11s. 10d. to the decimal of a £.

84. Find the cost of 3 pk. 2 qt. of meal at $0.60 a bushel.

85. What will be the cost of 2 qt. 1 pt. of kerosene at $0.12 a gallon?

86. How much will a man earn in 5 days 7 hours at the rate of $15 per week of 6 days of 10 hours each?

87. At $60 an acre, what must be paid for 2 A. 113$\frac{1}{8}$ sq. rd. of land?

88. At $200 a lb., what is the value of 10 oz. 10 pwt. 10 gr. of gold?

89. At $4.8665 per £, what is the value of £9 12s. 6d. English money?

90. Change 5 francs 50 centimes French money to U. S. money. (Art. 281.)

91. Change 10 marks 20 pfennigs German money to U. S. money. (Art. 281.)

ADDITION AND SUBTRACTION.

293. *Illustrative Examples.* (1) Add 2 mi. 172 rd. 6 ft., 1 mi. 310 rd. 14 ft., and 95 rd. 11 ft. (2) From 5 rd. 3 yd. 1 ft. take 1 rd. 4 yd. 2 ft.

WRITTEN WORK.

(1) ADDITION.

	2 mi.	172 rd.	6 ft.
	1	310	14
		95	11
	4 m.	258 rd.	$14\frac{1}{2}$ ft.
or,	4 m.	258 rd.	14 ft. 6 in.

(2) SUBTRACTION.

5 rd.	3 yd.	1 ft.	
1	4	2	
3 rd.	$3\frac{1}{2}$ yd.	2 ft.	
	$\frac{1}{2}$ =	1 ft.	6 in.
3 rd.	4 yd.	0 ft.	6 in.

(1) The numbers are written so that units of the same order are expressed in the same column, and the adding begins with the units of the lowest denomination, as in simple addition.

The sum of the feet is 31 feet = 1 rod $14\frac{1}{2}$ feet. Writing the $14\frac{1}{2}$ feet under the line in the place of feet, and adding 1 rod with the rods of the given numbers, the sum is 578 rods, equal to 1 mile 258 rods. The sum of the miles is 4. *Ans.* 4 mi. 258 rd. 14 ft. 6 in.

(2) The subtracting begins with the units of the lowest denomination, as in subtraction of simple numbers. As there is but 1 foot in the minuend, 1 of the 3 yards is changed to 3 feet, making with the 1 foot, 4 feet; 4 feet less 2 feet is 2 feet. As there are but 2 yards left in the minuend, 1 of the 5 rods is changed to $5\frac{1}{2}$ yards, making with the 2 yards, $7\frac{1}{2}$ yards; $7\frac{1}{2}$ yards less 4 yards is $3\frac{1}{2}$ yards. 4 rods less 1 rod is 3 rods. Changing the $\frac{1}{2}$ yard to feet we have 1 foot 6 inches, which added to 2 feet makes 3 feet 6 inches or 1 yard 6 inches. *Ans.* 3 rd. 4 yd. 0 ft. 6 in.

NOTE.—The operations upon compound numbers are similar to those upon simple numbers, the difference being that in operations upon compound numbers *varying scales* are used instead of the uniform scale of tens. Therefore, no special rules are necessary for addition, subtraction, multiplication, and division.

294. Examples for Written Work.

92. How much time is there in 2 h. 40 min., 3 h. 17 min., and 4 h. 45 min.?

93. Add together 10° 25′ 40″, 4° 9′ 5″, and 15° 45′ 18″.

94. Add together £5 17s. 11d., £8 10s. 4d., 18s. 6d., and £3 7s. 6d.

95. How much wood is there in 3 piles containing, respectively, 5 cords 3 cord feet, 8 cords 4 cord feet, and 10 cords 7 cord feet.

96. How much wheat is there in 1 bu. 3 pk. 7 qt., 2 bu. 1 pk. 5 qt., 1 bu. 6 qt., and 3 bu. 2 pk. 6 qt.?

97. How much land is there in four pastures, the first containing 7 A. 64 sq. rd., the second 15 A. 82 sq. rd. 260 sq. ft., the third 5 A. 17 sq. rd. 160 sq. ft., and the fourth 19 A. 89 sq. rd. 36 sq. ft.?

98. Add 320 cu. yd. 20 cu. ft. 1000 cu. in., 29 cu. yd. 24 cu. ft. 968 cu. in., 500 cu. yd. 728 cu. in.

99. Add $\frac{2}{3}$ bu. and 5 qt.

NOTE. — Change $\frac{2}{3}$ bushels to quarts.

100. Add $\frac{1}{8}$ mi. to 78 rd. 26 ft.

101. Add $\frac{1}{8}$ T. to 176 lb.

102. Add $\frac{4}{5}$ y. to 97 d.

103. $\frac{5}{8}° + 4.735° + 8° 17' 32'' = ?$

104. £$\frac{5}{8}$ + £1.65 + £4 12s. 6d. = ?

105. $\frac{7}{22}$ sq. mi. + $\frac{45}{88}$ A. = ?

106. From a firkin of butter containing 44 lb. were sold 18 lb. 10 oz. How much remained?

107. From a cask of oil containing 54 gal. were drawn 5 gal. 2 qt. How much remained?

108. A person who had 7 bu. 3 pk. 2 qt. of grapes sold 3 pk. 4 qt. to one person and 2 pk. 5 qt. to another. What quantity had he left?

109. How much remains of a journey of 54 mi. after 7 mi. $85\frac{1}{2}$ rd. have been traveled?

110. The longest day in Boston is 15 h. 16 min. What is the length of the shortest night?

111. Having 10 sovereigns in my purse, I spent in Liverpool £1 10s. 6d., £2 5s. 10d., and 11s. 6d. How much money had I left?

112. The Hoosac Tunnel is 25,031 ft. long; Mt. Cenis tunnel is $7\frac{5}{8}$ mi. long. What is the difference in feet?

113. The entire excavation for a cellar was 888.88 cu. yd.; the masonry in the cellar walls measured 111 perches, a perch being $16\frac{1}{2}$ ft. by $1\frac{1}{2}$ ft. by 1 ft. What was the number of cubic yards inside the walls?

295. How is the difference in latitude found between two places on the same side of the equator? On opposite sides?

Latitude of:

Chicago, Ill., 42° N.	Rome, Italy, 41° 5′ 54″ N.
Washington, D.C., 38° 51′ 20″ N.	Cape Good Hope, 32° 24′ 3″ S.
San Francisco, 37° 47′ 55″ N.	Cape Horn, 55° 58′ 4″ S.
Quebec, 46° 48′ 17″ N.	London, 51° 30′ 48″ N.

Find the difference in latitude between:

114. Washington and San Francisco.

115. Chicago and Rome. **116.** London and Quebec.

117. Cape Horn and Cape Good Hope.

118. Cape Horn and San Francisco.

119. Washington and Rome.

120. Select from your map two places on the same side of the equator, and find their difference of latitude.

121. Select two places on opposite sides of the equator, and find their difference of latitude.

296. How is the difference of longitude found between two places on the same side of the meridian of Greenwich? On opposite sides?

NOTE. — Whenever the difference of longitude exceeds 180°, it should be subtracted from 360°. Why? Show this by a diagram.

Longitude of:

New York, 74° 0′ 3″ W.	Paris, Fr., 2° 20′ $22\frac{1}{2}$″ E.
Boston, 71° 3′ 30″ W.	Canton, 113° 14′ 0″ E.
London, 0° 5′ 48″ W.	San Francisco, 122° 26′ 48″ W.

Find the difference of longitude between:

122. Boston and New York. **124.** London and Paris.

123. New York and London. **125.** Canton and San Francisco.

297. To find the difference of time between two dates.

Illustrative Example. What is the time in years, months, and days from May 11, 1897, to August 7, 1899?

SOLUTION.—From May 11, 1897, to May 11, 1899, is 2 years; to July 11 is 2 months more. From July 11 to July 31 is 20 days, and to August 7 is 7 days more. *Ans.* 2 yr. 2 mo. 27 da.

298. From the preceding illustration may be derived the following

Rule.

To find the number of years, months, and days between two dates: *First find the number of entire years between the two dates, then the number of full months remaining, and lastly, the remaining days, including the day of the later date.*

NOTE.—The method of finding difference of time by compound subtraction will be found in the Supplement, Art. 15.

Oral Exercises.

299. *a.* What time elapsed between the birth of Benjamin Franklin, Jan. 17, 1706, and that of George Washington, Feb. 22, 1732?

b. America was discovered by Columbus Oct. 12, 1492; the settlement at Jamestown took place May 13, 1607. What time elapsed between these two events?

c. Lafayette was born Sept. 6, 1757, and died May 19, 1834. What was his age at the time of his death?

d. What was Lafayette's age at the time of the Declaration of Independence, July 4, 1776?

e. How old was Washington at the time of his death, which occurred Dec. 14, 1799?

f. How old was Lafayette when Washington died?

g. What time elapsed between the landing of the Pilgrims, Dec. 22, 1620, and the Declaration of Independence?

h. How many years have elapsed since the Declaration of Independence?

i. If a man was born March 5, 1856, how old will he be if alive at the close of the ninetéenth century?

300. *Illustrative Example.* Find the exact number of days from Jan. 7, 1896, to April 10, 1896.

Solution.—There are 24 days remaining in January, 29 days in February (leap year), 31 days in March, and 10 days in April.

$24 + 29 + 31 + 10 = 94$. *Ans.* 94 days.

Note.—Any year is a leap year when the number denoting the year is divisible by 4 and not by 100, and when it is divisible by 400. (See Supplement, Art. 39.)

Find the exact number of days.

j. From April 10 to July 5.

k. From Sept. 3 to Nov. 12.

l. From Oct. 4 to Jan. 1.

m. From Dec. 2 to Feb. 1.

n. From Dec. 6, 1895, to March 8, 1896.

o. From Oct. 24, 1895, to May 12, 1896.

MULTIPLICATION AND DIVISION.

301. *Illustrative Examples.* (1) Multiply 5 oz. 7 pwt. by 4. (2) Divide 5 A. 8 sq. rd. by 3.

WRITTEN WORK.

(1)
$$\begin{array}{r} 5 \text{ oz. } 7 \text{ pwt.} \\ 4 \\ \hline 1 \text{ lb. } 9 \text{ oz. } 8 \text{ pwt.} \end{array}$$

(2)
$$3\overline{)5 \text{ A. } \quad 8 \text{ sq. rd.}}$$
$$1 \text{ A. } 109\tfrac{1}{3} \text{ sq. rd.}$$

(1) 4 times 7 pwt. equals 28 pwt., which equals 1 oz. 8 pwt. Write 8 pwt., and keep the 1 oz. to add to the product of ounces. 4 times 5 oz. equals 20 oz., which, with the 1 oz. kept, equals 21 oz., or 1 lb. 9 oz. *Ans.* 1 lb. 9 oz. 8 pwt.

(2) 5 acres divided by 3 gives 1 acre, with 2 acres remaining. We write the 1 acre and change the 2 acres to square rods, which equals 320 sq. rd. This, with 8 sq. rd., equals 328 sq. rd. 328 sq. rd. divided by 3 gives $109\frac{1}{3}$ sq. rd. *Ans.* 1 A. $109\frac{1}{3}$ sq. rd.

302. Examples for Written Work.

126. How much wood is there in 5 loads, each containing 1 cd. 5 cd. ft.?

127. How much coal is there in 4 loads, each containing 1 ton 1400 lb.?

128. If it takes 3 lengths of a string 36 ft. 9 in. long and 4 ft. 2 in. more to measure the depth of a lot, what is its depth?

129. Find the contents of 250 preserve jars if each jar contains 1 pt. 3 gi.

130. What quantity of peaches is brought into New York in one car load of 450 baskets, if each basket contains 1 pk. 6 qt.?

131. How much earth is removed in 15 loads, each containing 1 cu. yd. $20\frac{1}{2}$ cu. ft.?

132. How much glass is there in 24 windows of 12 panes each, if each pane contains 2 sq. ft. 8 sq. in.?

133. If 4 A. 8 sq. rd. is divided into 6 equal lots, what is the size of each?

134. A farmer brought 5 bu. 3 pk. of corn to mill. If the miller took $\frac{1}{18}$ part as toll, how much did he take?

135. When an express train travels 100 miles in 2 h. 30 min., what is its average time per mile?

136. If a horse is fed 12 quarts of oats a day, how many days will 6 bu. 4 pk. last him?

NOTE.—Change 6 bu. 4 pk. to quarts before dividing.

137. How many barrels, each holding 2 bu. 3 pk., will it take to hold 100 bushels of oysters, and how many pecks remain?

138. A dealer has 3 hogsheads of mineral water, each containing 63 gallons, which he wishes to put into bottles holding 1 gal. 3 qt. 1 pt. each. How many bottles can he fill?

139. Allowing for a horseshoe 14 ounces of iron, how many horseshoes can be made from 60 pounds?

140. How many cups, each weighing 7 oz. 15 pwt., can be made from 60 pounds of silver?

141. Allowing $2\frac{1}{2}$ oz. of gold to a case, how many watch-cases can be made from 2 lb. 10 oz. of gold, and how many rings, each 3 pwt., can be made from the gold remaining?

142. If sound moves 1120 feet per second, how long will it be in crossing a river 3 miles wide?

143. How many times will a carriage wheel, 7 feet 4 inches in circumference, turn in going 0.125 of a mile?

144. The distance from Boston to Portland is 110 miles. How many times will an engine wheel 18 feet 6 inches in circumference turn in going and returning?

145. How many revolutions an hour will a wheel make that turns 300° 40′ in every second of time?

146. In how many days can 12 dozen garments be stitched, allowing 2 h. 15 min. to each, and 8 working hours to a day?

LONGITUDE AND TIME.

303. As the earth turns upon its axis once in 24 hours, it follows that $\frac{1}{24}$ of 360°, or 15° of longitude, must pass under the sun in 1 hour, and $\frac{1}{60}$ of 15°, or 15′, must pass under the sun in 1 minute of time, and $\frac{1}{60}$ of 15′, or 15″, must pass under the sun in 1 second of time. Hence,

A difference in longitude

Of 15° makes a difference of 1 hour in solar time.
Of 15′ makes a difference of 1 minute in solar time.
Of 15″ makes a difference of 1 second in solar time.

All places on any given meridian have the same solar time, which is faster than the time of all places west of that meridian, and slower than the time of all places east of it.

This difference of time caused such inconvenience to the railroads and the traveling public, that in November, 1883, the principal cities and railroads of the United States adopted what is called "**Standard Time.**" This time is now generally used throughout the country.

Standard Time.

304. By the system of standard time, four meridians 15° apart are established as central meridians; reckoning from Greenwich, these are the 75th, the 90th, the 105th, and the 120th. *The local time of each of these meridians is taken as the standard time for all places within 7½ degrees on either side.* Thus there are four belts, in each of which the time by the clock is one hour ahead of that of the belt next west of it, and one hour behind that of the belt next east of it. These time belts are called the Eastern, the Central, the Mountain, and the Pacific belts.

a. Trace on your map the 75th meridian; the 90th meridian. Near what large city does each pass?

b. In the same way trace the 105th meridian.

c. The 120th meridian forms the western boundary of what state?

d. Trace the dividing line between the Eastern and the Central belts, and tell what states lie mostly in the Eastern belt.

e. What states lie mostly in the Central belt? The Mountain belt? The Pacific belt?

f. When it is 10 o'clock by standard time in the Eastern belt, what is the time in the Central belt? The Mountain belt? The Pacific belt?

Longitude and Solar Time.

305. From Art. 303 is derived the following

Rule.

To find the difference of longitude between any two places when the difference of solar time is known: *Multiply the difference of time between the two places, expressed in hours, minutes, and seconds, by 15. The product will express the difference of longitude in degrees, minutes, and seconds.*

Examples for Written Work.

306. What is the difference in longitude between two places, the difference in their solar time being

147. 4 h. 17 m.? **149.** 6 h. 13 m. 10 s.?
148. 2 h. 9 m.? **150.** 1 h. 5 m. 25 s.?

In what longitude from Greenwich is a place whose noon compared with that of Greenwich is

151. 3 hours earlier? **153.** 1 hour 12 minutes later?
152. 5 minutes later? **154.** 4 hours 8 minutes earlier?

155. A and B sailed together from San Francisco. A kept his watch by San Francisco time, and B set his by the sun every day. After 10 days, A's watch was 4 hours 39 minutes faster than B's. In what longitude were they then, the longitude of San Francisco being 122° 26′ 15″ west?

307. From Art. 303 is also derived the following

Rule.

To find the difference in time between any two places when the difference in longitude is known: *Divide the difference in longitude, expressed in degrees, minutes, and seconds, by 15. The quotient will express the difference of solar time in hours, minutes, and seconds.*

Examples for Written Work.

308. Using the longitude given in Article 296, find the difference in time between

156. Boston and Paris. **159.** London and San Francisco.
157. Paris and Canton. **160.** Canton and San Francisco.
158. New York and London. **161.** New York and Canton.

The longitude of Washington is 77° 2′ 48″ W. When it is 12 o'clock by solar time in Washington, what is the time

162. In Paris? **164.** In Canton?
163. In London? **165.** In San Francisco?

What are the kinds of measure of extension? Show how from long measure the measures of surface arise. Show how the units of cubic measure arise.

What is the standard unit of length? Of weight? Of liquid measure? Of dry measure?

Define a circle: circumference; arc· diameter; radius; angle. Name the different kinds of angles. How are angles measured? How are compound numbers added, subtracted, multiplied, and divided? What is the difference between these operations upon compound numbers and upon simple numbers? How is time between two dates found? What years are leap years? How is difference of latitude found? Of longitude?

Describe Standard Time. In which of the four great time belts do you live? What is the difference of longitude between the central meridian of this belt and the city or town in which you live? When it is 12 o'clock by the sun where you reside, what is the standard time?

Review Exercises.

309. In place of x in the following, supply its value:

a. 1 ft. $= \frac{1}{x}$ yd.

b. 1 in. $= \frac{1}{x}$ yd.

c. 1 sq. ft. $= \frac{1}{x}$ sq. yd.

d. 1 sq. in. $= \frac{1}{x}$ sq. yd.

e. 1 cu. ft. $= \frac{1}{x}$ cu. yd.

f. 1 cu. in. $= \frac{1}{x}$ cu. ft.

g. 1 qt. $= \frac{1}{x}$ gal.

h. 1 pt. $= \frac{1}{x}$ gal.

i. 1 gi. $= \frac{1}{x}$ gal.

j. 1 pk. $= \frac{1}{x}$ bu.

k. 1 qt. $= \frac{1}{x}$ bu.

l. 1 oz. Avoir. $= \frac{1}{x}$ lb.

m. 1 oz. Troy $= \frac{1}{x}$ lb.

n. 1 pwt. Troy $= \frac{1}{x}$ lb.

o. 1 gr. Troy $= \frac{1}{x}$ lb.

p. 1 cu. in. $= \frac{1}{x}$ gal.

q. 1 gr. Troy $= \frac{1}{x}$ lb. Avoir.

r. Circumference = diam. $\times\ x$.

Drill Table No. 7. (See Supplement, Art. 1.)

310. For additional practice in compound numbers.

	A.	B.	C.	D.
1.	T.	lb.	$2^{\text{T.}}$ $1428^{\text{lb.}}$ $6^{\text{oz.}}$	$7^{\text{lb.}}$ $8^{\text{oz.}}$
2.	l. T.	lb.	$18^{\text{cwt.}}$ $2^{\text{qr.}}$ $7^{\text{lb.}}$	$4^{\text{cwt.}}$ $3^{\text{qr.}}$ $11^{\text{lb.}}$
3.	lb.*	pwt.	$3^{\text{lb.}}$ $7^{\text{oz.}}$ $10^{\text{pwt.}}$	$11^{\text{oz.}}$ $14^{\text{pwt.}}$
4.	m.	ft.	$34^{\text{rd.}}$ $3^{\text{yd.}}$ $2^{\text{ft.}}$	$5^{\text{yd.}}$ $1^{\text{ft.}}$
5.	sq. m.	sq. rd.	$48^{\text{A.}}$ $9^{\text{sq. rd.}}$	$2^{\text{sq. rd.}}$ $4^{\text{sq. yd.}}$
6.	cu. yd.	cu. in.	$6^{\text{cu. yd.}}$ $15^{\text{cu. ft.}}$ $1506^{\text{cu. in.}}$	$14^{\text{cu. ft.}}$ $329^{\text{cu. in.}}$
7.	cd.	cu. ft.	$92^{\text{cd.}}$ $6^{\text{cd. ft.}}$ $12^{\text{cu. ft.}}$	$18^{\text{cd. ft.}}$ $14^{\text{cu. ft.}}$
8.	gal.	gi.	$4^{\text{gal.}}$ $2^{\text{qt.}}$ $1^{\text{pt.}}$	$3^{\text{qt.}}$ $1^{\text{pt.}}$ (liquid)
9.	bu.	pt.	$5^{\text{bu.}}$ $1^{\text{pk.}}$ $2^{\text{qt.}}$	$4^{\text{pk.}}$ $7^{\text{qt.}}$ $1^{\text{pt.}}$
10.	circ.	(′)	280° 2′ 28″	98′ 14″
11.	y.	hours.	$2^{\text{y.}}$ $7^{\text{d.}}$ $18^{\text{h.}}$	$348^{\text{d.}}$ $3^{\text{h.}}$
12.	rd.	in.	$15^{\text{rd.}}$ $11^{\text{ft.}}$ $8^{\text{in.}}$	$2^{\text{ft.}}$ $9^{\text{in.}}$
13.	A.	sq. yd.	$18^{\text{sq. rd.}}$ $206^{\text{sq. ft.}}$ $9^{\text{sq. in.}}$	$9^{\text{sq. yd.}}$ $110^{\text{sq. in.}}$

Exercises upon the Table.

In each line 1, 2, 3, etc.:

166–178. Change C to units of the lowest denomination in the example.

179–191. Change 132,687 B to higher denominations.

192–204. Change $\frac{8}{11}$ A to B. **205–217.** Add 0.5784 A to D.

218–230. Change 0.4627 A to B. **231–243.** Add $\frac{3}{7}$ A to C.

244–256. Change the numbers of lower denominations in C to a fraction of the highest.

257–269. Change D to a decimal of the highest denomination in C (4 places).

270–282. What part of A is D? **283–295.** Take D from C.

296–308. Multiply D by 15. **309–321.** Divide C by 7.

* Troy.

SECTION XII.

MENSURATION OF SURFACES AND SOLIDS.

Oral Exercises.

311. *a.* How is the area of any rectangular surface found? (Art. 108.) Illustrate by an example and diagram.

b. If the length is given in rods and the width in feet, what must first be done?

c. If a board is 1 ft. 6 in. long and 4 in. wide, what is its area in square inches?

d. When the area and one dimension of a rectangle are given, how can the other dimension be found?

e. What must be the length of a board 8 in. wide to contain 100 sq. in.?

f. What must be the width of a floor 12 ft. long to contain 132 sq. ft.?

g. Chinese matting is 36 in. wide. How many square yards are there in a roll 72 ft. long?

h. Brussels carpeting is $\frac{3}{4}$ yd. wide. How long must a roll be to contain 60 sq. yd.?

312. *i.* How is the volume of any rectangular solid found? (Art. 115.) Show this by an example.

j. When the contents and two dimensions of a rectangular solid are given, how do you find the other dimension?

k. What must be the depth of a cistern 4 ft. long and 3 ft. wide to contain 60 cu. ft. of water?

l. What must be the height of a room 20 ft. long and 15 ft. wide to contain 3000 cu. ft. of air?

SQUARES AND OTHER RECTANGLES.

313. Examples for Written Work.

1. How many bricks 8 in. long and 4 in. wide, laid flatwise, must be used to build a walk 4 ft. wide and 100 ft. long?

2. What must be paid for a concrete walk 5 rd. long and 5 ft. wide, at 90¢ per square yard?

3. What is the price of a building lot in Brooklyn 25 ft. wide and 80 ft. deep, at $8 a square foot? What is the value of the land per front foot?

4. How much money will a man be worth who owns a quarter of an acre of land worth 20¢ a foot?

5. My neighbor's garden is 100 ft. square; mine contains 100 sq. ft. What is the difference in size?

6. A and B have each a garden containing 10,000 sq. ft. A's is 200 ft. by 50 ft., and B's is 100 ft. square. C is to fence both at 28¢ per running foot. How much should A pay? How much should B pay? (Make diagrams of both gardens.)

7. A building lot contains $\frac{1}{8}$ of an acre, is rectangular, and measures on the street 45 ft. What is its depth?

8. A rectangular park contains 17.76 acres and is 310.33 yd. long. What is its breadth?

9. What is the area of the upper surface and sides of a marble slab 3 ft. $3\frac{3}{4}$ in. by 2 ft. 2.6 in. and 2.5 in. thick?

10. A map drawn to a scale of one inch to $3\frac{1}{2}$ miles, is 5 ft. 2 in. by 3 ft. 7 in. What area in square miles does it represent?

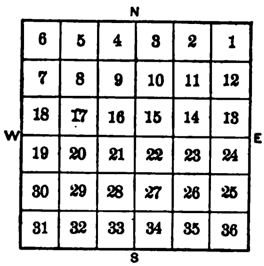
11. How many acres were covered by Machinery Hall at the World's Fair in Chicago, 1893, the main building measuring on the floor 846 ft. by 492 ft., and the annex 550 ft. by 49 ft.?

12. Find the cost for flooring the main building above, at 25¢ per hundred sq. ft. of floor for work, and $12 per thousand sq. ft. for boards, allowing 1 tenth additional stock for waste.

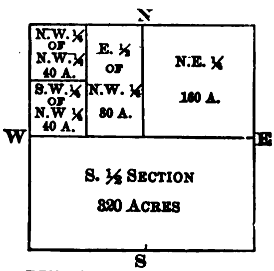
GOVERNMENT LANDS.

314. The United States public lands, before being offered for sale, are divided by parallels and meridians into rectangular tracts, called townships, each being as nearly as practicable 6 miles square, or 23040 acres in extent.

The lines bounding a township extend due north and south, and east and west, and a line on a parallel of latitude is always established as a *base*. A line of townships extending north and south is called a **range**. The ranges are designated by their numbers east or west of the principal meridian, and the townships in each range by their number north or south of the base line.



SECTIONS OF A TOWNSHIP



DIVISIONS OF A SECTION

Townships are subdivided into **sections** 1 mile square, or 640 acres, and sections into half-sections, quarter-sections, half-quarter sections, and quarter-quarter sections or lots.

Lots which for any reason are irregular in form are designated as Lot 1, 2, 3, 4, etc., of a particular section. City and village plots are subdivided into blocks, and these again into smaller lots.

1 township	= 36 sq. mi.	= 23040 acres	
1 section	= 1 "	= 640 "	
1 half-section	= $\frac{1}{2}$ "	= 320 "	
1 quarter-section	= $\frac{1}{4}$ "	= 160 "	
1 half-quarter section	= $\frac{1}{8}$ "	= 80 "	
1 quarter-quarter section	= $\frac{1}{16}$ "	= 40 "	

a. What must be paid for the N.E. quarter of section 11 of a South Dakota township at $3 per acre?

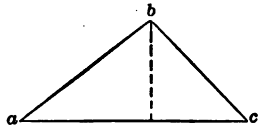
13. Among how many people may a township be divided so that each may receive a quarter-quarter section?

14. If a speculator buys the northern half of a section of land and sells at various times the N.E. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, the N.W. $\frac{1}{4}$ of N.E. $\frac{1}{4}$, the S.E. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, and the N.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$, how many acres has he left? Draw a diagram to show what parts of his half-section he has left.

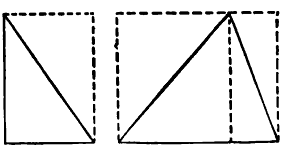
15. A real estate agent bought section 24 of township 2 north, range 8 west, at \$5 an acre. He sold the S. $\frac{1}{2}$ section at \$7 an acre, the N.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$ at \$8 an acre, the S.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$ at \$7.50 an acre, and the N.E. $\frac{1}{4}$ of N.W. $\frac{1}{4}$ at \$6.50 an acre. How many acres did he sell? What part of the section had he left? How much did he gain, valuing the portion he retained at its original cost?

TRIANGLES.

315. A plane figure bounded by three straight lines is a **triangle.** Any side upon which the triangle is supposed to stand, as *ac*, in the triangle *abc*, is its **base.** A line extending from the angle opposite to the base of the triangle and perpendicular to the base is the **height** or **altitude** of the triangle.



316. Every triangle is half a rectangle of the same base and height. Hence the area of a triangle is equal *to half the product of the number of units in the base by the number of like units in the height;* or as it is often expressed, *one half of the base multiplied by the height.*



a. Draw a triangle 3 inches long and 2 inches high, and show that it is half of a rectangle of the same base and height.

b. How many square inches are there in your triangle?

317. Examples for Written Work.

16. Find the area of triangles having the following dimensions. Give the answers in square feet.

Base 30 ft., height 15 ft.
Base $12\frac{1}{2}$ rd., height $14\frac{3}{8}$ ft.
Base 12 yd. 2 ft., height 5 yd. 1 ft.

17. What must be the height of a triangle that contains 2250 sq. ft., and whose base is 180 ft. in length?

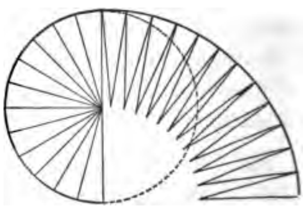
18. How many square feet are there in a triangular flower bed whose base is 18 ft. and the height 8 ft. 6 in.?

19. Mary is to crochet a half square breakfast shawl, each of its two equal sides being $1\frac{1}{4}$ yd. long. How many square feet will she crochet?

20. Find the surface of an octagonal spire, each of whose eight faces is $19\frac{1}{2}$ ft. at the base and $51\frac{3}{8}$ ft. high.

CIRCLES.

318. A circle may be considered as made up of triangles the sum of whose bases is the **circumference** of the circle and whose height is the **radius**. Hence the area of a circle equals *the product of the circumference by half the radius.*



What is the area of a circle

21. Whose circumference is 18.8496 ft. and radius 3 ft.?

22. Whose circumference is 23.562 ft. and diameter $7\frac{1}{2}$ ft.?

319. In every circle the circumference very nearly equals *the diameter multiplied by* 3.1416. How, then, when one dimension of a circle is given can the other be found?

Note. — For many purposes $3\frac{1}{7}$ or $\frac{22}{7}$, instead of 3.1416, is accurate enough. Work the examples in this article with the decimal value.

What is the length of the circumference of a circle

23. When the diameter is 2 ft.? 4 ft. 3 in.?

24. When the radius is 5 in.? 11 yd.?

NOTE. — Draw diagrams to illustrate the following examples:

25. What is the distance around a circular pond that is 100 ft. across?

26. What is the diameter of a circle, the circumference being 100 ft.? 50 rd.?

27. What is the radius of a circle whose circumference is 500 ft.? 12 ft. 9 in.?

Find the area of a circle when

28. The diameter = 4 rd.

29. The radius = 5 ft. 4 in.

30. The circumference = 560 ft.

31. The circumference = 480 ft.

32. Over how many square feet can a cow feed when tethered so that her head reaches 20 ft. from the stake?

33. The trunk of the largest tree now standing in California has a circumference of 106 ft. What is the distance through and what is the area of a cross section?

34. How many feet of surface has a circular mirror inside a frame which has a breadth of 4 in., and whose outside edge is the circumference of a circle whose diameter is 4 ft. 5 in.?

35. How many square feet are there in the frame of the above mirror?

320. RECTANGULAR SOLIDS.

36. How many cubic inches are there in a block of marble 1 ft. long, 8 in. wide, and 5 in. thick, and what is its weight at 0.098 of a pound to the cubic inch?

37. What is the weight of a block of granite 8 ft. long, $1\frac{1}{2}$ ft. wide, and $1\frac{1}{4}$ ft. thick, at 165 lb. to the cubic foot?

38. What must be the length of a beam 16 in. by 22 in. to contain $115\frac{1}{2}$ cu. ft.?

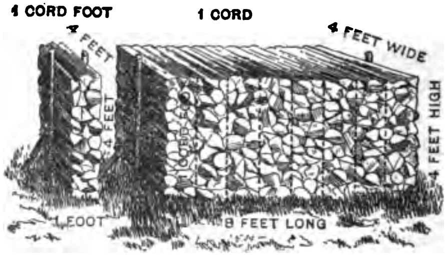
39. How many cubic yards of earth must be removed in digging a cellar 20 ft. long, 18 ft. wide, and 10 ft. deep?

WOOD MEASURE.

321. Wood is usually cut for marketing into 4 feet sticks. Two lengths of such wood piled 4 ft. high and 4 ft. wide contain how many cubic feet?

128 cu. ft. is a cord (cd.), used principally in measuring wood.
16 cu. ft., or $\frac{1}{8}$ of a cord, is a cord foot (cd. ft.).

The following illustration represents 1 cd. and 1 cd. ft.



Examples for Oral and Written Work.

322. *a.* How many cords of seaweed are in a pile 4 ft. high, 4 ft. wide, and 16 ft. long? 20 ft. long? 40 ft. long?

b. At $6 a cord for wood, what is the cost of 4 cd. ft.? of 12 cd. ft.? of 6 cd. ft.?

At $4 a cord for wood, find the cost of a load measuring

c. 4 ft. by 4 ft. by 4 ft. *e.* 8 ft. by 4 ft. by 1 ft.

d. 4 ft. by 4 ft. by 2 ft. *f.* 9 ft. by 4 ft. by 4 ft.

40. What is the cost at $3.50 per cord of a load of wood 8 ft. by 4 ft. by 3 ft.? 12 ft. by 6 ft. by 4 ft.?

41. If wood is cut in lengths of $4\frac{1}{4}$ ft. and piled 4 ft. high, how long must the pile be to contain a cord?

42. If wood cut 3 ft. 9 in. long was sold for 4 ft. wood, at $5.00 a cord, what deduction should be made in the bill for what was sold for 20 cd.?

LUMBER.

323. The unit of measure for boards, plank, joist, and timber is a square foot one inch in thickness called a **board foot.**

NOTE. — Unless otherwise specified, all square lumber is reckoned as at least 1 in. thick; all over 1 in. to $1\frac{1}{4}$ in. as $1\frac{1}{4}$ in.; all over $1\frac{1}{4}$ to $1\frac{1}{2}$ in. as $1\frac{1}{2}$ in.; all over $1\frac{1}{2}$ in. to 2 in. as 2 in.; all over 2 in., according to its thickness. Thus, a plank 20 ft. long, 9 in. wide, and 2 in. thick contains $20 \times \frac{9}{12} \times 2 = 30$ board feet.

Give the contents of the following lumber in board feet:

43. 3 boards 12 ft. by 12 in.
44. 2 boards 16 ft. by 6 in.
45. 6 boards 14 ft. by 8 in.
46. A 2 in. plank 16 ft. by 8 in.
47. Joist 20 ft. by 6 in. by 6 in.
48. Joist 30 ft. by 6 in. by 4 in.
49. Scantling 100 ft. by 3 in. by 2 in.
50. Scantling 75 ft. by 3 in. by 3 in.
51. Find the cost of boards $\frac{7}{8}$ of an inch thick required to lay a platform 75 ft. by 48 ft., at \$18 per M. (thousand ft.).

What is the cost of the following lots of square timber:

52. 200 ft. 8 in. by 6 in., at \$20 per M.?
53. 450 ft. 10 in. by 12 in., at \$24 per M.?
54. 285 ft. 14 in. by 14 in., at \$30 per M.?
55. 56 ft. 16 in. by 16 in., at \$$22\frac{1}{2}$ per M.?
56. How many feet are there in 12 boards, each 14 ft. long, 15 in. wide at one end, and 11 in. wide at the other?

NOTE. — For the width take the average of 15 in. + 11 in.

57. How many feet has a plank 20 ft. long and $2\frac{1}{2}$ in. thick, 14 in. wide at one end and 9 in. wide at the other?
58. What will be the cost of a pine plank 20 ft. long, 30 in. wide, $2\frac{1}{4}$ in. thick, at \$45 per M.?
59. An unfinished stone tower $27\frac{1}{2}$ ft. by $27\frac{1}{2}$ ft. was provided with a temporary roof, the four sides ending in a point 15 ft. from their bases. How many board feet would be required for the roof, not allowing for waste?

LATHS, SHINGLES, AND CLAPBOARDS.

324. **Laths** are 4 ft. long, $1\frac{1}{2}$ in. wide, and are usually laid $\frac{3}{8}$ in. apart at the sides, and close together at the ends.

60. How many square yards will be laid by a bundle of 100 laths?

61. How many bundles, 100 laths each, will be required to lath a ceiling 16 ft. by 15 ft.?

325. **Shingles** are 16 in. long and of various widths, but in reckoning, each width of 4 in. is considered one shingle; a bundle contains 250 shingles. Shingles are ordinarily laid from $\frac{1}{4}$ to $\frac{1}{3}$ to the weather.

NOTE. — A shingle 16 in. long, laid $\frac{1}{3}$ to the weather is $5\frac{1}{3}$ in. to the weather, and therefore covers $5\frac{1}{3}$ in. $\times$ 4 in. of surface. A shingle 16 in. long laid $\frac{1}{4}$ to the weather covers 4 in. $\times$ 4 in. of surface.

62. When shingles are laid 4 in. to the weather, how many shingles will cover the two sides of a roof each 40 ft. long and 25 ft. wide?

63. How many square feet of surface will 1000 shingles cover when laid $\frac{1}{3}$ to the weather? How many square feet will they cover if laid $\frac{1}{4}$ to the weather?

64. When 1 bundle of shingles is allowed for 30 sq. ft., how many bundles will be required for a roof 20 ft. long and 18 ft. wide, and what will they cost at $2.50 a thousand?

326. **Clapboards** are sometimes cut 4 ft. long and 6 in. wide, and sold in bundles of 25 each.

65. When clapboards are laid $3\frac{1}{2}$ inches to the weather, how many square feet will 6 bundles cover?

66. How many clapboards laid $3\frac{1}{2}$ in. to the weather will be required to cover 140 sq. ft.?

67. How many will be required to cover the side of a house 40 ft. long and 20 ft. high?

PAPERING.

327. Wall papers are usually $\frac{1}{2}$ yd. wide and sold in rolls each containing 8 yd. The paper is laid up and down the walls.

NOTE 1.—In reckoning the amount required to cover the walls, without allowing for windows and doors:

Find the distance in yards around the room, and reckon 2 strips for every yard. Divide this number of strips by the number that can be made from a roll. The quotient will be the number of rolls required.

NOTE 2.—In estimating the amount of paper necessary to cover a certain area, a whole roll is allowed for any part of a roll required

NOTE 3.—Bordering is sold by the yard.

68. How many rolls of paper will be required to paper a room 18 ft. long by $16\frac{1}{2}$ ft., 8 ft. from the baseboard to the ceiling, making no allowance for windows and doors?

69. How many rolls would be required for the ceiling of the above room?

70. How many rolls will be required to paper a hall $16\frac{1}{2}$ ft. by 6 ft., 7 ft. between baseboard and ceiling, deducting for a door and a window, each $3\frac{1}{2}$ ft. wide?

71. Measure and estimate the number of rolls of paper that would be required to paper one side of your schoolroom, deducting the width of windows and doors.

CARPETING.

328. *Illustrative Example.* How many yards of carpeting, laid lengthwise of the floor, will be required to carpet a room 18 ft. by 14 ft., no allowance being made for matching; the width of the carpeting being 1 yd. How many yards $\frac{3}{4}$ of a yard wide will be required?

SOLUTION.—(1) A floor 18 ft. long will require 6 yd. of yard wide carpeting to each breadth; the room being 14 ft. wide, it will require $4\frac{2}{3}$ breadths, practically 5 breadths. Hence, to cover the floor with yard wide carpeting will require 5 times 6, or 30 yards.

(2) Carpeting being $\frac{3}{4}$ of a yard wide, it will require as many breadths to carpet the above floor, laid lengthwise, as there are times $\frac{3}{4}$ yd. in 14 ft., which is $6\frac{2}{9}$ times; practically it will require 7 breadths. Hence, to cover the floor with carpeting $\frac{3}{4}$ yd. wide will require 7 times 6, or 42 yards.

NOTE 1. — Since $\frac{2}{9}$ of a breadth is less than a half, the breadth can be split, in which case $6\frac{1}{2}$ breadths will suffice. There is usually some waste in matching the patterns of carpets, which must be allowed for.

NOTE 2. — In each of the following examples, draw a plan of the floor showing how the carpeting should be laid down.

72. How many yards of carpet 1 yd. wide will be required to carpet the above room if it is laid crosswise?

73. How many yards of carpet $\frac{3}{4}$ yd. wide, laid crosswise, would be required to carpet the room?

74. Which way is it best to lay a carpet 1 yd. wide on a floor 21 ft. by 19 ft. so that no breadth may be split or turned under, and how many yards will be required, no allowance being made for matching figures?

75. If the above carpet is laid the other way and a breadth split, how many yards will be required?

76. If the floor in Example 74 is carpeted with Wilton $\frac{3}{4}$ of a yard wide, and laid lengthwise, how many yards will be required, one breadth being split?

77. Measure the floor of your schoolroom, and estimate the number of yards that would be required to carpet it with carpeting 1 yd. wide. Also find the number of yards required to carpet it with carpeting $\frac{3}{4}$ of a yard wide.

STONE AND BRICK WALLS.

329. Bricks of common size are 8 in. long, 4 in. wide, and 2 in. thick.

In laying walls the mortar occupies about $\frac{1}{5}$ of the space, so that 22 bricks to a cubic foot is a fair estimate.

Stone work is estimated by the perch, which is a rod ($16\frac{1}{2}$ ft.) long, $1\frac{1}{2}$ ft. wide, and 1 ft. thick, and which contains $24\frac{3}{4}$ cu. ft. In ordinary calculations the perch is considered to be 25 cu. ft.

330. Examples for Written Work.

78. How many bricks of the common size equal 1 cu. ft.?

79. How many bricks 4 in. by 8 in. will be required to pave a court 20 ft. long and 10 ft. wide?

80. How many perches, exact measure, are there in 4 stone piers 20 ft. by 12 ft., and 15 ft. high?

NOTE. — In the following examples, allow 22 bricks to a cubic foot.

81. What will be the cost, at $ 10.50 per M., of common brick required for building party walls 7 ft. high 8 in. thick to separate into three cellars the basement of a building 50 ft. wide inside measure?

82. What will be the cost, at $ 11 per M., of common brick to build the foundation and party walls of a block of three stores, each having a frontage of 25 ft., two adjacent stores to be 80 ft. deep, the other 75 ft. deep, the walls to be 7 ft. high and 1 ft. thick, no allowance for openings?

NOTE. — Allow exact length of front and rear walls, but deduct from the end and party walls two feet for the thickness of the front and rear walls.

83. What will be the cost, at 90¢ a perch (25 cu. ft.), for a trench wall of stone 2 ft. wide 18 in. deep under the walls of the above block?

84. Find the cost, at $ 2 per M., for laying the brick of a cistern 12 ft. by 15 ft. inside measure, the walls being 9 ft. high, $1\frac{1}{2}$ ft. thick.

To fill out the corners of the above cistern, how many times the thickness of the wall must be added to the total length of the four sides?

NOTE. — In estimating the amount of stock required for a building, a deduction of $\frac{1}{2}$ of the openings for doors and windows is usually made.

85. A bank building of brick is 40 ft. by 34 ft. outside; the walls are 15 ft. high and average 2 ft. thick. There are 6 windows, each 4 ft. by 7 ft., and 1 door 7 ft. 10 in. by 8 ft., for which areas a deduction of 1 half is to be made. What is the cost of brick, at $ 12 per M.?

CAPACITY OF BINS, CISTERNS, ETC.

331. In estimating roughly the capacity and volume of bins, cisterns, etc., there are in common use certain **units of approximate** value in capacity, volume, and weight.

1 T. of coal (2000 lb.)	= about 35 cu. ft.	
1 bbl. ($31\frac{1}{2}$ gal.)	= about $4\frac{1}{5}$ cu. ft.	
1 bu. (2150.42 cu. in.)	= about $1\frac{1}{4}$ cu. ft.	
1 bu. heaped measure	= about $1\frac{1}{2}$ cu. ft.	
1 gal. of water (231 cu. in.)	= about $\frac{2}{15}$ cu. ft.	= $8\frac{1}{3}$ lb. weight.
1 pt. of water		= about 1 lb.
1 cu. ft.	= about $\frac{4}{5}$ of a bushel	= about $7\frac{1}{2}$ gal.
1 cu. ft. of water	= 1000 oz.	= $62\frac{1}{2}$ lb.

In measuring bulky fruits and vegetables, as apples and potatoes, the measures are heaped. In measuring small fruits, grain, etc., the measures are evened, or stricken with a straight edge.

Examples for Written Work.

332. Roughly estimate the following:

86. How many bushels of grain can be put in a bin 4 ft. by 3 ft. by 5 ft.?

87. How many bushels of apples can be put in the above bin?

88. How many tons of coal can be put in a bin 7 ft. by 5 ft. by 2 ft.?

89. How many cubic feet are there in a cistern holding 1000 gal.?

90. How many pounds do 10 cu. ft. of water weigh?

91. What must be the depth of a box 15 in. by 20 in., to hold 3 bu. 3 pk. of rye?

92. A can of linseed oil 16 in. by 14 in. by 9 in. contains how many gallons?

93. What is the weight of the above oil, it being 0.94 as heavy as water?

94. A jar weighs 8 lb. when empty and 60 lb. when filled with water. How many gallons does it hold?

333. Miscellaneous Examples.

Including tests for advanced pupils.

95. What will 6400 sheets of paper cost at \$7.80 a ream?

96. How many rods of fencing are required to inclose a square lot, each side of which is $234\frac{1}{4}$ feet long?

97. How many furrows, each 20 inches wide, must be made in plowing lengthwise a lot of land 8 rd. 10 ft. wide?

98. What is the width of a rectangular field whose length is $217\frac{1}{4}$ feet, and the area is $44,644\frac{1}{8}$ square feet?

99. A quantity of silver weighed 4 lb. 10 oz. 3 pwt. before refining, and 3 lb. 11 oz. 2 pwt. 9 gr. afterwards. What weight was lost in the process?

100. If I burn 120 pounds of coal in a day, and buy my coal by the long ton at \$6.25 per ton, what is the cost for December and January?

101. John Adams was born Oct. 30, 1735, and died at the age of 90 y. 8 mo. 4 d. What was the date of his death?

102. Dating the commencement of the Civil War in the United States at April 12, 1861, and its close at May 26, 1865, how long did it continue?

103. Make a bill against William Rice, and receipt it, for $2\frac{1}{2}$ gross of pencils at \$2. . gross, $7\frac{1}{2}$ quires of paper at \$2.40 a ream, and 300 envelopes at \$0.90 per thousand.

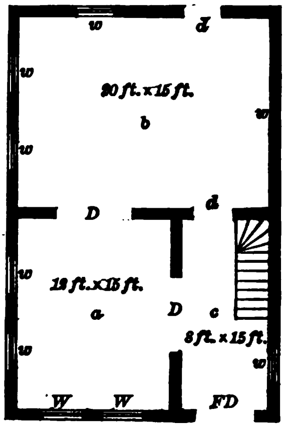
104. How many gallons of water will be contained in a tank 3 feet square, if the water is 5 ft. 4 in. deep?

105. Change 15 lb. 8 oz. Avoirdupois to Troy weight.

106. Add $\frac{3}{4}$ of the month of February, 1893, to $\frac{1}{8}$ of the days from March 21 to June 19.

107. If a factory can make 1200 yards of cloth per hour, how many yards could be made by working 10 hours a day from July 7th to January 4th, allowing for 26 Sundays?

108. How many perches of masonry (25 cu. ft.) are there in a wall 100 ft. long, 10 ft. high, $1\frac{1}{2}$ ft. thick? How many bricks, 22 to a cubic foot, will construct this wall?



Here is the floor plan of some rooms in a cottage. The rooms are 9 feet high.

Rooms. *a*, Study; *b*, Sitting-room; *c*, Hall.

Doors. *FD*, the front door, 5 ft. by 8 ft.; *D*, *D*, 6 ft. by 7 ft.; *d*, *d*, $3\frac{1}{2}$ ft. by 7 ft.

Windows. *W*, *W*, $3\frac{1}{2}$ ft. by $7\frac{1}{2}$ ft. The others, 3 ft. by 6 ft.

Baseboards. All the baseboards 9 inches high.

NOTE.—Plastering, painting, and kalsomining are usually computed by the square yard.

109. Find the number of square feet in the baseboard of the study, passing under all the windows.

110. Find the number of square feet in the windows and doors of the study.

111. How many square feet are in the walls and ceiling of the study, deducting for baseboards, doors, and windows?

112. At 40¢ per sq. yd., find the cost of plastering the study walls, deducting half the area of the doors and windows.

113. What is the cost of kalsomining the ceiling of the study and sitting room at 9¢ per square yard?

114. How many square feet of glazing in the above plan, *W*, *W* having each 4 panes 42 inches by 18 inches, and the others having each 4 panes 31 inches by 15 inches?

115. How many yards of carpet, 1 yard wide, will be required to carpet the study? How many yards $\frac{3}{4}$ of a yard wide will be required, no allowance being made for matching figures?

116. How many rolls of paper 18 inches wide, 3 strips to a roll, will be required to paper the study, deducting the width of the windows and doors, and reckoning a whole roll for any part of a roll required? (Art. 327.)

117. If an 18 inch border of hard wood is laid around the study, how many square feet of hard wood are laid?

NOTE. — The pupil will be aided by first drawing a plan of the flooring.

118. What must be the dimensions of carpeting to fit the remainder of the above floor, and how many breadths of Wilton $\frac{3}{4}$ of a yard wide, and laid lengthwise, will be required? How much will the carpet cost at \$1.625 per yard?

119. How many yards of bordering, 9 inches wide, will be required for a rug measuring before the border is put on 3 ft. by $2\frac{1}{2}$ ft.?

120. How many square yards of linoleum will be required for the hall, deducting for the staircase 10 ft. in length and $4\frac{1}{2}$ ft. in width?

Make similar problems about the sitting room and its furnishings.

121. If a piazza is built 8 ft. wide on the front of the house in the plan above, and 6 ft. wide on the side, how many square feet are there in the floor of the piazza, reckoning the corner also, the house measuring outside 32 ft. in length and 22 ft. in width?

122. If the excavation for the cellar of the above cottage was 22 ft. wide and 33 ft. long, what would be the cost of laying the stone wall 7 ft. high 18 in. thick, at 25 cents a perch of 25 cubic feet, no allowance being made for corners or openings?

123. This cottage stands upon a lot 12 rd. 2 ft. long, and 6 rd. 1 ft. wide. What was the cost of the land at 5 cents a square foot?

124. What was the cost of grading the above lot at \$1.75 per square of 100 square feet?

125. At 50 cents each, find the cost of planting trees, 25 feet apart in the boundary of the lot, beginning at a corner?

126. A pitched roof, each side of which is 25 ft. wide and 32 ft. long, is to be covered with slates 16 in. by 12 in. How many slates will it take, allowing them to lap one half?

127. What weight of slates will the above roof have to support, the average thickness of the slate being $\frac{1}{6}$ in., and the weight being 2.11 times as heavy as water? (Art. 331.)

128. How many cords of wood can be put into a shed whose inside measure is 10 ft. by 14 ft. 8 in. and 9 ft. high, a space of 6 ft. square from the floor upward being reserved?

129. From a pile of wood 58 ft. long, 4 ft. high, and 4 ft. wide, were sold at one time $2\frac{5}{8}$ cords, at another $3\frac{1}{4}$ cords. How much is the remainder worth at $5 a cord?

130. A man purchased 75 cords of wood for $360; he sold the following lots, $10\frac{1}{2}$ cords, 15 cords, and $11\frac{3}{4}$ cords, all at $5 per cord. How much did he gain on what he sold?

131. At 20 cents per cord for each sawing, what would be the cost of sawing the remainder of the 75 cords, so that every stick should make four pieces?

132. If the first Atlantic cable weighed 19 cwt. a mile, what was the weight of the entire cable in long tons, the distance from Valentia to Newfoundland being 2435 miles?

133. A farmer divided one half of his land of 540 acres 16 square rods equally between his two daughters, and the balance, after setting off $16\frac{3}{4}$ acres, equally between his two sons. What was the share of a daughter? of a son?

134. What must I pay for a dozen silver spoons, each weighing 2 oz. 9 pwt., at $1.25 per ounce?

135. How many bushels of buckwheat will a bin contain that is 8 ft. long, 5 ft. wide, and 3 ft. deep? (Art. 331.)

136. To what height will 62 bu. 2 pk. of grain fill a bin 10 ft. long and 8 ft. wide?

137. In grading my lot which is 100 ft. square, I used the material dug from a cellar 60 ft. by 24 ft., and 8 ft. deep. What was the depth of the grading of the lot?

138. How many square feet does the surface of a block contain which is 5 ft. 6 in. long, 3 ft. wide, and 2 ft. 4 in. thick?

139. A yacht race took place in English waters Aug. 4, 1894, which resulted as follows. Yachts started at 10 o'clock 30 min. 5 sec. The Vigilant completed the course, which was 48 miles, at 2 o'clock 36 min. 15 sec., the Britannia at 2 o'clock 41 min. 43 sec. By how much did the Vigilant beat her rival? What was her average rate per hour?

140. How much corduroy $\frac{3}{4}$ yd. wide will cover the top and sides of a block 3 ft. long, 9 in. wide, and 6 in. high? No allowance for waste.

141. At 15¢ per pound, what is the cost of lead, 5 lb. to a square foot, to line a tank 6 ft. by 4 ft., and 5 ft. deep?

142. The earth turns upon its axis once in 24 hours. How many degrees does it turn in $16\frac{3}{8}$ hours?

143. It is said that 13,000,000 bricks were put into the Art Palace at the World's Fair in 1893. How many cubic feet of brick were there? (See Art. 329.)

144. How many common bricks 22 per cubic foot will it take to build a wall 38 ft. long, 8 ft. high, and $1\frac{1}{4}$ ft. thick, allowing 1 fifth of the wall for mortar?

145. The circular dome of the Horticultural building at the World's Fair was 180 ft. in diameter. What was its circumference? (Art. 319.)

146. What must be the diameter in yards of a trotting park that the circumference may be 1 mile around?

147. What will be the area of the above park in acres?

148. At $36 per thousand, what is the cost of chestnut plank $2\frac{1}{4}$ in. thick to cover a circular cesspool, the cover to be 9 ft. in diameter, and $\frac{1}{8}$ of the material purchased being allowed for waste?

149. How many paving stones 8 in. square will be required to pave 10 rd. of a street that is 40 ft. wide?

150. Estimate the cost of feeding a yoke of oxen during the winter of 1895 and 1896, hay being $13.75 per ton, one ox weighing 1772 lb., the other 1431 lb., allowing $\frac{1}{50}$ of their weight in hay per day?

151. What was gained on 200,000 ft. of lumber by buying at $9.80 per thousand and selling at $1\frac{7}{8}$¢ per foot?

152. How many feet of boards will be required to inclose and separate, with a tight board fence 4 ft. high, 3 rectangular lots lying side by side, each 5 rd. wide in front and 10 rd. deep, the fronts being in the same straight line?

153. From a barrel of 42 gallons of kerosene costing 8¢ per gallon, there were sold at 10¢ per gallon 15 gal. 2 qt., 18 gal. 3 qt. 1 pt., and 4 gal. 3 qt. 1 pt. The rest wasted by evaporation and leakage. What was the gain or loss on the barrel?

154. How much will it cost to tin a roof 9 ft. by 17 ft., allowing 12 in. all around additional for gutters at $8 per square of 100 ft.?

155. A walk along a block of 50 house lots averaging 4 rd. in front, was laid with 1650 flagstones 4 ft. square. What was the width of the walk?

156. In a schoolroom 28 ft. 6 in. by 30 ft. 8 in. and 11 ft. 10 in. high, there are 48 pupils. How many cubic feet of air are allowed for each pupil?

A schoolroom should have from $\frac{1}{4}$ to $\frac{1}{6}$ as much lighting surface as floor surface. Calculate these surfaces in the following rooms still in existence, and find the fractional part which the lighting surface is of the floor surface.

157. First room, 40 ft. × 28 ft.; 6 windows, each 12 lights 10 in. by 12 in. of glass.

158. Second room, 24 ft. × 28 ft.; 4 windows, each 12 lights 10 in. by 15 in. of glass.

159. Third room, 35 ft. × 32 ft.; 6 windows, each 12 lights 12 in. by 15 in. of glass.

160. Calculate the floor and lighting surface in your own schoolroom, and find how much more or less than $\frac{1}{8}$ of the floor surface the lighting surface is.

161. In a shower, 2772 cu. in. fell on a flat roof 14 ft. long and 11 ft. wide. What was the depth of the rainfall?

162. If a vessel beats to windward $5\frac{1}{3}$ knots per hour, how long will it take her to get ahead 15 mi.? (Art. 277.)

163. If a bird can fly 1° in 1 h. 8 min. 15 sec., in what time could it fly around the world at the same rate?

164. What decimal of a mile in depth is a body of water which, when sounded, was found to be 220 fathoms deep?

165. In what time will a vessel go through a strait 2 mi. long, if she is carried ahead by tide 30 ft. a minute, by wind 25 ft. a minute, and by steam 100 ft. a minute? In what time can she go through the strait against wind and tide?

How is the area of a *rectangle* found? When the area and one dimension are given, how is the other dimension found? How are the United States public lands divided preparatory to selling them?

What is a *triangle?* What is its base? its altitude? How is the area of a triangle found?

To obtain a rule for finding the area of a circle, of what plane figures may we consider it as made up? How is the area of a circle found? When the circumference of a circle is given, how can you find the diameter? the radius?

How do you find the contents of a *rectangular solid?* When the volume and two dimensions of a rectangular solid are given, how do you find the third?

What can you tell about *wood measure?* about *measurement of lumber?* of *laths? shingles? clapboards?* How are *wall papers* usually sold? What have you learned about papering a room? about carpeting floors?

What is the common size of bricks? How many bricks are estimated to a foot? How many cubic feet are there in a perch of stone? What units of approximate value in *capacity*, *volume*, and *weight* can you name that are in common use?

SECTION XIII.

PERCENTAGE.

334. *Illustrative Examples.* (1) A nursery man set out young trees in his nursery, but lost 15 out of every 100 of them from a drought. How many hundredths of his trees did he lose?

(2) Another nurseryman lost 9 out of every 50 of his trees. What part of his trees did he lose? How many hundredths?

(3) Another lost 4 trees out of every 25. How many hundredths of his trees did he lose? Compare his loss with that of each of the others.

In these questions the number 100 trees is used as a standard of comparison. It is usual in such questions to employ the phrase per cent, or the sign %, in place of the words "hundredths," "out of every 100," etc. Thus we should say that the first nurseryman lost 15 per cent (15%) of his trees; the second lost 18 per cent (18%), and the third 16 per cent (16%).

NOTE.—The phrase "per cent" is from the Latin *per* (by) and *centum* (hundred).

Oral Exercises.

335. *a.* A man having \$100 lost \$10. What per cent of his money did he lose?

b. An uncle promised to give his nephew \$5 for every \$25 the nephew would save out of his earnings. How much would be given for \$100 saved? The money given would be equal to what per cent of the money saved?

c. A farmer had 200 sheep, but sold 28 of them. What part of his sheep did he sell? How many hundredths? What per cent?

d. School was in session 200 days last year, and Amelia was present 180 days. What part of the time was she present? What per cent of the time?

e. In a certain school of 400 pupils, 100 pupils are under 10 years of age. What part are under 10 years of age? What per cent?

f. In a grove of trees, 3 trees in every 5 are oak. How many trees in 100 are oak? What per cent are oak?

g. The rest of the trees are pine. What per cent of the trees are pine?

h. Jack is catching fish, but loses 1 out of every 4 that bite. How many does he lose out of 100 that bite? What per cent does he lose?

336. The base and rate being given to find the percentage.

Illustrative Example. A nurseryman set out 2520 young trees, but lost 15 per cent of them from drought. How many trees did he lose?

WRITTEN WORK.

$$\begin{array}{r} 2520 \\ \underline{0.15} \\ 12600 \\ \underline{252} \\ 378.00 \end{array}$$

Losing 15 per cent is the same as losing 0.15 of his trees. 0.15 of 2520 is 378.

Ans. 378 trees.

337. Percentage is that part of a number which is found by taking a number of hundredths of the number.

In the example, 378 trees, being 15 hundredths of 2520 trees, is a percentage of 2520 trees.

338. The **base** of the percentage is the whole number a part of which is taken as a percentage.

In the example, 2520 trees is the **base**.

339. The number of hundredths the percentage is of the base is the **rate per cent,** or, briefly, the **per cent.**

Thus, in the example, the *rate per cent* is 15, and the *per cent* of trees lost is 15.

340. The word **rate** used alone means the fraction by which the base is multiplied to produce the percentage.

Thus, in the example, the *rate* is 15 per cent, or 15 %, or 0.15, or $\frac{15}{100}$, or $\frac{3}{20}$.

NOTE.—The distinction between *rate* and *rate per cent* is important; and neglect of it is often a source of confusion. The *rate* may be expressed in various forms, as 25 per cent, 25 %, 0.25, $\frac{25}{100}$, $\frac{5}{20}$, $\frac{1}{4}$, $\frac{3}{12}$, $\frac{15}{60}$, etc. But the *rate per cent* is only the numerator of the fraction which, with the denominator 100, would express the rate. Thus, the *rate* being $\frac{25}{100}$, the *rate per cent* is 25.

341. From the foregoing examples is derived the following

Rule.

To find the percentage: *Multiply the base by the rate.*

342. The rule is thus expressed as a formula:

Percentage = Base × Rate.

Oral Exercises.

343. *a.* What is 4% of $700? 5% of $600?
b. What is 2% of 900 bushels? 12% of 200 days?
c. Find 1% of $2000; of $4000; of $7000.
d. Find 2% of $1000; of $3000; of $5000.
e. Find 3% of $1000; of $2000; of $5000.
f. Find 4% of $5000; of $7000; of $9000.
g. Find 5% of $4000; of $6000; of $8000.
h. Find 6% of $9000; of $12000; of $20000.
i. Find 8% of $7000; of $9000; of $12000.
j. What is 1% of $213? of $340? of $768?

k. What is 2% of \$150? of \$225? of \$320?
l. What is 5% of \$120? of \$160? of \$210.
m. What is 8% of \$200? of \$500? of \$250?
n. What is 10% of 90 sheep? 1% of 50 pounds?
o. What is 7% of 60 yards? 8% of 25 tons?
p. What is 9% of 80 cents? 6% of \$5?
q. What is 9% of \$10? 8% of 6 ounces?
r. What is 7% of 9 quarts? 12% of \$300?
s. Find 3% of 40 acres; 4% of 90 dollars.
t. Find 2% of 300 bushels; 5% of 20 gallons.

344. The decimal form is not always the simplest form in which the rate can be expressed.

Thus $25\% = 0.25 = \frac{25}{100} = \frac{1}{4}$; and it is shorter to multiply by $\frac{1}{4}$ (that is, divide by 4) than to multiply by 0.25.

Again, $12\frac{1}{2}\% = 0.125 = \frac{125}{1000} = \frac{1}{8}$; and it is shorter to multiply by $\frac{1}{8}$ (that is, divide by 8) than to multiply by 0.125.

345. In the following examples, express the rate as a decimal and as a common fraction in smallest terms, and decide which form will make shorter work in multiplying.

a. 25%.	*e.* $37\frac{1}{2}\%$.	*i.* $66\frac{2}{3}\%$.	*m.* 10%.	*q.* 60%.
b. 50%.	*f.* $62\frac{1}{2}\%$.	*j.* $8\frac{1}{3}\%$.	*n.* 20%.	*r.* 90%.
c. 75%.	*g.* $87\frac{1}{2}\%$.	*k.* $16\frac{2}{3}\%$.	*o.* 30%.	*s.* 110%.
d. $12\frac{1}{2}\%$.	*h.* $33\frac{1}{3}\%$.	*l.* $83\frac{1}{3}\%$.	*p.* 40%.	*t.* 150%.

346. *Illustrative Examples.* What per cent of a number is $\frac{1}{2}$ of it? $\frac{1}{4}$ of it? $\frac{3}{4}$ of it?

Solution. — $\frac{1}{2}$ changed to hundredths $= \frac{50}{100} = 0.50 = 50\%$, *Ans.*

u. What per cent of a number is $\frac{1}{3}$ of the number? $\frac{2}{3}$? $\frac{1}{5}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$?

v. What per cent of a number is $\frac{1}{6}$ of the number? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{11}$? $\frac{1}{12}$? $\frac{1}{20}$? $\frac{1}{25}$?

w. What per cent of a number is $\frac{4}{5}$ of the number? $\frac{5}{6}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{3}{10}$? $\frac{7}{10}$? $\frac{9}{10}$?

x. What per cent of a number is $\frac{5}{12}$ of the number? $\frac{7}{12}$? $\frac{11}{12}$? $\frac{3}{20}$? $\frac{7}{20}$? $\frac{9}{20}$? $\frac{3}{25}$? $\frac{7}{50}$? $\frac{23}{50}$?

Examples for Written Work.

347. *Illustrative Example.* A piece of land was sold for \$563.84, and 27% of the price was paid in cash. How much was paid in cash?

WRITTEN WORK.

$$\begin{array}{r} \$563.84 \\ \underline{0.27} \\ 3946\,88 \\ \underline{11276\,8} \\ 152.23\,68 \end{array}$$

The base, \$563.84, is multiplied by the rate, 0.27, to produce the percentage. The answer is to be expressed to the nearest cent.

Ans. \$152.24.

348. What is

1. 35% of \$263.50?
2. 57% of \$9856?
3. 68% of 648 tons?
4. 95% of 185 days?
5. 88% of 225 gallons?
6. 78% of 1250 men?

7. The annual rent of a house is fixed at 8% of its cost. If the house cost \$4500, how much is the rent?

8. In an army corps of 9250 men, 82% were reported fit for duty. How many men were reported fit for duty?

9. A dealer purchased 2480 tons of coal, but sold 25% of it the same day. How many tons did he sell?

10. A merchant bought flour at \$4.96 per barrel, and sold it so as to gain $12\frac{1}{2}$%. How much money did he gain on one barrel?

11. Find $37\frac{1}{2}$% of \$16,000.
12. Find $62\frac{1}{2}$% of \$800.
13. Find 75% of \$432.
14. Find $16\frac{2}{3}$% of \$6000.

15. A merchant sold goods to the amount of $ 5286 to a customer who afterwards became bankrupt, and paid only 42 %. How much did the merchant get for his goods?

16. From the amount of a bill of goods 5 % is deducted for cash payment. How much is deducted if the bill amounts to $ 396.42?

17. Add together 10 % of $ 25.34, 20 % of $ 19.83, 30 % of $ 28.64, and 40 % of $ 33.42.

18. Which is the greater number, and how much, 66⅔ % of 2481, or 60 % of 2760?

19. Find the difference between 87½ % of $ 482.96 and 90 % of $ 469.80.

20. The firm of Holbrook and Stevens, dealers in leather, being embarrassed in business, made an assignment, and the assignee in making settlement of claims against the firm was able to pay 67 %. How much did one creditor receive whose claim was $ 643.92?

21. Another creditor, whose claim was $ 10,592.60?

22. Another creditor, whose claim was $ 8000?

23. A trader bought a lot of standing grass, and agreed to pay for cutting it and making it into hay 30 % of what he should get for the hay when sold in the market. He sold 12 tons at $ 21.50, 6 tons at $ 18.75, and 15 tons at $ 19.25 per ton. How much did he pay the haymakers?

24. A store which cost $ 6500 is rented for a year at 12½ % of its cost, through an agent who receives 5% of the rental for services. How much does the agent receive?

25. A dealer in grain bought 12,000 bushels of wheat at 73 ¢. He sold 50 % of it at 78 ¢, 20 % of it at 71 ¢, and the rest of it at 65 ¢ per bushel. Did he gain or lose on the whole, and how much?

26. The silver ore from a certain mine is assayed and found to yield 8 % of pure silver. How many pounds does a ton (2240 pounds) of this ore yield? What is the value of this yield at 63¾ cents an ounce Troy? (Art. 267.)

349. To change a fraction to a per cent.

Illustrative Example. What per cent of a number is $\frac{74}{289}$ of it?

WRITTEN WORK.

(1)

```
289)74.00(0.25 175/289
    57 8
    16 20
    14 45
     1 75
```

(2)

```
289)74.00(0.25605
    57 8
    16 20
    14 45
     1 750
     1 734
        1600
        1445
         155
```

(1) $\frac{74}{289}$ changed to hundredths $= 0.25\frac{175}{289}$. The result, $25\frac{175}{289}\%$, is accurate, but not in convenient form.

(2) A more convenient form, where only an approximate result is wanted, is found by continuing the division one, two, three or more places, according to the degree of approximation required.

Always give the answer to the *nearest* tenth, the *nearest* hundredth, or the *nearest* thousandth, as may be required. Thus, the answer to this example, expressed to the nearest tenth, is 25.6%; expressed to the nearest hundredth, 25.61%; and to the nearest thousandth, 25.606%.

350. Change the following fractions to per cents, expressing the results to the nearest tenth of one per cent:

27. $\frac{85}{344}$. **29.** $\frac{128}{519}$. **31.** $\frac{225}{582}$. **33.** $\frac{153}{294}$.
28. $\frac{92}{431}$. **30.** $\frac{68}{362}$. **32.** $\frac{97}{312}$. **34.** $\frac{231}{252}$.

Change the following fractions to per cents, expressing the results to the nearest hundredth of one per cent:

35. $\frac{145}{295}$. **37.** $\frac{92}{188}$. **39.** $\frac{48}{237}$. **41.** $\frac{342}{351}$.
36. $\frac{63}{97}$. **38.** $\frac{99}{385}$. **40.** $\frac{441}{892}$. **42.** $\frac{324}{945}$.

Change the following fractions to per cents, expressing results to the nearest thousandth of one per cent:

43. $\frac{77}{81}$. **44.** $\frac{87}{98}$. **45.** $\frac{389}{721}$. **46.** $\frac{356}{718}$.

47. The sum of the following fractions, $\frac{237}{3196}$, $\frac{943}{3196}$, $\frac{861}{3196}$, $\frac{522}{3196}$, $\frac{633}{3196}$, is 1, as may easily be seen by adding the numerators. Find what per cent (to the nearest tenth) each fraction is equal to, and see how near the sum of these per cents comes to 100%.

351. The base and rate per cent being given, to find the amount or the remainder.

Illustrative Examples. (1) A had $ 3250, and by trading gained a sum of money equal to 24% of this. How much money did he gain? What amount of money did he then have?

(2) B had $ 3250, and by trading lost a sum of money equal to 16% of this. How much money did he lose? What was the remainder of his money?

WRITTEN WORK.

(1) Base,	$ 3250	(2) Base,	$ 3250
Rate,	0.24	Rate,	0.16
	13000		19500
	650		325
Percentage, gain,	780.00	Percentage, loss,	520.00
	3250		3250
Amount,	$ 4030	Remainder,	$ 2730.

Ans. A gained $780, and then had $ 4030.

Ans. B lost $520, and then had $ 2730.

If only A's amount and B's remainder had been required, the examples might have been worked by regarding the amount or the remainder as a percentage of the base and computing it directly, thus:

(1) $\$\,3250 \times 1.24 = \$\,4030.00$, amount.

(2) $\$\,3250 \times 0.84 = \$\,2730.00$, remainder.

(1) The money A had at first is the base, and is 100% of itself. Adding the gain, equal to 24% of the base, we have the amount equal to 124% of the base. (2) The money B had at first is the base, and is 100% of itself. Subtracting from this the loss, equal to 16% of the base, we have the remainder equal to 84% of the base.

352. From these examples are derived the following

Rules.

1. To find the amount: *Add the percentage to the base;* or, *Multiply the base by 1 plus the rate.*

2. To find the remainder: *Subtract the percentage from the base;* or, *Multiply the base by 1 minus the rate.*

353. These rules are thus expressed as formulas:

1. **Amount = Base + Percentage = Base × (1 + Rate).**

2. **Remainder = Base − Percentage = Base × (1 − Rate).**

354. Examples for Written Work.

48. A tree 25.3 feet high last year has grown 32% in height this year. What is its height now?

49. What will be its height next year if it again grows 32%?

50. A merchant investing $48,000 in business, gains in three years 72% of his capital. How much has he at the end of that time?

51. A lent B $850 on condition that B should repay the money at the end of a year, together with a sum of money equal to 6% of the loan for the use of it. What amount of money should B pay to A at the end of the year?

52. What is 128% of $36,520?

53. What is 135% of 26,500 head of cattle?

54. If a piece of cloth 53 yards long shrinks 13% when wet, what is its length when wet?

55. How much remains to be paid on a bill of goods amounting to \$342.85 when 37% of it has been paid?

56. What is left of an estate worth \$37,500 when 68% of it has been squandered?

57. A grocer has \$15,268.76 charged on his books against customers, but expects to lose 12% of this in bad debts. How much does he expect to collect?

58. From a cargo of 500 casks of molasses the leakage is allowed to be $7\frac{1}{2}$%. The casks contain on the average $42\frac{1}{2}$ gallons each. What is the net amount of molasses after leakage has been deducted?

59. A distance of $1\frac{3}{4}$ miles is to be walked in 2 hours. How many feet remain when 65% of the whole distance has been walked? How many minutes remain when 65% of the whole time has been spent?

60. A ranchman has 12,000 sheep upon his ranch. Supposing the rate of increase to be 60% each year, how many will he have at the end of one year? of two years?

61. The rate of increase in the population of a certain city being 2.45% each year, what will be the total population a year hence if it is now 500,000? What will it be three years hence?

Oral Exercises.

355. The percentage and base being given, to find the rate.

Illustrative Example. What per cent of 180 is 90?

Solution.—90 is $\frac{1}{2}$ of 180; and $\frac{1}{2}$ is 50%.

a. What per cent of 40 gallons is 10 gallons?

b. What per cent of 1 gross is 4 dozen?

c. What per cent of time is lost, if 6 days out of 20 are lost?

d. A milkman buys milk at 5¢ and sells it at 7¢ a quart. What per cent does he gain?

e. Fred had 90 chickens, but during a cold rain storm 18 of them died. What per cent of his chickens did he lose?

f. A young tree 15 feet high last year grew this year 9 feet. By what per cent did its height increase?

g. What per cent of 50 is 40? Of 40 is 50?

356. *Illustrative Examples.* (1) A school boy was present 361 half-days during the year, and school kept 380 half-days. What was the boy's per cent of attendance?

(2) In the same school there was a girl who attended 375 half-days. What was her per cent of attendance?

WRITTEN WORK.

(1)	(2)
380)361.00(0.95	380)375.000(0.9868
342 0	342 0
19 00	33 00
19 00	30 40
	2 600
	2 280
	3200
	3040
	160

(1) The boy was present $\frac{361}{380}$ and the girl $\frac{375}{380}$ of the time. Changing these fractions to hundredths (Art. 349), we find that the boy was present 0.95, or 95% of the time. *Ans.* 95%.

(2) The girl's per cent of attendance was 98.7, or 98.68, according to the degree of approximation with which we wish to express it. *Ans.* 98.7%, or 98.68%.

357. From these examples is derived the following rule.

To find the rate per cent: *Divide the percentage by the base, carrying the division to hundredths.* Or,

To find the rate: *Divide the percentage by the base.*

358. The rule is thus expressed as a formula:

Rate = Percentage ÷ Base.

NOTE.—This rule may be established in another way, thus: The percentage is the product of the base and the rate. Therefore, if the percentage be divided by the base (one of its two factors), the quotient will be the rate (the other factor).

359. Examples for Written Work.

62. What per cent of a long ton (2240 lb.) is a short ton (2000 lb.)? What per cent of a short ton is a long ton?

63. From casks containing 12,236 gallons of oil, 523 gallons leaked out. What per cent of the oil was lost?

64. What per cent of an acre of land is covered by a house 40 feet square? by a garden 8 rods square?

65. What per cent of $ 926.50 is $ 117.25?

66. What per cent of $ 23,856 is $ 8,922?

67. A quantity of white paper was made from 2500 pounds of cotton rags and 700 pounds of linen rags. What per cent of the paper is linen?

68. A farm consists of 230 acres of woodland, 135 acres of pasture, 50 acres of marsh land, and 85 acres of tillage land. What per cent of the whole farm is each part?

69. Three men, A, B, and C, formed a partnership. A put in $ 12,000, B $ 15,000, and C $ 18,000. What per cent of the whole capital was each partner's share?

70. During one week in school 250 words were given out to be spelled. William spelled 235 words correctly, Francis 241, Mabel 246, and Caroline 248. What per cent of the words did each spell correctly?

71. In making brown bread, if $4\frac{1}{2}$ cups of rye meal are mixed with $6\frac{1}{2}$ cups of corn meal, what per cent of each kind of meal does the bread contain?

72. To 80 gallons of spirit containing 95% pure alcohol are added 40 gallons of water. What per cent of the mixture is pure alcohol?

73. The pupils in a certain school are divided into eight grades. In the first grade there are 123 pupils; in the second 111; in the third 97; in the fourth 88; in the fifth 86; in the sixth 83; in the seventh 66; and in the eighth 48. What per cent of the pupils are in each grade? How near to 100% does the sum of your results come?

74. If 35 gallons of spirit containing 78% pure alcohol is mixed with 48 gallons of spirit containing 83% pure alcohol, what per cent of pure alcohol will the mixture contain?

75–88. In the following table are given the population in 1890 and the population in 1880 of the ten largest cities in the United States and of a few others that increased rapidly between the same years. Find the per cent of increase (to the nearest tenth of one per cent) in the population of each city.

Cities.	Population 1890.	Population 1880.
New York, N.Y. . . .	1,515,301	1,206,299
Chicago, Ill.	1,099,850	503,185
Philadelphia, Pa. . . .	1,046,964	847,170
Brooklyn, N.Y.	806,343	566,663
St. Louis, Mo.	451,770	350,518
Boston, Mass.	448,477	362,839
Baltimore, Md.	434,439	332,313
San Francisco, Cal. . .	298,997	233,959
Cincinnati, Ohio . . .	296,908	255,139
Cleveland, Ohio . . .	261,353	160,146
Minneapolis, Minn. . .	164,738	46,887
Omaha, Neb.	140,452	30,518
St. Paul, Minn.	133,156	41,473
Denver, Colo.	106,713	35,629

Oral Exercises.

360. The percentage and rate being given, to find the base.

a. A grocer sold 50 barrels of flour, which was 25% of all he had. How many barrels of flour had he?

Solution. — If 50 barrels was 25% of all he had, 1% was 2 barrels, and 100% was 200 barrels. *Ans.* 200 barrels.

b. In an orchard there are 60 cherry trees. How many trees are there in the orchard if the cherry trees are 20% of the whole number?

c. If 5% of my money is $30, how much have I?

d. $40 is 8% of what sum? $24 is 6% of what sum?

e. Of how much time is 48 days 12%?

f. 35 is 5% of what number?

g. If $360 is 20% more than the price of a piano, what is the price of the piano?

SOLUTION. — If $360 is 20% more than the price of the piano, it must be 120% of that price; 1% of the price is $\frac{1}{120}$ of $360, or $3; and 100%, or the whole price, is $300.

h. $240 is 20% more than what sum of money?

i. 250 yards is 25% more than what distance?

j. If $360 is 10% less than the rent of a house, what is the rent of the house?

SOLUTION. — If $360 is 10% less than the rent, it must be 90% of the rent; 1% of the rent is $\frac{1}{90}$ of $360, or $4; and 100%, or the whole rent, is $400.

k. $240 is 20% less than what sum of money?

361. *Illustrative Example.* A man sold 208 acres of land, which was 32% of all he owned. How much did he own?

WRITTEN WORK.

$208 \div 0.32 = 650$. *Ans.* 650 acres.

Since 208 is a percentage, produced by multiplying the base by the rate, 0.32 (Art. 341), the base can be found by dividing 208 by 0.32.

362. Hence the following

Rule.

To find the base: *Divide the percentage by the rate.*

363. The rule is thus expressed as a formula:

Base = Percentage ÷ Rate.

364. Examples for Written Work.

89. 815 pounds is 5% of what weight?

90. $94.05 is 5% of how much money?

91. If 15% of the cost of a factory is $7500, what is the whole cost?

92. 54 inches is 3% of what distance?

93. 428 hours is 8% of how much time?

94. $680.40 is 105% of what sum of money?

95. 8343 square feet is 9% of how many square yards?

96. 125% of what sum of money is equal to $1035?

97. $\frac{9}{16}$ is 3% of what number?

98. $\frac{7}{10}$ is $3\frac{1}{2}$% of what number?

99. Of what number is 1728 cubic inches a percentage, if the rate is 36%?

100. The school census of a certain city shows 78,324 children of school age (5 to 15 years); if these children form 20% of the whole population, what is the population?

101. Of what number is 348.3 miles a percentage, if the rate is $13\frac{1}{2}$%?

102. A horse was sold for $258, which was an advance of 20% on the cost. How much did the horse cost?

103. A grocer sold sugar for 115% of its cost and made $\frac{3}{4}$ of a cent a pound. How much did the sugar cost?

104. A house is sold for $4500, which is 10% less than it cost to build. How much did it cost to build?

105. $540 is 10% less than the cost of a pair of horses. How much did the horses cost?

106. A stock of goods has been damaged by smoke and water to the extent of 40%, and is now valued at $4500. What was their original value?

107. After a spendthrift has wasted 70% of his property he has $9000 left. How much had he at first?

108. A boy is 14 years old, and his age is $12\frac{1}{2}$% less than that of his brother. How old is the brother?

109. After the wages of a factory girl had been cut down 10%, she received \$1.44 a day. What were her wages before the reduction?

110. The number of pupils present in school on a certain day is 651, which is 7% below the number belonging to the school. What is the number belonging to the school?

111. A piece of cloth having shrunk 10% by being wet is 45 yards long. What was its length before it was wet?

112. A broker charges $\frac{1}{4}$% for buying railroad shares. How many shares at \$100 each did he buy for me, if I paid him \$62.50?

113. A man's weight is 180 pounds, and he is 20% heavier than his brother. What is the weight of his brother?

114. 1860 is 25% more than what number?

115. \$2000 is 5% less than what sum of money?

116. From my deposit in a savings bank I drew out 25%; of this I have used for repairs on my house, \$114.51, which is 55% of what I drew out. What remains in the bank?

117. A mechanic paid out $27\frac{1}{2}$% of his year's wages for rent, $7\frac{1}{2}$% for coal; 43% for family expenses; laid by $12\frac{1}{2}$% in the savings bank; and had the rest, amounting to \$142.50, for pocket money. How much did he earn in the year? How much did he spend for rent? for coal? for family expenses? How much did he save?

118. The wages of stone masons were raised 10%, and afterwards reduced 10%, when they received \$3.96 a day. What did they receive before these changes?

119. The wages of carpenters were reduced 10%, and subsequently raised 10%, when they received \$3.96 a day. How much did they receive before these changes?

120. A town, by levying a tax of $1\frac{1}{5}$% on the taxable property of its citizens, raises \$240,000. What is the whole amount of taxable property in the town? What is the amount of property owned by a citizen who pays a tax upon it of \$168?

PROFIT AND LOSS.

Oral Exercises.

365. *a.* By selling goods at 25% above cost, how much is gained if the cost was \$12? \$32? \$50?

b. By selling goods at 10% below cost, how much is lost if the cost was \$20? \$60? \$85?

c. At what price must tea which cost 40¢ a pound be sold to gain 20%? 25%? $12\frac{1}{2}$%? $33\frac{1}{3}$%?

d. At what price must damaged flour which cost \$6 a barrel be sold to lose 10%? 5%? 35%? $66\frac{2}{3}$%?

e. A quantity of wheat was bought for \$750. What would be gained if the market price should advance 10%? What would be lost if the market price should fall 5%?

f. What per cent is gained if sheep, costing \$5 a head are sold for \$6? for \$7? for \$10?

g. What per cent will be lost if cheese, costing 15¢ a pound, is sold for 12¢ a pound? for 10¢? for $7\frac{1}{2}$¢?

h. A lumber merchant paid \$27 per thousand feet for boards at the mill; paid \$5 per thousand for freight, insurance, and other expenses, and sold the same at \$40 per thousand. What per cent did he gain?

i. What was the cost of cloth on which, if sold at \$1.20 a yard, there will be a gain of 50%? $33\frac{1}{3}$%? 20%?

j. What was the cost of hay on which, if sold at \$12 a ton, there will be a loss of 25%? 40%?

k. What was the cost of flour if a loss of 24¢ on a barrel means a loss of 6%? 4%? 3%?

l. What was the cost of a horse upon which, if sold, at \$20 above cost, there is a gain of 10%? 5%? 4%?

366. The difference between the cost of anything and the price for which it is sold is a **profit** or a **loss**.

367. A profit or a loss is usually reckoned as a **percentage**, the **cost** being taken for the **base**.

368. Examples for Written Work.

121. A sheep farm in Texas which cost $7950 was sold so as to gain 17%. What was received for it?

122. A lot of timber was bought for $852. For what must it be sold to gain 35%?

123. A dealer bought 2000 pairs of boots for $7000. For what price a pair must he sell them to gain 30%?

124. A dealer sold three car loads of corn at 15% profit, and gained $279. How much did the corn cost?

125. By selling a schooner for $27,600, there was a gain of 15% on the cost. What was the cost?

126. A tract of land, having increased in value 150%, is now worth $22,500. What was it worth before?

127. A piano was sold at a loss of $28\frac{1}{2}$% for $286. How much did the piano cost?

128. A grocer pays 42¢ a pound for butter, and sells it for 50¢. What per cent does he gain?

129. What is the per cent of gain if a lot of live stock costing $5400 is sold so as to gain $1269?

130. Paper bought at $2.10 per ream is sold at 25¢ per quire. What per cent is gained?

131. A merchant bought 320 barrels of beef at $12.00 per barrel; paid 65¢ per barrel for storage and insurance, and sold the whole for $4594.48. What per cent did he gain?

132. A grain dealer bought wheat at 80¢, barley at 65¢, corn at 62¢, and oats at 46¢ per bushel. At what price must he sell each kind of grain to gain 20%?

133. A hardware dealer bought 6 gross door knobs at $18 per gross, and sold half of them at 20¢ and half at 25¢ apiece. How much did he gain, and what per cent?

134. Two horses were sold for $180 each. On one there was a gain of 20%, and on the other a loss of 20%. Was there gain or loss on the whole transaction, and if any, how much, and what per cent?

135. If $ 2000 was paid for goods, half of which were sold for $ 645, and the rest for $ 585, what per cent was lost?

136. What was the original value of the sixteenth part of a ship which, selling at an advance of 35%, brings $ 3456? How much did the whole ship increase in value?

137. A beef packer bought 150 head of cattle at $ 37.25 per head, 40 head at $ 39.50, and 80 head at $ 39.75. He sold them all for $ 11,175.30. What per cent did he gain?

138. A man buys books at 40% below the list price and sells them at 15% below. What per cent does he gain?

139. What per cent is gained by buying coal by the long ton (2240 lb.) and selling it by the short ton (2000 lb.), at the same price per ton?

140. A storekeeper marks his goods 25% above cost, and then takes off 10% from his asking price in making sales. What per cent does he gain?

COMMISSION.

369. An **agent** is a person authorized to transact business for another person, who is called the **principal.**

Agents have different names according to the nature of the authority given them, or the kind of business they transact. A **factor** or **commission merchant** buys and sells merchandise for others but usually has temporary possession of the goods, which he is bound to care for with reasonable prudence (by insuring, preserving from damage, etc.) as if they were his own. A **broker** is employed to negotiate purchases and sales of merchandise or other property, and usually has no possession of it; but a **stock-broker** has possession and care of the stocks and bonds which he buys and sells. A **collector** is an agent employed to collect debts.

370. When factors, commission merchants, brokers, or other agents are allowed for their services a percentage on the amount of the purchases, sales, or collections, this percentage is called **commission.**

371. The remainder, after the commission and other charges for the care and sale of the goods have been deducted from the gross receipts, is the **net proceeds.**

372. When commission is reckoned as a percentage, *the base is the amount of money expended or received by the agent in transacting business for the principal.*

373. Examples for Written Work.

141. A factor in Australia purchased for the Saxonville Mills in Massachusetts a quantity of wool for which he paid \$9500. What is his commission at $1\frac{1}{8}\%$?

142. A commission merchant sold 4250 yards of calico at $3\frac{7}{8}$¢ per yard, 3580 yards gingham at $8\frac{5}{8}$¢ per yard, and 2520 yards blue jean at $9\frac{1}{8}$¢ per yard. What was his commission at $2\frac{1}{4}\%$?

143. An auctioneer, whose terms are 5% commission, sells 75 yards carpet at \$1.25, 60 yards carpet at 95¢, 1 rug \$28, 1 rug \$47.50, and 5 pairs lace curtains at \$5.50. What is the amount of his commission?

144. A commission and shipping house in Havana write me that, following my instructions, they have purchased 63,000 pounds of raw sugar f. o. b. (free on board ship) at $2\frac{1}{16}$¢, and request me to remit an amount of money that will cover this purchase and their commission of $2\frac{1}{4}\%$? What amount must I remit?

145. A real estate broker, who charges 4% commission, receives \$224 for selling a house and preparing the papers necessary for the transfer of the property. What price is paid for the house? How much does the former owner receive?

146. A collection agent collected \$2537.50 on overdue accounts, and he remitted this amount, less his commission of 15%, to his client. How much did he remit to his client?

147. If $8240 is sent to an agent to cover the amount of his purchases and his commission thereupon of 3%, what is the amount of his purchases?

NOTE. — The amount sent is 103% of the amount of the purchases.

148. What per cent is an agent charging if, out of a remittance of $3308.85, he reserves $68.85 before making the purchases required of him?

149. If out of a remittance of $2328.75 I keep my commission of 3½% on the purchase, how much barley at 45¢ a bushel can I buy with the remainder?

150. A commission merchant, after selling the goods consigned to him, remitted the amount received less his commission of 4½% to the consignor. If the latter received $2244.25, what was the amount of the sales?

151. As commission merchant I sell for my principal 4500 thousand feet of lumber at $13.50 per thousand to B. B. Field & Co., and take their notes on 1, 2, and 3 months' time, which notes I indorse, thus becoming responsible for their payment. For making the sale my commission is 1%; and for guaranteeing payment my charge is 3½%. What do I earn by the whole transaction?

152. A commission house, whose sole business is to sell the entire product of certain cotton factories, and to guarantee payments for the same, sells during the year cotton cloths to the amount of $3,597,846.75. What does the house receive, the commission for selling being 2½%, and that for guaranteeing payments 2½% more?

COMMERCIAL DISCOUNT.

374. **Commercial** or **trade discount** is a percentage deducted from the price of goods or from the amount of a bill of goods for a cash payment, or to enable the purchaser to sell at a profit without asking more than the market price, or for any other reason. To induce earlier settlement, debts and claims are sometimes reduced by a percentage.

375. Examples for Written Work.

153. What is the asking price of cloth which, after a discount of 5% has been made, sells for $1.14 per yard?

154. A merchant bought goods on 60 days' credit, but offered cash for a 10% discount on the bill of $2500. How much would he pay if his offer should be accepted?

155. A shipper agrees to send his goods by a certain railroad if he can be allowed a discount (called a rebate) of 20% upon his freight bills. How much will he save on a bill amounting to $532.75?

156. What is the gross amount of a freight bill which, after a rebate of 15%, amounts to $198.05?

157. What is the net amount of a bill of $257.50 for goods at list prices subject to trade discounts of 20% and 10%, and 5% off for cash?

NOTE.—This means that a discount of 20% is first made from the face of the bill; then from the remainder a discount of 10% is made; and finally from the last remainder a discount of 5% is made. These are successive discounts.

158. A bill for hardware amounting in gross to $2537.75 is subject to successive discounts of 40%, 10%, and 5%. What is the net amount?

159. A bill for books amounting to $523 is reduced to what net amount by discounts of 20%, 10%, and 5%?

160. A dealer bought 100 baby carriages at $5.50 each, trade discounts 30% and 10%, and 5% off for cash. What was the net amount of the bill?

161. A retail trader buys certain goods at discounts of 30%, 10%, and 8%, and sells them at list prices. What per cent does he gain?

162. A merchant buys goods at discounts of 40%, 10%, and 5% from list prices, and sells at 5% below list prices. What per cent does he gain?

163. If the price of paper is $1.08 per ream after discounts of 20% and 10%, what is the list price?

164. Which is better for the buyer, and how much, a direct discount of 25%, or successive discounts of 10%, 10%, and 5%?

165. What single discount is equivalent to the successive discounts 25%, 20%, and 5%?

166. A dealer having bought goods at list prices less discounts of 20%, 10%, and 5%, sells them at 35% above net cost prices. At what per cent below list prices does he sell?

INSURANCE.

376. A, the owner of a house, pays B a small percentage of its value, B, on his part, contracting to make good to A any loss that may arise within a specified time from damage or destruction of the house by fire.

Such a contract is a contract of **insurance.** B is the **insurer**, or **under-writer**, and A is the **insured.** Also A's house is said to be insured.

377. **Insurance** is a contract by which the insurer undertakes to pay the insured for any loss or damage resulting from an uncertain event.

NOTE. — Among such events are fires in buildings, storms at sea, sickness, personal injury, death, etc. Hence arise different kinds of insurance, as fire insurance, marine insurance, accident insurance, health insurance, life insurance and many other kinds.

378. The written contract is called the **policy.** The price paid for insurance is the **premium.**

379. The premium is a percentage of which the amount insured is the base.

NOTE 1. — When property is insured, the amount insured on the property is made somewhat less than the full value of the property.

NOTE 2. — Policies are renewed yearly, or once in five years, or at other stated times, and the premium is paid in advance.

NOTE 3. — There may be a charge, usually of one dollar, for writing the policy; but no notice is taken of this in the following examples.

380. Examples for Written Work.

167. What must be paid for a policy of $ 2500 on the books and furniture in a house, the rate being $\frac{3}{8}\%$?

168. My house, valued in the policy at $ 4000, is insured for 5 years at a premium of 2%. What is the premium, and what does my insurance cost me each year?

169. A hotel is insured for $ 90,000 at $2\frac{1}{8}\%$ for 3 years. What is the cost of insurance each year?

170. A stock of goods in a store valued at $ 193,239 is insured for $\frac{2}{3}$ its value at $1\frac{1}{4}\%$. What is the amount of the premium?

171. What premium is paid for insurance on a lot of silks on board steamer invoiced at $ 25,500 and insured for $\frac{2}{3}$ the invoiced value at $\frac{3}{4}\%$?

172. A shipowner owns 50 ships, which may be supposed to be all of equal value. Which would be the more expensive for him, to pay 2% a year for insurance on all his ships, or to lose a ship every two years?

173. What is the rate of insurance when a policy for $ 26,500 costs $ $496.87\frac{1}{2}$ premium?

174. What is the rate of insurance when a policy for $ 325,000 costs $ 6093.75 premium?

175. What is the value of a stock of goods, if $ 280 is paid for insurance on $\frac{2}{3}$ of its value at $1\frac{3}{4}\%$?

176. A lot of lumber which cost $ 30,000 is insured for $ 20,000 at $5\frac{1}{2}\%$. If it should be totally consumed by fire, what would be the loss to the owner? To the insurers?

177. A cattle dealer paid $ 187.50 for insurance on 500 head of cattle at $1\frac{1}{4}\%$. What was the value of the cattle per head?

178. What is the average rate of insurance paid by the owner of a schooner who obtains one policy for $ 5000 at $1\frac{3}{4}\%$, another for $ 7500 at $1\frac{1}{8}\%$, another for $ 4500 at $1\frac{7}{8}\%$, and another for $ 3000 at $2\frac{1}{8}\%$, all for one year?

179. A cargo of wool from Australia containing 1000 bales, averaging $987\frac{1}{2}$ pounds to a bale, and valued at 23¢ a pound, is insured for the voyage at $\frac{7}{8}\%$ on $\frac{3}{4}$ of its value. What is the amount of the premium?

180. January 1, 1892, a machinist took out a health policy, paying $1.75 premium on the first day of each month. April 5, 1893, he fell sick, and received $15 a week for 5 weeks. How much did he receive more than he had paid out in premiums?

181. A man insured his life for $10,000, paying a premium each year of $31.50 per $1000. How much more or less than the total amount paid in premiums will his heirs receive if he lives to make 10 annual payments? 20 annual payments? 40 annual payments?

182. A ship was insured for $\frac{3}{4}$ of her value at $2\frac{1}{8}\%$ a year. After the annual premium, $1275, had been paid four times the ship was lost. Find the loss to the insurers and to the insured.

183. A marine insurance company has $450,000 at risk on cotton afloat, at $2\frac{1}{2}\%$ premium. Learning that the vessels carrying this cotton are overdue and that severe storms at sea are reported, the agent of the company decides to reinsure $\frac{2}{3}$ of the risk in other companies at $3\frac{1}{4}\%$. Will the company gain or lose by the whole business, and how much, supposing the cotton arrives safely?

184. An insurance company takes a risk of $90,000 on a factory at $2\frac{7}{8}\%$, and reinsures this risk with another company at $2\frac{1}{4}\%$. What is the amount of premium retained by the first company?

185. A merchant has policies of insurance on his stock of goods as follows: one for $10,000 at $\frac{5}{8}\%$, one for $12,500 at $\frac{3}{4}\%$, one for $15,000 at $\frac{7}{8}\%$, and one for $2500 at 1%. The goods cost him $50,000. If a fire should totally destroy the goods, what would be his loss, including the sums paid for insurance?

186. The combined experience of many fire insurance companies through a long series of years has shown, let us suppose, that one dwelling house in 800 is lost by fire each year. What fractional part of the sum received for premiums on dwelling house insurance must a company expect to pay out for losses, if the rate of insurance is $1\frac{7}{8}\%$ on five-year policies? What clear profit would the company make each year on $5,000,000 at risk on dwelling houses?

TAXES.

381. The money necessary to meet the expenses of a town, city, county, or state government, is raised by levying taxes on the persons and on the property of the citizens.

382. A tax levied on the person is called a **poll tax** or a **capitation tax.** It is a tax of so much *per head* (*poll*). A tax levied on property (real and personal estate) is a **property tax**; and a tax on annual income, an **income tax.**

383. Real estate is property in land and buildings, including whatever is attached to and belongs with them. It may be described generally as immovable property. Movable property, as money, ships, merchandise, horses, cattle, carriages, etc., is called **personal property.** Real estate is taxed in the county, town, or city where it is situated; personal property is taxed where its owner lives.

384. The method of raising money by taxation so that each citizen pays his fair share varies in different states, but may be described in general terms as follows:

The citizens assembled in town meeting, or representatives chosen by the citizens assembled in city council, or in a county board, determine by vote what sum of money shall be raised to meet the expenses of the local government. This sum is called the **tax levy.**

Officers called assessors estimate the value of each piece of real estate, recording their estimates against the owners' names. The sum of all these estimates is called the valuation of real estate. The

value of the personal property owned by each citizen is estimated and recorded in like manner; and the sum of these estimates is the valuation of personal property. The sum of these two valuations is the **total valuation.**

That part of a person's annual income which is subject to taxation is usually included in the estimate of his personal property.

The assessors are now ready to apportion the tax levy. The amount that is to be raised by the poll taxes is found by counting the number of citizens required by law to pay a poll tax, and multiplying the amount of one poll tax by this number. This result being subtracted from the tax levy leaves the amount that must be raised on property. This is called the **property tax levy.**

To provide a margin for abatements, uncollected taxes, etc., the property tax levy is increased in some states by adding a small percentage called **overlay.**

The property tax levy increased by the overlay and divided by the total valuation gives the **tax rate.**

Finally, each property owner's tax is computed *by multiplying the assessed value of his property by the tax rate.*

In some states, the state tax is apportioned among the towns and cities in proportion to their several valuations. The county tax is apportioned in the same way among the towns and cities in the county. Each town and city includes its share of the state tax and of the county tax in its own tax levy; so that the citizen pays his state, county, and town or city tax in one sum to the town or city collector of taxes. In other states the county and not the town levies and collects state and county taxes.

Learn how the state, county, town or city, and school-district taxes are levied and collected in your own state.

385. *Illustrative Example.* The following item is taken from a newspaper. "The assessors of Oldtown have computed the valuation of property subject to taxation in the town, and report: Personal property, $1,237,688; real estate, $7,633,594; total, $8,871,282; increase over last year, $121,217; town tax levy, $112,448; state tax, $6590; county tax, $8675; rate, $14.30 per thousand; polls, 2815; population, 10,594." — *Oldtown Herald.*

How was the tax rate of this town computed?

Computation of the Tax Rate.

Town tax levy,	$112,448
State tax,	6,590
County tax,	8,675
Total tax levy,	127,713
Subtract 2815 poll taxes, $2 each,	5,630
Property tax levy,	122,083
Add 4 % overlay,	4,883.32
	$126,966.32

Valuation:

Real,	$7,633,594
Personal,	1,237,688
Total,	8,871,282

```
8,871,282)126,966.32000(0.01431
          88 712 82
          38 253 500
          35 485 128
           2 768 3720
           2 661 3846
             106 98740
              88 71282
```

```
 $8871282
   0.0143
 26613846
35485128
8871282
$126859.3326
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The rate 0.01431 might be stated as 1.431 %, or as 14.31 mills on a dollar, but the usual form is $14.31 on a thousand. The assessors in Oldtown, however, find it convenient to drop the 1 cent and make an even rate, $14.30 on a thousand. This rate on a valuation of $8,871,282 produces $126,859.33, which exceeds the property tax levy by $4776.33. This last amount is the actual overlay.

The warrant to the collector of taxes is:

For property taxes,	$126,859.33
For poll taxes,	5,630.00
Total,	$132,489.33

386. Examples for Written Work.

187. In Oldtown what is the amount of A. G. Brown's tax bill, who owns a house and garden valued at $5400; personal property, $1265; and who pays one poll tax?

188. What is the amount of D. L. Goodrich's tax bill, who owns a farm and buildings valued at $11,750; horses, cattle, tools, farm products, and other personal property valued at $7,650; and who pays one poll tax?

189. What is the amount of M. J. Wood's tax bill, who lives in another town, but owns in Oldtown 265 acres of woodland valued at $25 per acre?

190. The rate of taxation in Norwood being $12.20 per thousand, what is the valuation of a farm on which a tax of $137.25 is paid?

191. Mr. Corey finds by his tax bill that he has been assessed $148.20 on his real estate, and $59.28 on his personal property. The rate being $9.50 per thousand, what valuation have the assessors placed on his real estate and what on his personal property?

192. In Newton the valuation of real estate for the year 1894 was $34,139,350; and that of personal property was $9,860,835. Polls, 7763, taxed at $2. The city tax levy was $585,405.93; state tax, $34,640; county tax, $37,882.78. Find the tax rate allowing an overlay not exceeding 5%.

193. The total valuation of real and personal property in Boston, in 1894, was $928,092,456. The total tax levy was $11,713,090.97. The number of polls, taxed $2 each, was 139,743. Allowing 4% overlay, what was the rate? Taking $12.80 per thousand, the actual rate, find the actual overlay, and the total warrant to the collector.

194. Find the tax levy, total valuation, number of polls, and poll tax in the town or city where you live; compute the tax rate; and then find the tax bills for persons supposed to own given amounts of property.

CUSTOMS OR DUTIES.

387. The expenditures of the United States ar met by taxes upon merchandise imported into the country. These taxes are called **customs** or **duties**.

The **custom house** is a building belonging to the United States, where all business connected with incoming and outgoing vessels and their cargoes is transacted with officers of the government. A **port of entry** is a place where a custom house has been established. It is usually a seaport, but may be any place on or near the boundary where merchandise is brought into the country. The duties levied on imported merchandise are paid at the custom house.

The rates of duty to be paid on the various kinds of merchandise imported are fixed by an Act of Congress, usually referred to as "The Tariff Act." To learn the rate of duty on each kind and quality of merchandise, an importer must refer to the schedules contained in the last "Tariff Act."

388. Duties are either **specific** or **ad valorem**; the former being based on the quantity, the latter on the value of the merchandise imported.

A few of the *specific duties* levied by the Tariff Act of 1894 are as follows: 1¢ per pound on window glass, 50¢ per cubic foot on rough marble, 3¢ per dozen on eggs, $2 per ton on hay, $1\frac{1}{2}$¢ per yard on cotton cloth, 40¢ per ton on coal, and $\frac{6}{10}$¢ per pound on wrought iron bars.

Some of the *ad valorem* duties are 15% on straw, 20% on beef, pork, and mutton; 25% on watches, clocks, and wooden furniture; 35% on stained glass, porcelain, crockery, paper, confectionery, and jewelry; 40% on carpets, woolen cloth, and clothing; 50% on silks, velvets, laces, cloaks, ulsters, stockings, and shirts.

Sometimes the duties levied on an article are *both specific and ad valorem.* Thus the duty on collars and cuffs is 30¢ per dozen and 30% ad valorem; on cologne water $2.00 per gallon and 50% ad valorem; on cigars $4.00 per pound and 25% ad valorem; on spirit varnish 25% and $1.32

per gallon for the alcohol in it; and on pearl buttons 1¢ per line (= $\frac{1}{40}$ inch) button measure per gross and 15% ad valorem.

NOTE 1.—In estimating specific duties, certain allowances are made: (1) for waste or impurities, called *draft;* (2) for weight of boxes, casks, etc., called *tare;* (3) for waste of liquids, called *leakage;* (4) for breaking of bottles, etc., called *breakage*. The weight of goods before allowances are made is the **gross weight**; and the weight after deducting allowances is the **net weight**.

NOTE 2.—The value of imported goods appears from the invoice; which is a list of the goods with statement of the quantities and of the prices in the country from which they were imported.

389. Examples for Written Work.

NOTE.—For rates of duty not given, see Article 388.

195. How much is the duty on 200 tons (2240 lb. to a ton) of wrought iron bars.

196. How much is the duty, at 1½¢ per pound, on 15 boxes of raisins, 24 lb. to a box, tare 6½ lb. to a box?

197. At 35% how much is the duty on 50 dozen watch crystals, invoiced at $1.80 per dozen, breakage 3%?

198. What is the duty on 10 bbl. spirit varnish, 32 gallons to a barrel, invoiced at $6 per gallon, allowance for leakage being 10% and analysis showing that the varnish contained 43.34 per cent alcohol?

199. Find the amount of duties on 1000 yards English carpet, invoiced at 4 shillings a yard in London, and imported at New Orleans.

NOTE.—The pound sterling = $4.8665.

200. How much is the duty on 25 tons of hay and 10 tons of straw (the latter valued at $6 per ton) brought from Canada into the United States?

201. Watches valued at 400 francs each in Geneva would be subject to how much duty in Boston?

NOTE.—The franc = $0.193.

202. How much must be paid in New York for duties on 5 velvet cloaks, costing 600 francs each in Paris?

203. Reckon the duty at New York on 15 gallons of cologne, valued in Cologne at 33 marks per gallon.

NOTE. — The mark = $0.238.

204. A ship load of tables, chairs, bedsteads, bureaus, bookcases, and wardrobes was brought into New York from Berlin. How much duty was paid if the invoice showed a valuation of 23,000 marks in Germany?

205. On 100 dozen linen collars imported from London and invoiced at 5 shillings 6 pence per dozen, how much duty must be paid at the Custom House in Chicago?

206. How much duty must be paid in Baltimore on 25 gross pearl buttons measuring 11 lines, and costing in Vienna 70 florins per gross?

NOTE. — The florin = $0.406.

207. What is the cost in Philadelphia of 22,892 lb. of sugar, invoiced at Havana for $858.45, on which is paid $324.60 for freight and carting, and duties of $\frac{1}{8}$¢ per lb. and 40% ad valorem, allowance for tare being 8%?

390. Miscellaneous Examples.

208. What is $\frac{1}{5}$% of $59.85? $1\frac{2}{5}$% of $89.75?

209. If a percentage is 756, and the rate $3\frac{1}{2}$%, what is the base?

210. Change $107\frac{1}{2}$% to a fraction in smallest terms.

211. Express $1\frac{3}{20}$ decimally; also $2\frac{7}{50}$; and $3\frac{9}{25}$.

212. Change $\frac{1}{9}$ to a per cent; also $\frac{1}{17}$; and $\frac{3}{23}$.

213. If 35% of a number exceeds 28% of it by 301, what is the number?

214. What per cent is 375 of 2500? of 7500?

215. What is the cost of a horse which is sold at a gain of 25% for $240?

216. A yacht formerly valued at $8500 has depreciated 30%. What is her present value?

217. A house worth $7500 is insured for $\frac{2}{3}$ of its value at $\frac{1}{2}\%$. What is the premium?

218. What amount of insurance may I procure on my house for $25, if the rate is $\frac{5}{8}\%$?

219. What must be paid a collector for collecting $19,470, if he has been promised $2\frac{1}{2}\%$?

220. A collector is paid $2\frac{1}{2}\%$ commission. What amount did he collect, if his commission was $56.65?

221. How many shares of railroad stock, now selling at 15% advance on the par value, $100, can be bought for $4840.50, brokerage $\frac{1}{4}\%$?

222. Insurance was procured on the ship Alice to Cape Town and return in the sum of $15,000 at $2\frac{1}{4}\%$, and on her return cargo, valued at $12,500, at $1\frac{1}{4}\%$. What was the amount of the premium paid?

223. What is the duty at 50% ad valorem on 527 yards of silk valued at $2.75 per yard?

224. What is the duty at 30¢ per dozen and 30% ad valorem on 500 dozen linen collars valued at 95¢ per dozen?

225. A speculator bought manufacturing stocks at $80 per share and sold them at $104. What per cent did he make on his investment?

What per cent is gained or lost

226. By buying at $100 and selling at $120?

227. By buying at $120 and selling at $100?

228. By buying at $90 and selling at $110?

229. By buying at $110 and selling at $90?

230. When 80 shares of stock, par value $100, are sold for $8100, at what per cent above par does it sell?

231. A merchant sold a lot of wool for $1886, gaining 15%. For what should he have sold it to gain 25%?

232. A grocer sold 400 bbl. of flour at $9\frac{1}{2}\%$ profit, gaining $142.50. What did the flour cost per barrel?

233. A speculator bought 300 shares of mining stock at $5\frac{1}{2}\%$ below par, and sold them at $1\frac{1}{2}\%$ above par. How much did he gain, the par value of a share being $ 10?

234. If 200 shares of a railroad are bought at $ $105\frac{1}{2}$ and sold at $ $115\frac{3}{4}$ per share, what is the amount gained? what per cent is gained on the investment?

235. A watch is sold for $ 190 at a loss of 24%. What should it have sold for to make the loss only 5%?

236. If I buy 20 shares of stock, the par value of which is $ 100 per share, at 14% above par, and sell it at 4% below par, what per cent do I lose?

237. A carpet mill valued at $ 225,000 is insured for $\frac{3}{4}$ of its value by two companies, the first taking $\frac{3}{8}$ of the risk at $\frac{7}{10}\%$, and the second the rest at $\frac{5}{8}\%$. What was the whole premium?

238. A provision dealer's sales in one year amounted to $ 81,000. Two thirds of this amount was received for beef, on which his profit was 10%, and one third for pork, on which his profit was 15%. What was the cost to him of the whole amount of beef and pork sold?

239. If an insurance company takes a risk of $ 9000 at $1\frac{1}{4}\%$, and reinsures one half of it in another company at $1\frac{3}{8}\%$, how much does the first company make if no loss occurs?

240. The capital stock of a bank is $ 500,000, and the net earnings for the last quarter are $ 12,248.50. What is the largest even rate of dividend that the directors can declare, and what sum will be carried to the surplus?

241. A railroad corporation increases its capital stock from $ 10,000,000 to $ 25,000,000. The stockholders are allowed to purchase at $ 90 per share as many shares of the new stock as they hold of the old stock, and the remaining shares are sold in open market at an average price of $ $101\frac{1}{2}$ per share. How much can a holder of 250 shares of the old stock make by selling his new stock in the market?

SECTION XIV.

INTEREST.

391. *Illustrative Examples.* (1) A merchant borrowed $3000 for a year, and used it in his business. At the end of the year he paid back the money and paid for its use a sum equal to 6% of the money borrowed. How much did he pay for its use? *Ans.* $180.

392. The price paid for the use of money is **interest.** The money for the use of which interest is paid is the **principal.**

393. The **rate of interest** is the fraction which a year's interest is of the principal. It is usually expressed as a **rate per cent per annum.**

In the example, what is the principal? the interest? the rate of interest?

394. The rate of interest fixed by law is the **legal rate.** Interest at a higher rate than the legal rate is **usury.**

(2) If $180 is paid for the use of $3000 for one year, how much ought to be paid for the use of the same sum of money for two years? three years? half a year?

(3) If 6% of $3000 is paid for the use of $3000 one year, what per cent of that sum ought to be paid for its use two years? three years? half a year?

395. Interest is reckoned as a percentage, the **principal** being the **base** and the **rate** being the **rate per annum** *increased or diminished in proportion to the time.*

Thus, in example (3) above, the rate per annum is 6%; the rate for two years is 12%; for three years, 18%; for half a year, 3%.

INTEREST FOR YEARS.

Oral Exercises.

396. *a.* At 5% a year, what is the rate for 2 years? for 3 years? 4 years? ½ a year? 1½ years? 2½ years?

b. At 7% per annum, what is the rate for 2 years? for 3 years? half a year?

c. At 8% per annum, what is the rate for 3 years?

d. At 5% per annum, what is the interest on $3000 for one year? for two years?

e. At 7% per annum, what is the interest on $400 for one year? for 2 years? 3 years?

f. At 6% per annum, what is the interest of $4000 for one year? for 3 years? half a year? one third of a year?

g. What is the interest on $700 for 2 years at 6% per annum?

At 6% per annum, what is the interest

h. Of $500 for 3 years?

i. Of $700 for 2 years?

j. Of $900 for 4 years?

k. Of $1200 for 2 years?

l. Of $1500 for 2 years?

m. Of $300 for 10 years?

n. Of $600 for 5 years?

o. Of $800 for 2 years?

p. Of $1000 for 3 years?

q. Of $2000 for 4 years?

Examples for Written Work.

397. *Illustrative Example.* What is the interest of $3250 for 3 years at 6%?

WRITTEN WORK.

$$\$3250 \times 0.18 = \$585.00 \; \textit{Ans.}$$

The principal, $3250, is multiplied by 0.18, the rate for 3 years, and the product is the interest for that time. *Ans.* $585.00.

The rate of interest being 6%, what is the interest

1. Of $1244 for 3 y.?
2. Of $478 for 6 y.?
3. Of $12,617 for 2½ y.?
4. Of $2400 for 2⅓ y.?
5. Of $156.50 for 5 y.?
6. Of $237.75 for 4 y.?
7. Of $1253.37 for 2 y.?
8. Of $327.05 for 4½ y.?

INTEREST FOR MONTHS (6%).

398. *Illustrative Examples.* (1) At 6% per annum, what is the interest of $237.63 for 2 months? (2) Of $523.18 for 8 months?

(1)

WRITTEN WORK.

Principal,	$237.63
Interest,	$2.3763

(1) The rate for 2 months, or $\frac{1}{6}$ of a year, is $\frac{1}{6}$ of 6%, or 1%; and 1% of the principal is found by moving the decimal point two places to the left. *Ans.* $2.38.

(2)

WRITTEN WORK.

Principal,	$523.18
	4
Interest,	$20.9272

(2) The interest for 8 months is 4 times the interest for 2 months. Multiplying the principal by 4, and then moving the decimal point two places to the left, gives the same result as finding two months' interest (as in Ex. 1), and multiplying that by 4. *Ans.* $20.93.

399. From these examples we derive the following

Rule.

To find the interest, at 6%, for any number of months: *Multiply the principal by half the number of months, and move the decimal point two places to the left.*

Examples for Written Work.

400. At 6% a year, or 1% for 2 mo., find the interest

9. Of $320 for 2 mo.
10. Of $430 for 4 mo.
11. Of $325.70 for 2 mo.
12. Of $526 for 10 mo.
13. Of $365 for 14 mo.
14. Of $962.75 for 16 mo.
15. Of $494.09 for 18 mo.
16. Of $500 for 20 mo.
17. Of $3000 for 16 mo.
18. Of $6000 for 6 mo.
19. Of $8000 for 8 mo.
20. Of $927 for 1 mo.
21. Of $625 for 3 mo.
22. Of $350 for 7 mo.
23. Of $763.20 for 5 mo.
24. Of $2500 for 9 mo.
25. Of $9000 for 15 mo.
26. Of $5000 for 5 mo.
27. Of $8000 for 17 mo.
28. Of $800 for 25 mo.

INTEREST FOR DAYS (6%).

401. *Illustrative Examples.* (1) At 6% a year, what is the interest of $2400 for 6 days? (2) Of $630 for 16 days?

(1)

WRITTEN WORK.

Principal,	$2400
Interest,	$2.400

(1) The rate for 60 days (2 months) being 0.01, the rate for 6 days is $\frac{1}{10}$ of 0.01 which is 0.001. One thousandth of the principal is found by moving the decimal point three places to the left. *Ans.* $2.40.

(2)

WRITTEN WORK.

Principal,	$630
	16
	3780
	63
	6)$10080
Interest,	$1.680

(2) The interest for 16 days might be found by first finding the interest for 6 days (moving the decimal point), then dividing that interest by 6, and lastly multiplying the quotient by 16. But it usually takes fewer figures to reverse the order of these steps, and the result is the same. So the principal is multiplied by 16, the product divided by 6, and the decimal point moved three places to the left. *Ans.* $1.68.

402. From these examples we derive the following

Rule.

To find the interest, at 6%, for any number of days: *Multiply the principal by the number of days, divide by 6, and move the decimal point three places to the left.*

NOTE.—If the number of days happens to be divisible by 6, a step is saved by multiplying the principal by one sixth of the number of days and then moving the point three places to the left.

Examples for Written Work.

403. At 6% a year, or 0.001 for 6 da., find the interest

29. Of $920 for 6 days.
30. Of $426 for 12 days.
31. Of $2365 for 18 days.
32. Of $3840 for 15 days.
33. Of $927 for 63 days.
34. Of $231 for 33 days.
35. Of $328 for 93 days.
36. Of $972 for 123 days.
37. Of $525 for 90 days.
38. Of $632 for 42 days.
39. Of $925.50 for 1 day.
40. Of $123.75 for 2 days.

At 6% per annum find the interest

41. Of $9280 for 3 days.
42. Of $2800 for 4 days.
43. Of $2250 for 5 days.
44. Of $92.60 for 53 days.
45. Of $32,500 for 17 days.
46. Of $426.76 for 91 days.
47. Of $625.72 for 88 days.
48. Of $237.58 for 11 days.

INTEREST FOR YEARS, MONTHS, AND DAYS (6%).

Oral Exercises.

404. *a.* The rate of interest being 6%, what is the rate for 2 years? for 3 years? for 4 years? for 5 years? for 6 years? for 8 years?

b. If the interest for 2 months is 1%, or 0.01, of the principal, what part of the principal is the interest for 4 months? for 6 months? for 8 months? for 5 months? for 7 months? for any number of months?

The interest for any number of months is one half as many hundredths of the principal as there are months.

c. If the interest for 6 days is 0.001 of the principal, what part of the principal is the interest for 12 days? 18 days? 24 days? 9 days? 15 days? 16 days? 17 days? 19 days? any number of days?

The interest for any number of days is one sixth as many thousandths of the principal as there are days.

405. *Illustrative Examples.* (1) What is the rate, at 6% per annum, for 3 years, 8 months, 24 days?

(2) For 2 years, 7 months, 15 days?

SOLUTION (1).

The rate for 3 y. = 0.18
for 8 mo. = 0.04
for 24 d. = 0.004
0.224

Ans. 0.224

SOLUTION (2).

The rate for 2 y. = 0.12
for 7 mo. = 0.035
for 15 d. = $0.002\frac{1}{2}$
$0.157\frac{1}{2}$

Ans. $0.157\frac{1}{2}$

Examples for Written Work.

406. At 6% per annum, what is the rate

49. For 2 y. 3 mo. 27 d.?

50. For 1 y. 5 mo. 24 d.?

51. For 1 y. 7 mo. 16 d.?

52. For 1 y. 9 mo. 13 d.?

53. For 2 y. 0 mo. 7 d.?

54. For 3 y. 2 mo. 19 d.?

55. At 6% per annum, what is the interest of $1 for 3 y. 6 mo. 12 d.? for 2 y. 8 mo. 24 d.? for 2 y. 5 mo. 22 d.? for 1 y. 11 mo. 7 d.? for 1 y. 1 mo. 10 d.? for 1 y. 3 mo. 6 d.?

407. *Illustrative Example.* What is the interest of $2386 for 2 years 5 months and 23 days at 6%?

WRITTEN WORK.

$$
\begin{array}{ll}
\text{The Rate} & \\
\quad \text{for 2 y.} & = 0.12 \\
\quad \text{for 5 mo.} & = 0.025 \\
\quad \text{for 23 d.} & = \underline{0.003\frac{5}{6}} \\
\quad \text{for the whole time} & = 0.148\frac{5}{6}
\end{array}
\qquad
\begin{array}{lr}
\text{Principal,} & \$2386 \\
\text{Rate for time,} & \underline{0.148\frac{5}{6}} \\
 & 6)\underline{11930} \\
 & 1988\frac{1}{3} \\
 & 19088 \\
 & 9544 \\
 & \underline{2386} \\
\text{Interest,} & 355.116\frac{1}{3}
\end{array}
$$

The interest for 2 years is 0.12 of the principal; for 5 months it is 0.025 of the principal; and for 23 days it is $0.003\frac{5}{6}$ of the principal. Hence the interest for the whole time is $0.148\frac{5}{6}$ of the principal. Multiplying the principal by $0.148\frac{5}{6}$ gives the interest. *Ans.* $355.12

408. From the foregoing examples is derived the following

Rule.

To compute interest for years, months, and days at 6%: *Add together six times as many hundredths as there are years, one half as many hundredths as there are months, and one sixth as many thousandths as there are days, and by the decimal so found multiply the principal.*

At 6% per annum, what is the interest

56. Of $300 for 2 y. 5 mo. 24 d.?
57. Of $46.18 for 1 y. 3 mo. 27 d.?
58. Of $496.50 for 2 y. 4 mo. 28 d.?
59. Of $800.40 for 3 y. 6 mo. 19 d.?
60. Of $526.60 for 1 y. 8 mo. 18 d.?
61. Of $629.90 for 2 y. 9 mo. 13 d.?

409. The sum of the principal and the interest is called the **amount.**

Examples for Written Work.

410. Find the amount, at 6% per annum,

62. Of $900 for 2 y.
63. Of $10,000 for 7 y.
64. Of $80,000 for 5 y.
65. Of $5000 for 3 y.
66. Of $1500 for 4 y.
67. Of $5000 for $2\frac{1}{2}$ y.
68. Of $491.60 for 5 y.
69. Of $968.50 for $1\frac{1}{2}$ y.
70. Of $126.00 for $1\frac{2}{3}$ y.
71. Of $1250.00 for $3\frac{1}{2}$ y.

Find the amount, at 6% per annum,

72. Of $92 for 4 mo.
73. Of $320 for 6 mo.
74. Of $460 for 2 mo.
75. Of $85 for 8 mo.
76. Of $620 for 10 mo.
77. Of $230 for 3 mo.
78. Of $500 for 5 mo.
79. Of $800 for 7 mo.
80. Of $650 for 11 mo.
81. Of $98.75 for 6 mo.
82. Of $236.18 for 7 mo.
83. Of $528.75 for 8 mo.
84. Of $88.25 for 9 mo.
85. Of $956.48 for 3 mo.

Find the amount, at 6% per annum,

86. Of $472.30 for 57 days; for 16 days.
87. Of $35.00 for 35 days; for 27 days.
88. Of $96.75 for 48 days; for 7 days.
89. Of $128,962.36 for 3 days; for 1 day.
90. Of $3,468,269.68 for 2 days; for 5 days.
91. Of $985.50 for 33 days; for 63 days.
92. Of $485.00 for 93 days; for 123 days.

INTEREST AT ANY RATE OTHER THAN 6%.

411. *Illustrative Example.* What is the interest of $6280 for 1 year 6 months 25 days at 7%? at 5%? at 4%? at $7\frac{1}{2}$%?

WRITTEN WORK.

Principal,	$6280
	$0.094\frac{1}{6}$
	$1046\frac{2}{3}$
	25120
	56520
Interest at 6%,	$$591.366\frac{2}{3}$
($\frac{1}{6}$)	$$98.561\frac{1}{9}$
Interest at 7%,	$$689.927\frac{7}{9}$
Interest at 5%,	$$492.805\frac{5}{9}$

The interest is computed first at 6%, because this is a convenient rate. This interest increased by $\frac{1}{6}$ of itself is the interest at 7%, and diminished by $\frac{1}{6}$ of itself is the interest at 5%.

In the same way the interest at 6% diminished by $\frac{1}{3}$ of itself becomes the interest at 4%, and increased by $\frac{1}{4}$ of itself becomes the interest at $7\frac{1}{2}$%.

Ans. $689.93; $492.81; $$394.244\frac{4}{9}$; $$739.208\frac{1}{3}$.

Oral Exercises.

412. What part of the interest at 6% should be added or subtracted to obtain the interest

a. At 8%? *c.* At 5%? *e.* At $7\frac{1}{2}$%?
b. At 9%? *d.* At 4%? *f.* At $4\frac{1}{2}$%?

If the part to be added or subtracted is not an easy one to find, the interest at 6% may be divided by 6 and the quotient multiplied by the given rate per cent; or, better, the interest at 6% may be multiplied by the given rate per cent and this product divided by 6.

Examples for Written Work.

413. What is the interest

93. On $500 for 3 years at 5%?
94. On $325 for 2 years at 4%?
95. On $926 for 4 years at 7%?
96. On $840 for 6 years at $2\frac{1}{2}$%?
97 On $96.50 for 2 years at $4\frac{1}{2}$%?

What is the interest

98. On $ 325 for 2 months at 4% ?
99. On $ 1296 for 8 months at 5% ?
100. On $ 92.63 for 9 months at 3% ?
101. On $ 638.20 for 7 months at $3\frac{1}{2}$% ?
102. On $ 1100.75 for 5 months at $2\frac{1}{2}$% ?
103. On $ 1600 for 14 months at $7\frac{1}{2}$% ?
104. On $ 900 for 1 month at 3% ?
105. On $ 853.60 for 3 months at 7% ?

106. Find one day's interest, at 4%, on $ 3482.85; on $ 3985.20; on $ 2842.56; on $ 1583.16; and on $ 1263.40. Add the results. Add the five given sums of money and reckon one day's interest on the whole at 4%. How does the last result compare with the sum of the first five results?

What is the interest

107. On $ 4286.60 for 5 days at 5% ?
108. On $ 6384.00 for 18 days at 4% ?
109. On $ 940 for 63 days at 7% ?
110. On $ 862 for 93 days at $4\frac{1}{2}$% ?
111. On $ 98.50 for 33 days at 9% ?
112. On $ 105.75 for 123 days at 8% ?
113. On $ 628.20 for 35 days at $8\frac{1}{2}$% ?
114. On $ 350.00 for 28 days at $3\frac{1}{2}$% ?
115. On $ 3480.00 for 1 day at 1% ?

What is the interest

116. On $ 126.50 for 1 y. 3 mo. 24 d. at 4% ?
117. On $ 925.62 for 2 y. 5 mo. 15 d. at 7% ?
118. On $ 1362.56 for 17 mo. 13 d. at $4\frac{1}{2}$% ?
119. On $ 263.75 for 2 y. 3 mo. 16 d. at $2\frac{1}{2}$% ?
120. On $ 562.00 for 3 y. 6 mo. 28 d. at 8% ?
121. On $ 916.50 for 4 y. 2 mo. 16 d. at $3\frac{1}{2}$% ?
122. On $ 2685.00 for 1 y. 6 mo. 17 d. at 5% ?
123. On $ 3480.00 for 8 mo. 18 d. at 9% ?

INTEREST FROM DATE TO DATE.

414. When interest is to be reckoned from one date to another, the time may differ by one or two days, according to the different modes of reckoning adopted. In this book, for the sake of uniformity, the method shown in Art. 298 is the only one used; except that, if the time is less than two months, the exact number of days is counted.

Examples for Written Work.

415. Find the time in years, months, and days from April 19th, 1891, to March 4th, 1904. *Ans.* 12 y. 10 mo. 14 d.

NOTE. — Observe that the year 1904 is a leap year.

Find the number of years, months, and days

124. From January 21st, 1891, to April 29th, 1894.

125. From September 12th, 1889, to December 26th, 1893.

126. From April 1st, 1888, to November 25th, 1895.

127. From August 20th, 1890, to February 27th, 1892.

128. From February 8th, 1892, to January 29th, 1895.

129. From July 18th, 1893, to March 18th, 1896.

130. From May 10th, 1892, to August 25th, 1895.

131. From June 15th, 1894, to October 6th, 1895.

132. From November 23d, 1893, to June 10th, 1894.

133. What is the interest on $ 920 from October 19th, 1893, to April 25th, 1895, at 6% ?

134. What is the amount of $ 528.16 on interest at 6% from August 22d, 1894, to September 27th, 1896 ?

135. What is the amount of $ 600 on interest at 6% from June 20th to October 2d, 1894 ?

136. What is the amount, at 6%, of $ 12,625 on interest from March 25th, 1890, to January 1st, 1895 ?

137. What is the interest on $ 2500 from November 11th, 1889, to August 15th, 1895, at 7% ?

138. What is the interest on $130.27 from February 7, 1894, to December 1, 1898, at 8% ?

139. What is the interest on $19.60 from October 25, 1895, to July 28, 1899, at 10% ?

140. What is the interest on $322.65 from August 13, 1892, to April 2, 1894, at $5\frac{1}{2}$% ?

141. What is the interest on $625 from the 20th of January, 1891, to the 1st of May, 1895, at $4\frac{1}{2}$% ?

142. What is the interest on $2340 from March 15th, 1894, to February 28th, 1895, at 5% ?

143. What is the interest on $955.65 from 10th April, 1893, to 2d October, 1894, at $2\frac{1}{2}$% ?

144. What is the interest on $244.96 from April 19, 1890, to May 30, 1893, at $7\frac{1}{2}$% ?

145. What is the interest on 564.28 from May 15, 1893, to January 6, 1895, at $6\frac{3}{8}$% ?

146. What is the interest on $628.50 from June 30, 1892, to October 31, 1894, at 4.7% ?

147. What is the interest on $526.80 from January 30, 1893, to March 19, 1895, at $5\frac{3}{10}$% ?

148. On the 12th of October, 1894, I gave two notes, one for $450 with interest at $4\frac{1}{2}$%, and the other for $960 with interest at 4%. What amount will be required to pay these notes November 5th, 1894?

149. Mr. Wheeler owes $500 payable October 15th, 1894, but is told that interest at 5% will be deducted if he will pay the debt March 25th, 1894. How much will he save by paying at the earlier date?

150. Messrs. Fowle & Belknap bought goods at various dates during the year, agreeing to pay interest at 4% on the amount of each purchase from its date to the 1st of the following January. The purchases were as follows: April 15th, $624.50; May 20th, $926.75; June 25th, $244.00; August 19th, $428.25; September 9th, $325.00; and November 3d, $916.20. What amount was due January 1st?

ACCURATE INTEREST.

416. The common methods of computing interest set forth in the foregoing rules and examples, are based on the assumption that the year consists of 12 equal months of 30 days each, or 360 days. This assumption, though convenient, and for common purposes accurate enough, is not accurate enough when applied to large sums of money. To be accurate, the time should be reckoned in years and days, and one day should be regarded as $\frac{1}{365}$ of a year. The governments of the United States and Great Britain pay accurate interest. Also bankers and merchants are adopting the custom of computing accurate interest when the principal is large. The use of Interest Tables makes this method as easy as any other.

417. *Illustrative Example.* What is the accurate interest, at 6%, of $ 5,817,922.85 for 7 days?

SOLUTION. — Since 7 days equals $\frac{7}{365}$ of a year, the interest for 7 days equals $\frac{7}{365}$ of a year's interest. *Ans.* $ 6694.60.

NOTE. — By the common method, the answer would have been $ 6787.58, or $ 92.98 more.

418. From this example is derived the following

Rule.

To compute accurate interest for any time less than a year: *Count the exact number of days from date to date, and take as many 365ths of a year's interest as there are days.*

419. Examples for Written Work.

151. Find the accurate interest, at 6%, on $ 7982.50 from January 15th to June 15th, 1895.

152. Find the accurate interest on $ 9865.00 from January 20th, 1894, to March 28th, 1895, at 6%.

153. Find the accurate interest on $ 892.75 from June 12th to September 12th, 1895, at 6%.

154. Find the accurate interest on $ 4860 from July 1st, 1895, to December 1st, 1896, at 7%.

155. Find the accurate interest on $ 50,000 from December 15th, 1893, to October 25th, 1894, at 5%.

156. Find the accurate interest on $ 200,000 from February 1st to May 1st, 1894, at 4%.

157. Find the accurate interest on $ 34,816 from December 20th, 1895, to September 30th, 1896, at 3%.

158. Find one day's accurate interest, at 4%, on each of the following sums: $ 9365.40, $ 8753.72, $ 9284.60. Add the results, and compare the sum with one day's accurate interest on the whole of the money.

159. Find the difference between the accurate interest and the interest by the common method, on $ 4,285,629 from April 4th to July 17th of the same year, at 6%.

PARTIAL PAYMENTS.

420. *Illustrative Example.*

> $592 50/100. Albany, N.Y., June 1, 1894.
>
> For value received, I promise to pay to the order of William Mason, Five Hundred Ninety-two 50/100 Dollars, on demand, with interest at 6 per cent.
>
> Charles Hammond.

This is the written promise of one person, Charles Hammond, to pay another person, William Mason, or any one to whom William Mason may order it paid, a certain sum of money, $ 592.50, for value received. Such a promise is called a **promissory note**, a **note of hand**, or simply a **note**. The sum named in the note, as $ 592.50, is the **face** of the note.

421. To discharge the interest and in part pay the above note, a payment of $ 250 was made December 1, 1894. What balance then remained due? *Ans.* $ 360.28.

Suppose this balance of $ 360.28 to have remained on interest from December 1, 1894, to December 1, 1895, when a payment of $ 200 was made, what sum then remained due? *Ans.* $ 181.90.

422. Payments in part of a note or other debt, like the payments described above, are **partial payments.** A record of the sum paid, with the date of the payment, is made upon the back of the note. Such a record is an **indorsement.**

The method adopted by the Supreme Court of the United States, and by most of the states, for computing interest in case of partial payments, requires (1) *That a payment be applied first to discharge accrued interest, and then, if the payment is large enough, to reduce the principal.* (2) *That no unpaid interest be added to the principal to draw interest.*

For Annual Interest, see Supplement, Art. 16; for the Vt., Conn., and N.H. Rules for partial payments with annual interest, see Supplement, Arts. 17–19.

423. *Illustrative Example.*

$900. *Worcester, Mass., June 25, 1894.*

For value received I promise to pay M. D. Washburn, or order,
Nine Hundred Dollars,
on demand, with interest at 6 per cent.

Cyrus A. Whitcomb.

On the back of this note were the following indorsements:

Oct. 17th, 1894, Paid $150.	*Aug. 19th, 1895, Paid $300.*
May 21st, 1895, Paid $20.	*July 1st, 1896, Paid $100.*

Find what was due on this note December 20, 1896, interest at 6%.

WRITTEN WORK.

Principal from June 25, 1894		$900.00
Interest to Oct. 17, 1894 (3 mo. 22 d.) . . .		16.80
Amount		916.80
First payment Oct. 17, 1894		150.00
New principal from Oct. 17, 1894		766.80
Interest to May 21, 1895 (7 mo. 4 d.),	$27.35	
Second payment May 21, 1895 . . .	20.00 *	
Interest on $766.80 from Oct. 17, 1894, to Aug. 19, 1895 (10 mo. 2 d.)		38.60
		805.40
Second and third payments		320.00
New principal from Aug. 19, 1895		485.40
Interest to July 1, 1896 (10 mo. 12 d.) . . .		25.24
Amount		510.64
Fourth payment		100.00
New principal from July 1, 1896		410.64
Interest to Dec. 20, 1896 (5 mo. 19 d.) . . .		11.57
Amount due Dec. 20, 1896		422.21

424. The above is in accordance with

The United States Rule for Partial Payments.

1. *Find the amount of the principal to the time when the payment or the sum of the payments equals or exceeds the interest; take from this amount a sum equal to the payment or payments.*

2. *With the remainder as a new principal, proceed as before, to the time of settlement.*

---

* This payment is too small to cover the accrued interest, and so it is set aside until the third payment is made, when it is added to that payment and with it subtracted from the amount then due.

425. Examples for Written Work.

160. November 15, 1893, I gave my note promising to pay on demand, with interest at 6%, $490. March 7, 1894, I paid $225. What remained due August 28, 1894?

161. I held a note, secured by a mortgage on real estate, for $800, bearing interest at 5% from June 10, 1891. July 15, 1892, I received $240; May 22, 1893, I received $150. What remained due June 10, 1894?

162. A note for $1500, dated November 12, 1891, was indorsed as follows: January 9, 1892, $180; June 18, 1892, $425; October 3, 1892, $500. What amount was due April 16, 1894, interest at 6%?

163. What balance was due August 1, 1895, on a promissory note for $1235 on interest at 6% from October 1, 1889, and indorsed, January 20, 1890, $175.50; April 25, 1891, $350; May 10, 1892, $450; and July 15, 1894, $200?

164.

$1200 CLEVELAND, May 19, 1892.

For value received, We jointly and severally promise to pay William Hunter, or order, Twelve Hundred Dollars, on demand, with interest at four per cent.

DAVID LAMSON.
HUGH BRICKETT.

Indorsements: October 12, 1892, $250; February 23, 1893, $250; July 14, 1894, $250; December 20, 1894, $250. What was due July 1, 1895?

165. July 16, 1894, John Allen borrowed of Albert Nichols $3500, and gave his note for that sum with interest at 7%. September 22, 1894, a payment of $1250 was made, and December 10, 1894, another payment of $1250. January 1, 1895, a new note was given for the balance then due, with interest at 5%. Find the balance due and write the new note in proper form.

426. When partial payments are made upon notes that have but a short time to run, interest is often computed by the following, called

The Merchants' Rule for Partial Payments.

1. *Compute interest on the principal from the time it begins to draw interest to the time of settlement, and also on each payment from the time it is made to the time of settlement.*

2. *Take the difference between the sum of the principal and its interest and the sum of the payments and their interests; this difference will be the balance due.*

427. *Illustrative Example.* Find how much was due on the following note May 15, 1895.

ROCHESTER, N. Y., January 15, 1895.

Four months after date, I promise to pay to the order of Alfred King & Co., One Thousand Dollars, with interest at six per cent, value received. STEPHEN FOSTER.

Indorsements: January 28, 1895, $120; February 16, 1895, $150; March 3, 1895, $100; March 25, 1895, $80.

WRITTEN WORK.

Principal on interest from Jan. 15, 1895 . .		$1000.00
Interest to May 15, 1895 (4 mo.)		20.00
Amount of note		1020.00
Payment, Jan. 28	$120.00	
Interest to May 15 (3 mo. 17 d.),	2.14	
Payment Feb. 16	150.00	
Interest to May 15 (2 mo. 29 d.),	2.23	
Payment March 3	100.00	
Interest to May 15 (2 mo. 12 d.),	1.20	
Payment March 25	80.00	
Interest to May 15 (1 mo. 20 d.),	0.67	
		456.24
	Ans.	$563.76

428. Examples for Written Work.

166.

$1250 PHILADELPHIA, July 8, 1894.

Six months after date, I promise to pay Samuel Henderson & Co., or order, One Thousand Two Hundred Fifty Dollars, with interest at six per cent, value received.

STEPHEN KNIGHT.

Indorsements: August 9, 1894, $150; September 12, 1894, $250; October 18, 1894, $200; November 21, 1894, $180; December 20, 1894, $120.

What balance was due January 8, 1895?

167.

$15,000 WASHINGTON, D. C., March 4, 1895.

Three months after date, I promise to pay to the order of D. Ryder & Co., at the National Revere Bank, Boston, Fifteen Thousand Dollars, with interest at six per cent, value received.

GEORGE W. NORRIS.

Indorsements: April 15, 1895, $2562.15; April 28, 1895, $3840.00; May 16, 1895, $4287.67; May 24, 1895, $3922.86.

What was due on this note June 4, 1895?

168.

$2500 BOSTON, January 1, 1894.

One year from date, I promise to pay to the order of Brown, Martin, & Co., Twenty-five Hundred Dollars, for value received, with interest at five per cent.

HENRY PIPER.

On this note are indorsements showing the payment of two hundred dollars on the first day of every month till January 1, 1895, when the balance then due was paid. How much was this last payment?

PROBLEMS IN INTEREST.

429. From examples thus far given, we learn that interest is the product of three factors, principal, rate per annum, and time. For a short expression of this fact, let letters stand for numbers as follows:

$i =$ the interest, in dollars.
$p =$ the principal, in dollars.
$r =$ the rate per annum, a decimal fraction.
$t =$ the time, in years.

Then we may say

$$i = prt,$$

meaning that *interest equals principal multiplied by rate multiplied by time.* For in algebraic notation two or more letters written in immediate succession, as prt, denote the product of the numbers the letters stand for.

As an example, suppose the interest on \$480 for 3 years at 5% is required.

$$\begin{array}{r} \$\,480 = p \\ \underline{0.15} = rt \\ 2400 \\ \underline{480} \\ i = \$\,72.00 = prt \end{array}$$

Otherwise,

$$\$\,480 \times 0.5 \times 3 = \$\,72.00$$
$$p \times r \times t = i$$

Working this example in the ordinary way, we first multiply 0.05, the rate per annum $(= r)$ by 3, the number of years $(= t)$, and obtain 0.15, the rate for the whole time $(= rt)$. We then multiply the principal $(= p)$ by 0.15 $(= rt)$, and obtain the interest $(= prt)$.

430. In the formula

$$i = prt$$

are represented four numbers, any three of which being given, the fourth can be found. Hence arise four problems:

1. Given p, r, and t, to find i.
2. Given p, r, and i, to find t.
3. Given p, i, and t, to find r.
4. Given i, r, and t, to find p.

The first of these problems represents all the examples heretofore given which required the interest to be found. The second, third, and fourth require further illustration.

431. Principal, rate, and interest given, to find the time.

If each member of the equation

$$i = prt$$

is divided by pr,* the result will be

$$\frac{i}{pr} = t,$$

which means that *if the given interest is divided by the product of the principal and the rate, the quotient will be the time.*

Examples for Written Work.

432. *Illustrative Example.* In what time will $380 yield $28.50, interest at 5%?

SOLUTION. $\frac{28.50}{380 \times 0.05} = \frac{57}{38} = 1\frac{1}{2}$. *Ans.* $1\frac{1}{2}$ years, or 1 y. 6 mo.

In what time will

169. $600 gain $30 at 6%?

170. $400 gain $48 at 4%?

171. $800 gain $84 at 7%?

172. $3200 gain $160 at 5%?

173. $816 gain $217.60 at $7\frac{1}{2}$%?

174. $53.50 gain $10.165 at 8%?

175. In what time will $4820 amount to $5133.30 at 4%?

NOTE.—To find the interest, subtract the principal from amount. Then find the time as in the previous examples.

176. In what time will $326, at 7%, amount to $385.332?

177. How long must a note for $15,000 run to amount to $20,000 at 8%?

178. In what time will $100 amount to $200 at 6%?

179. In what time will $500 amount to $1000 at 5%?

180. In what time will any sum of money at interest double itself at 5%? at 6%?

---

* Thus, $\frac{i}{pr} = \frac{prt}{pr} = t$.

181. In what time will a sum of money at interest double itself at 2%? at 4%?

182. In what time will a sum of money at interest double itself at 3%? at 10%?

183. Show in general that the time in which a sum of money on interest at the rate r will double itself is $\frac{1}{r}$ years.

433. Principal, interest, and time given, to find the rate.

If each member of the equation

$$i = prt$$

is divided by pt, the result will be

$$\frac{i}{pt} = r,$$

which means that *if the interest is divided by the product of the principal and the time, the quotient will be the rate.*

Examples for Written Work.

434. *Illustrative Example.* At what rate will \$360 gain \$40.80 interest in 1 year 5 months?

SOLUTION. $\frac{\$40.80}{\$360 \times 1\frac{5}{12}} = \frac{40.80}{510} = 0.08.$ *Ans.* 8%.

184. At what rate will \$10,000 gain \$5000 in 10 years?

185. At what rate will \$12 gain \$4 in 3 y. 4 mo.?

186. At what rate will \$450 gain \$49.87½ in 1 y. 7 mo.?

187. At what rate will \$250 gain \$35 in 2 y. 9 mo. 18 d.?

188. At what rate will \$57.20 gain \$4.719 in 1 y. 6 mo.?

189. At what rate will \$75 gain \$23.62½ in 3 y. 6 mo.?

190. At what rate will \$1000 gain \$500 in 6 y. 8 mo.?

191. The amount of \$150 for 2 y. 6 mo. was \$157.50. What was the rate?

NOTE. — The interest is found by subtracting the principal from the amount.

192. When a note for \$2000 amounts to \$2116.66$\frac{2}{3}$ in 7 months, what is the rate?

193. At what rate will the interest equal the principal in 20 years?

194. At what rate will the interest equal the principal in 12 years?

195. Show in general that the rate at which a sum of money will double itself in t years is $\frac{1}{t}$.

435. Interest, rate, and time given, to find the principal.

If each member of the equation

$$i = prt$$

is divided by rt, the result will be

$$\frac{i}{rt} = p,$$

which means that *if the interest is divided by the product of the rate and the time, the quotient will be the principal.*

Examples for Written Work.

436. *Illustrative Example.* What principal will gain \$61.25 in 2 y. 6 mo. at 5%?

SOLUTION. $\frac{\$61.25}{0.05 \times 2\frac{1}{2}} = \frac{61.25}{0.125} = 490.$ *Ans.* \$490.

196. What principal at 5% will gain \$100 in 5 years?

197. What principal at 6% will gain \$72 in 2 years?

198. How much money must be put on interest at 3% to yield \$229.50 in 2 y. 6 mo.?

199. To gain \$341 in 7 mo. 15 d., what sum of money must be put on interest at 4%?

200. What sum of money put on interest at 4% will gain \$100,000 in 100 years?

201. What principal on interest 11 mo. 14 d. at 6% will gain \$8.256?

202. How much money on interest at 6% for 8 mo. 17 d. will gain \$18.3755?

203. How many 5% bonds of \$1000 each must be purchased to secure an income of \$1000 semi-annually?

204. A certain fund is invested in $4\frac{1}{2}$% bonds and yields \$297 quarterly. How much is invested?

437. The *amount* has been defined as the sum of the principal and the interest. Letting a stand for the amount, we may say $a = p + i$. But as $i = prt$, we may also say

$$a = p + prt \tag{1}$$

This equation may be written in the form

$$a = p(1 + rt) \tag{2}$$

For if p is multiplied by $1 + rt$, the result will be $p + prt$.

As an example suppose the amount of \$420 on interest $3\frac{1}{2}$ years at 6% is required.

This may be solved in two ways:

(1)

$$\begin{array}{rl}
 & \$420 = p \\
 & \underline{0.21} = rt \\
 & \overline{420} \\
 & \underline{840} \\
i = & 88.20 = prt \\
 & \underline{420.00} = p \\
a = & 508.20 = p + prt
\end{array}$$

(2)

$$\begin{array}{rl}
 & \$420 = p \\
 & \underline{1.21} = 1 + rt \\
 & \overline{420} \\
 & 840 \\
 & \underline{420} \\
a = & 508.20 = p(1 + rt)
\end{array}$$

438. Amount, rate, and time given, to find the principal.

If each member of the equation

$$a = p(1 + rt)$$

is divided by $1 + rt$, the result will be

$$\frac{a}{1 + rt} = p,$$

which means that *if the amount is divided by 1 plus the product of the rate and the time, the quotient will be the principal.*

Examples for Written Work.

439. *Illustrative Example.* What sum of money put on interest at 5% will amount to $299 in 3 years?

SOLUTION.

$a = \$299$	$rt = 0.15$	$\frac{\$299}{1.15} = \$260$
$r = \ 0.05$	$1 + rt = 1.15$	

Ans. $260.

205. What sum of money put on interest now at 5% will amount to $1000 in 10 years?

206. What principal on interest at 6% will amount to $1740 in 7 y. 6 mo.?

207. What is the principal on interest at 4% that will amount to $10,000 in 20 years?

208. A man who is under obligation to pay $2000 at the end of 8 years provides for his obligation by putting on interest now, at 4%, such a sum of money as will amount to $2000 in 8 years. How much does he put on interest?

PRESENT WORTH AND TRUE DISCOUNT.

440. The **present worth** of a sum of money due at some future time is that sum which, put on interest now, will amount to the given sum in the given time.

NOTE. — The last five examples are examples of present worth, the answer to each being the sum which, put on interest at the given rate, will amount to the given sum in the given time.

The difference between a sum of money due at a future time and its present worth is the **true discount.**

209. What is the true discount at 6% of $4200 due in 2 y. 6 mo.?

210. A young man has the option of receiving a legacy of $5000 at the age of 25 or its present worth at the age of 21. The rate of interest being 5%, what is the value of his legacy at the age of 21?

211. A man purchased a tract of woodland for $120,000, paying $20,000 cash, and agreeing to pay the rest in four equal annual payments at the end of one, two, three, and four years, with the option, however, of paying the present worth of the last three installments at the end of the first year. How much will it cost him to settle the whole claim at the end of the first year, the rate of interest being 7%?

COMPOUND INTEREST.

441. *Illustrative Example.* A loan of $800 was made with interest at 6% payable annually, unpaid interest to be added to the principal and draw interest at the same rate. No interest was paid for four and one half years, at the end of which time the loan with accumulated interest was paid. What was the whole amount paid?

WRITTEN WORK.

Principal	$800.000
Interest for 1st year	48.000
Amount, or principal for 2d year	848.000
Interest for 2d year	50.880
Amount, or principal for 3d year	898.880
Interest for 3d year	53.933
Amount, or principal for 4th year	952.813
Interest for 4th year	57.169
Amount, or principal for 5th year	1009.982
Interest for half a year	30.299
Amount	1040.281
Ans. Whole amount paid	$1040.28
Deduct loan	800.00
Leaving accumulated interest	$240.28

442. Interest accumulated by adding unpaid interest to the principal at stated intervals, the whole to form a new principal, is **compound interest**.

In the example, what is the principal? the amount? the compound interest?

Interest is compounded annually, semi-annually, quarterly, or for any other regular periods, as may be agreed upon. In the absence of an agreement, however, the law generally does not permit the collection of compound interest.

443. From the foregoing example may be derived the following

Rule.

To compute compound interest:

1. *Find the amount of the given principal for the first period of time. With this as a new principal, find the amount for the second period of time, and so continue for the whole time. The last amount is the amount required.*

2. *The last amount less the original principal is the compound interest.*

Examples for Written Work.

444. At compound interest annually, what is the amount

212. Of \$300 for 2 years at 6%?

213. Of \$520 for 5 years at 5%?

214. Of \$600 for $4\frac{1}{2}$ years at 7%?

215. Of \$2000 for 3 years 10 months at 6%?

216. Of \$5000 for 2 years 7 months at 8%?

217. What is the compound interest of \$3500 for 2 years 6 months at 6%, interest compounded semi-annually?

Note. — Take the interest at 3% for five periods of time.

218. What is the compound interest of \$900 for 2 years at 7%, interest compounded semi-annually?

219. What is the compound interest of \$700 for 1 year 6 months at 4%, interest compounded quarterly?

220. A boy at the age of 18 years puts \$100 in a savings bank. Every half year the bank adds 2% to the deposit for interest. What will the deposit amount to when the boy is a man 25 years old?

221. What is the difference at the end of five years between the amount of \$1000 at 6%, interest compounded annually, and the amount of the same sum of money at the same rate of interest but compounded semi-annually?

222. Show that the amount of \$1 at compound interest for 5 years, interest compounded annually at 6%, is equal to $(1.06)^5$ dollars; also that the amount of any number of dollars, say \$50 for the same time and at the same rate, is equal to $\$50 \times (1.06)^5$.

223. Show in general that, if a represent the amount, p the principal, r the rate, and n the number of years (or periods), that

$$a = p(1 + r)^n.$$

NOTE.—If $p = \$1$, the formula becomes

$$a = (1 + r)^n,$$

and this is the formula used in the making of the Compound Interest Table given in the next Article.

445. The work of computing compound interest may be shortened by the use of the

COMPOUND INTEREST TABLE.

Showing the amount of \$1 at compound interest from 1 year to 10 years, at 3, 4, 5, 6, and 7 per cent.

Years.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1.	1.030000	1.040000	1.050000	1.060000	1.070000
2.	1.060900	1.081600	1.102500	1.123600	1.144900
3.	1.092727	1.124864	1.157625	1.191016	1.225043
4.	1.125509	1.169859	1.215506	1.262477	1.310796
5.	1.159274	1.216653	1.276282	1.338226	1.402552
6.	1.194052	1.265319	1.340096	1.418519	1.500730
7.	1.229874	1.315932	1.407100	1.503630	1.605781
8.	1.266770	1.368569	1.477455	1.593848	1.718186
9.	1.304773	1.423312	1.551328	1.689479	1.838459
10.	1.343916	1.480244	1.628895	1.790848	1.967151

446. *Illustrative Example.* What is the compound interest of $1000 for 2 y. 4 mo. at 7% ?

WRITTEN WORK.

Amount of $1 at 7% for 2 years	$1.1449
Amount of $1000 at 7% for 2 years	1144.90
	$1.02\frac{1}{3}$
Amount of $1144.90 for 4 mo. } Amount of $1000 for 2 y. 4 mo. }	1171.6143
	1000.
Compound interest	$171.61 *Ans.*

The amount of $1000 for 2 years is first found, and the amount of that amount for four months is then found by multiplying by $1.02\frac{1}{3}$.

447. Examples for Written Work.

224. What is the compound interest of $450 for 2 y. 6 mo. at 5%, interest payable annually?

225. What is the compound interest of $1200 for 5 y. at 4%, interest payable annually?

226. What is the compound interest of $2500 for 4 y. at 8%, interest payable semi-annually?

227. What is the compound interest of $300 for 3 y. 2 mo. at 8%, interest payable semi-annually?

228. What is the compound interest of $380 for 1 y. 10 mo. 22 d. at 6%, interest payable semi-annually?

229. If at the age of 25 years, a person puts $1000 on interest, compounding annually at 6%, what will be the amount due him when he is 40 years old?

230. What is the difference between the compound interest of $10,000 for 5 y. at 5%, interest payable annually, and the simple interest on the same sum of money for the same time at the same rate?

231. Find the compound interest of $5000 for 17 y. at 5%, interest payable annually.

NOTE. — First find by the table the amount for 10 years, then find the amount of that amount for 7 years more.

BANK DISCOUNT.

448. *Illustrative Example.*

> 425\frac{50}{100}$. Providence, August 20, 1894.
>
> Sixty days after date I promise to pay to the order of __________ James Davis ______________
>
> Four hundred twenty-five and $\frac{50}{100}$ Dollars at the Merchants National Bank
>
> Value received.
>
> No. 24 Due October 19/22 Charles M. Stone

Suppose Davis wants the money promised by this note at once, August 20th. He can offer the note at a bank for the money. If the officers of the bank are satisfied that Stone's credit is good, so that he can be depended on to pay the note when it becomes due, or that Davis' credit is good, so that he can be depended on to pay the note if Stone fails to do so, the bank will pay Davis $425.50 less the interest on that sum for 63 days at the current rate. Supposing the current rate to be 7% per annum, Davis will get for the note $425.50 less $5.21, or $420.29. The note is then said to have been **discounted** by the bank.

Before giving the note to the bank Davis must **indorse** it, that is, write his name across the back of it, and so become responsible to the bank for its payment when it becomes due if Stone should fail to pay it at that time.

The bank reserves interest for 63 days instead of 60, as specified in the note, because the promisor, or maker, is allowed three days, called **days of grace**, beyond the time specified in the note, within which to make the promised payment. This note is *nominally* due October 19th, but not *legally* due until October 22d. On this last day it is said to *mature*.*

* In Alaska, Cal., Conn., Idaho, Ind. Ter., Ill., Maine, Mass., Md., Mo., Mont., N.H., N.J., N.Y., No. Dak., Ohio, Ore., Pa., Utah, Vt., Wis., days of grace have been abolished. In these states a note is nominally and legally due on the same day.

If the note had been offered for discount on some later day, say September 10th, then the time which the note would have had to run would have been from September 10th to October 22d, which is 42 days. The discount for 42 days would have been $3.47.

449. **Bank discount** is the interest reserved by a bank for advancing money on a promissory note, draft, or bill of exchange before it becomes due. The **proceeds, avails,** or **cash value** of a note is the face of the note less the discount. The time a note has to run from the day of discount to maturity is called the **term of discount.**

NOTE 1. — It will be seen that bank discount is simple interest on the face of a note for the term of discount; and that the bank not only receives interest on a larger sum of money than it lends, but it takes that interest in advance.

NOTE 2. — If a note is written so as to be payable a certain number of *days* after date, it matures after that full number of days and three days of grace (where days of grace are allowed) have expired. Thus a thirty days' note dated February 16, 1895, is due March 18/21, and matures March 21st.

NOTE 3. — If a note is written so as to be payable a certain number of *months* after date, calendar months are to be understood; and the note is nominally due on the day of the month corresponding to its date. Thus a note dated February 16th and payable two months after date is due April 16/19. If the month in which the note matures is a short month, and has no day corresponding to the date of the note, the note is nominally due on the *last* day of that month, and matures on the third day of the following month.

450. From what has been stated may be derived the following

Rules for Bank Discount.

1. To find the term of discount: *Count the number of days from the day of discount to the day of maturity.*

2. To find the discount on a non-interest bearing note: *Compute the interest on the face of the note for the term of discount at the given rate.*

3. To find the proceeds: *Subtract the discount from the face of the note.*

4. To find the discount and the proceeds of an interest bearing note: *First find the amount of the note at its maturity, and use this amount as the basis of* discount *as if it were the face of a non-interest bearing note.*

NOTE 1. — The common, or 360-day, method of computing interest is usually employed for discount.

NOTE 2. — The term of discount is usually reckoned by counting the actual number of days from the day of discount to and including the day of maturity. In this book it is always so reckoned, except in the case of a note made payable a certain number of months after date and discounted on the day of its date. In such a case, the discount will be reckoned by months and days.

Examples for Written Work.

451. Find the bank discount and proceeds of the following notes, discounted at date, with days of grace, at 6%:

232. A 30 days' note for $750.00.
233. A 60 days' note for $900.00.
234. A 20 days' note for $380.60.
235. A 90 days' note for $580.50.

Find the discount and proceeds of notes as follows:

	Face.	Date.	Time.	Date of Discount.	Rate.
236.	$780.00	May 9, 1895	60 days	June 3, 1895	6%
237.	$425.63	Aug. 4, 1894	90 days	Oct. 6, 1894	6%
238.	$268.00	Nov. 8, 1894	3 months	Jan. 2, 1895	7%
239.	$382.37	Jan. 1, 1895	15 days	Jan. 5, 1895	8%
240.	$126.65	Oct. 7, 1896	2 months	Nov. 1, 1896	5%
241.	$89.00	July 3, 1895	100 days	Sept. 10, 1895	7%
242.	$642.30	Sept. 12, 1894	30 days	Oct. 1, 1894	8%
243.	$575.82	Dec. 1, 1896	4 months	Dec. 14, 1896	6%

244.

$592\frac{25}{100}$ RICHMOND, February 10, 1894.

Three months after date, I promise to pay to the order of James McBride, Five Hundred Ninety-two and $\frac{25}{100}$ Dollars at the Manufacturers' National Bank, value received.

Discounted, at date, at 6%. WILLIAM GANNON.

245.

$1240 RALEIGH, June 16, 1894.

Sixty days after date, I promise to pay to the order of Stephen Gardiner, Twelve Hundred Forty Dollars, at the Atlas National Bank, in Boston, Mass., value received.

Discounted July 1, 1894, at 5%. BENJAMIN HAMMOND.

246.

$648\frac{24}{100}$ ATLANTA, GA., December 31, 1894.

Six months after date, I promise to pay George Dexter, or order, Six Hundred Forty-eight and $\frac{24}{100}$ Dollars, for value received. HIRAM BUTTERWORTH.

Discounted February 21, 1895, at 7%.

247.

$846\frac{75}{100}$ LOUISVILLE, KY., June 6, 1894.

Four months after date, for value received, I promise to pay Peter Francis, or order, Eight Hundred Forty-six and $\frac{75}{100}$ Dollars, at the office of the Girard Trust Co.

Discounted June 8, 1894, at 8%. WILLIAM WHARTON.

248.

$1048 ALBANY, N. Y., July 6, 1894.

Thirty days after date, for value received, we promise to pay to the order of Ingalls, Peters & Co., One Thousand Forty-eight Dollars, at the Commercial Bank.

JACOB VAN DUZER & CO.

Discounted July 8, 1894, at 7%. No grace.

249.

$956\frac{84}{100}$ NEW ORLEANS, December 30, 1894.

Three months after date, for value received, we promise to pay to the order of David Kimball, Nine Hundred Fifty-six and $\frac{84}{100}$ Dollars, at the Peoples' Bank.

BENNETT, HURD & CO.

Discounted January 15, 1895, at 8%.

250.

$3500 NEW YORK, January 15, 1894.

Ninety days after date, for value received, I promise to pay to the order of Edwin Carleton, Three Thousand Five Hundred Dollars.

WM. F. ROSCOE.

Discounted January 20, 1894, at 8%. Accurate interest. No grace.

251.

$2500 DETROIT, MICH., May 12, 1894.

Six months after date, for value received, I promise to pay to the order of Henry G. Davidson, Two Thousand Five Hundred Dollars, with interest at eight per cent per annum.

SAMUEL F. CRANE.

Discounted June 8, 1894, at 9%.

252.

$2853\frac{75}{100}$ CHARLESTON, November 20, 1894.

Four months after date, we promise to pay John G. Farlow, or order, Two Thousand Eight Hundred Fifty-three and $\frac{75}{100}$ Dollars, at our office, with interest at five per cent per annum, value received.

NORRIS BROTHERS & CO.

Discounted January 9, 1895, at $4\frac{1}{2}$%.

253.

$2000 St. Paul, Minn., March 22, 1895.

On September 1, 1895, we jointly and severally promise to pay to the order of Albert M. Weeks, Two Thousand Dollars, value received, with interest at six per cent per annum.

John F. Shaw.
Thaddeus Martin.

Discounted April 10, 1895, at $6\frac{1}{2}\%$.

BILLS OF EXCHANGE.

452. *Illustrative Example.*

£500. Boston, Aug. 23, 1894.

Sixty days after sight of this first of Exchange, second and third unpaid, pay to the order of Brown, Smith & Co.,
Five Hundred Pounds sterling,
value received, and charge the same to the account of
Kidder, Peabody & Co.
To Messrs. Baring Brothers & Co.
London.

This is an order made and signed by Kidder, Peabody & Co., bankers, of Boston, directing Baring Brothers & Co., bankers, of London, to pay Brown, Smith & Co., merchants, of England, five hundred pounds. An American merchant, we may suppose, had imported cloth from Brown, Smith & Co., valued at £500; but instead of sending this money to England, he paid an equivalent amount to Kidder, Peabody & Co., who gave him this order, called a bill of exchange, directing the London bankers to pay the English merchants £500.

The American merchant sent the bill to the English merchants, who, on receiving it September 5th, presented it to the London bankers for acceptance. This acceptance is a promise to pay the bill at maturity, and is signified by the bankers writing the word "accepted," the date,

and their name across the face of the bill. The English merchants could now have the bill discounted, just as if it were a promissory note; or they could keep it till maturity and receive full payment from the English bankers at that time. So the American merchant paid his debt, and the English merchants received their money, and yet no money was sent across the ocean.

But there remains the debt of the American bankers to the English bankers. How will this be adjusted? There will be no need of sending the money; because this particular debt will enter as merely one item into an open account which the two banking houses are keeping with each other. They are constantly drawing bills on each other, so that this particular bill will very soon be offset by bills which the English bankers will draw on the American bankers in favor of American merchants who have been sending goods to England.

453. There is another way in which the transaction just described might have been effected. The American merchant, whom we will call John Robinson, instead of sending a bill of exchange to the English merchants, might have waited to be drawn upon by them. Then the bill would have been in this form:

£ 500. *London, Aug. 10, 1894.*

Sixty days after sight of this first of Exchange, second and third unpaid, pay to the order of Baring, Brothers & Co.

Five Hundred Pounds sterling, value received, and charge the same to the account of

Brown, Smith & Co.

To John Robinson,
Boston, U.S.A.

Accepted Aug 28
John Robinson

The English cloth merchants, Brown, Smith, & Co., after having seen the cloth placed safely aboard ship and having received the bill of lading for it, write a bill of exchange as above and give it, with the bill of lading attached, to the Barings, receiving from that house the face

of the bill less a discount. The Barings send this bill of exchange with bill of lading attached to their correspondents, Kidder, Peabody, & Co. of Boston, with instructions to collect the bill and place the amount to the Barings' credit. The Boston bankers notify Robinson that by accepting the bill of exchange he can have his bill of lading, which is his evidence that the imported cloth belongs to him. Robinson pays the bill of exchange at maturity (or before maturity if allowed a discount) to the Boston bankers, and so the transaction is complete as far as the merchants are concerned. The bankers take care of their part of the transaction in the manner already explained.

Bills of exchange drawn by bankers on bankers, like the one first given above, are called *bankers' bills;* and those drawn by merchants on merchants, like the second one above given, are called *commercial bills.*

NOTE 1. — As a precaution against loss, bills of exchange are usually drawn in duplicate or in triplicate, and sent by different mails. The "second of exchange" bears the words "first and third unpaid," and the "third of exchange" bears the words "first and second unpaid." The payment of any one bill of a set makes the other two void.

NOTE 2. — Bills of exchange may be "on time" or "at sight." If the bill is to be payable on presentation, the words "at sight" or "on demand" are used.

Write the other two bills of the set to which each of the bills above given belongs.

454. Examples for Written Work.

254. Suppose an English merchant has sent you carpets, for which you have agreed to pay £600. Write one of a set of exchange, payable sixty days after sight, such as an American banker would draw on a London banker in favor of the English merchant, and sell to you for the purpose of making the payment. What would this exchange cost you at $4.85½ to the pound sterling? What would it cost you if written so as to be payable on demand, demand bills being quoted at $4.89 to the pound sterling? Why should demand (or sight) bills cost more than 60 days' bills?

255. Suppose an English merchant has sent you cutlery to the value of £175, and draws upon you for that amount at 30 days after sight. Write the first of exchange which he would draw for this purpose.

256. Write a 60 days' bill of exchange such as you might receive from an English merchant to whom you had sent goods to the value of $4000.

NOTE.—This would be an order by an English banker directing an American banker to pay you $4000.

257. Write a 60 days' bill of exchange such as you would draw upon an English merchant to whom you have exported 5000 bushels of wheat at $73\frac{1}{2}$ cents a bushel, in favor of the American banker to whom you sell the bill with the bill of lading attached. Would the banker pay you the full face of the bill? Why not?

455. Exchange with all parts of the world may be effected through London. This is because London is the great commercial center of the world; and its bankers draw upon and are drawn upon by bankers in all countries. Exchange with continental Europe can also be effected through the large commercial centers, as Paris, Vienna, Berlin, Hamburg, Amsterdam, etc. The whole commerce of the world, with insignificant exceptions, is carried on by bills of exchange. The commercial dealings between two countries may amount to thousands of millions of dollars, and yet the bills of exchange drawn in each country may be so nearly offset by bills drawn in the other that only a trifling amount of money is needed to pay the balance.

456. The rates of exchange between this country and the principal commercial centers are given daily in the newspapers. The following is taken from a New York paper:

"The market for sterling opened firmer in tone and still higher rates bid, the posted rates however remaining unchanged at $4.87\frac{1}{2}$ and

4.89. Closing was very firm, with bills scarce and eagerly sought, and higher rates ruled than any recently recorded. Rates for actual business were as follows: sixty days, $4.86\frac{3}{4}$ @ 4.87; demand, 4.88 @ $\frac{1}{4}$; cables, $4.88\frac{1}{4}$ @ $\frac{1}{2}$. Commercial bills were $4.85\frac{3}{4}$ @ 4.86.

"The following are the posted rates of the leading drawers of foreign exchange:

	60 days.	Demand.
Sterling	$4.87\frac{1}{2}$	4.89
Paris, francs	$5.15\frac{5}{8}$	$5.13\frac{3}{4}$
Antwerp, francs	$5.16\frac{1}{4}$	$5.13\frac{3}{4}$
Berlin, reichsmarks	$95\frac{5}{8}$	96
Amsterdam, guilders	$40\frac{3}{8}$	$40\frac{5}{8}$

"New York Exchange at Chicago to-day was 35¢ premium."

NOTE. — The rate for sterling shows the number of dollars and cents that must be paid for one pound sterling. The rate for francs shows the number of francs and centimes (hundredths of a franc) that can be bought for one dollar. The rate for reichsmarks (money of the German Empire) shows the number of cents that must be paid for four marks. The rate for guilders (Dutch florins) shows the number of cents that must be paid for one guilder.

457. The rates of foreign exchange are governed, as the prices for all other things are, by the law of supply and demand.

If our imports from a foreign country, England for example, have been greater in value than our exports to that country, then there will be a lively demand for bills on London with which to pay for these imports; but the supply of these bills will be short, because the exporters have fewer of them to offer. So exchange on London will rise. On the other hand if our exports have been greater in value than our imports, our exporters will have plenty of bills on London to offer for money. So exchange on London will fall. But the rise in the price of exchange on London will not go above a certain point; for if it did our merchants would find it cheaper to send gold to pay for their imports. Neither will exchange on London fall below a certain point; for if it did, our bankers would stop selling exchange and begin importing gold. This statement makes clear the following extract taken from the same New York paper:

"The par of sterling exchange is 4.86-67. The actual business rate of demand sterling bills at which gold can be exported to London without loss on regular commercial account is about 4.88 for bars and 4.89 for coin, and the rate for which it can be imported without loss is 4.83½."

It will be seen by comparing this with the former extract, that the rate for sterling exchange that day was nearly up to the gold-exporting point. Indeed, the same paper contained the statement that considerable amounts of gold had been purchased in New York for shipment to Europe.

458. Examples for Written Work.

258. Find the cost of a demand bill of exchange on London for £ 2000 at the posted rate. At the lowest rate for actual business above quoted.

259. Find the cost of a 60 days' bill of exchange on London for £ 865 at the lowest rate for actual business above quoted.

260. How much could have been obtained in New York for commercial bills on London amounting to £ 682 10*s.* at the highest rate above quoted for such bills?

261. My friend telegraphs me from Europe that he wishes me to have £ 1000 placed to his credit at a London banker's with the least possible delay. How much will it cost me to do this at the highest rate above quoted for "cables"?

262. Find the cost of a 60 days' bill of exchange on Paris for 2500 francs. Of a 60 days' bill on Antwerp for the same amount. Of a demand bill on either place for the same amount.

263. From Berlin, in Germany, I have imported books to the value of 1200 marks, and I have agreed to send a 60 days' bill of exchange for this amount. How much will the bill cost me?

264. The rate for sterling exchange in New York having risen to 4.89½, how much shall I save by purchasing gold in bars at 4.88 for the purpose of remitting £ 30,000?

265. My agent in Amsterdam telegraphs me that he has purchased on my account goods to the amount of 8500 guilders, and requests me to send him demand exchange for that amount. How much will the exchange cost me in New York?

DRAFTS.

459. *Illustrative Example.*

> *$ 2000.* *Boston, July 18, 1894.*
>
> *Thirty days after sight, pay to John Lafarge, or order,*
>
> *Two Thousand Dollars,*
>
> *and charge the same to the account of*
>
> *Richardson, Flint & Co.*
>
> *To Wm. Manderson & Co.,*
>
> *New Orleans.*
>
> Accepted Aug. 4. 1894. Wm. Manderson & Co.

Living in Boston and wishing to pay John Lafarge, of New Orleans, $2000, I purchase of Richardson, Flint & Co., bankers, of Boston, their draft (in form as given above) on Wm. Manderson & Co., bankers, of New Orleans. This draft I send to Lafarge, who presents it for acceptance, and, when the draft matures, receives the money from Wm. Manderson & Co.; or he receives the money at once if he allows a discount. By comparing this draft with the bill of exchange (page 269) it will be seen that they are in substance the same. Indeed, drafts are often called *inland* or *domestic bills of exchange*, for the reason that they are used to effect exchange between different parts of the same country, just as *foreign* bills of exchange are used to effect exchange between different countries. Drafts are not usually written in duplicate or triplicate, so that their language is simplified by the omission of the words required for that purpose.

The rates for domestic exchange vary slightly according to the pressure of supply and demand, the drafts being at a premium when

the supply is less than the demand and at a discount when the supply is greater than the demand. For example, the statement "New York exchange at Chicago to-day was 35¢ premium" (page 273) means that Chicago bankers have not an abundance of funds standing to their credit in New York to draw against; while at the same time the demand for drafts on New York is good. So the Chicago bankers put up the rate a little, charging 35 cents for a 1000-dollar draft and for other amounts, at the same rate. If the rate should go much higher, it would be cheaper to send money by express to New York than to buy drafts.

What conditions of business would cause New York exchange at Chicago to be at a discount? When New York funds are at a premium in Chicago, how should Chicago funds be in New York?

460. Examples for Written Work.

266. Find the cost of a Chicago draft for $12,000 on New York, payable at sight, when New York funds are quoted at 25¢ premium.

267. Find the cost in New York of a sight draft for $35,000 on San Francisco when San Francisco funds are at a discount of 15¢.

268. Find the cost of remitting $15,624.18 by sight draft from Boston to New Orleans when New Orleans funds are quoted in Boston at 75¢ premium.

269. What is the cost at Detroit of a sight draft for $36,782.75 on New York when New York funds are quoted in Detroit at a discount of 15¢?

STOCKS AND BONDS.

461. A **partnership** or **firm** is formed by two or more persons entering into an agreement to carry on business together, furnishing capital, and sharing profits and losses according to the terms of their agreement.

In the absence of legal notice to the contrary, the law holds each partner liable for the debts of the firm, not merely to the extent of the capital he has put in, but beyond this, if necessary, to the extent of all his other property.

462. A **corporation** is a company of persons authorized by law to hold property, transact business, sue and be sued in a company name just as an individual could.

Members of a corporation are not partners, and, generally, are not held responsible for the corporation debts beyond their several shares in the corporation property. Sometimes, however, there are statutes which increase the stockholders' liability, usually to double the amount of their several shares.

463. The **capital stock** of a business corporation is usually divided into **shares** of $100 each, and members subscribe for these shares, paying for them a part or the whole of their face value as called upon. The members are called **shareholders** or **stockholders**; and, as evidence of their property in the capital stock, hold papers called **certificates of stock.**

464. A **dividend** is the whole or a part of the net profits of the corporation business set aside for distribution among the stockholders. It is a percentage on the capital stock, and each stockholder receives the same percentage on the face value, called **par value,** of his stock.

Any part of the net profits retained by the corporation is called surplus. The surplus increases the real value of the shares, for it belongs ultimately to the shareholders.

465. An **assessment** is a sum of money which the stockholders are required to pay to meet the expenses or the losses of the corporation business. It is levied as a percentage on the capital stock, and each shareholder pays in a percentage on his stock at the same rate.

Sometimes a corporation raises more money for carrying on its business by issuing more shares of stock and promising a dividend to the purchasers of such stock in preference to the holders of the original stock, should the net profits be insufficient for a dividend to both. Hence arises the distinction between *common stock* and *preferred stock.*

466. Shares of stock are bought and sold in the stock market, the transaction consisting in a delivery of the certificates of stock to the purchaser, and a record of the trans-

fer on the books of the corporation. So the members of a corporation are constantly changing, some going out and others coming in. Persons who make a business of buying and selling stocks for their customers are called **stock brokers**; and their compensation is called **brokerage.**

When a stock is quoted at 90, the meaning is that a share, the par value of which is $100, can be bought for $90. When quoted at $108\frac{3}{4}$, the share would cost $108.75. In the first case the stock is at 10% discount, and in the other at $8\frac{3}{4}$% premium.

Stock brokerage is reckoned on the par value of stocks.

467. The government of the United States, the governments of the several states, and of counties, cities, towns, or school districts not infrequently borrow money, issuing as evidence of their indebtedness interest-bearing promissory notes called **bonds.**

468. Government bonds are usually offered by the government publicly for sale to the highest bidders.

The price obtained depends on the rate of interest offered, on people's confidence in the government's good faith and financial resources, and on the abundance or scarcity of money seeking investment.

When government bonds have passed into private hands they are bought and sold in the stock market.

469. Business corporations obtain loans by selling bonds in the same way that governments do; but a corporation usually secures its bonds by a mortgage on its property.

By consulting the newspaper the prices paid in the open market for different stocks and bonds can be learned; and these prices show what people think of them as investments.

It may be seen, for example, that a second mortgage bond is sold for less than a first mortgage bond of the same corporation; and shares of common stock are sold for less than shares of preferred stock. Again, by comparing the prices of bonds which pay the same rate of interest and have about the same number of years to run, it is easy to see which corporation or which government has the better credit in the money market.

The following quotations are taken from a daily paper:

At Broker's Board.

Bonds.

Atch. 4s	$72\frac{1}{8}$
Cen. P. 1sts	$101\frac{3}{4}$
Erie 2nds	75
Gen. El. 5s	$91\frac{3}{8}$
N. J. Cen. g. 5s	117
St. L. & S. F. gen. mtg.	$93\frac{1}{2}$
Tex. P. 1sts	$86\frac{1}{4}$
do 2nds	$26\frac{1}{4}$
U. Pac. 1sts	$104\frac{7}{8}$
Wabash 1st 5s	$104\frac{1}{4}$

Governments.

U. S. 4s reg.	$113\frac{3}{4}$
do 4s c.	$114\frac{1}{4}$
do 5s	$118\frac{3}{8}$

Railroads.

Atchison	$6\frac{1}{4}$
Balt. & Ohio	74
Boston & Albany	208
C. C. C. & St. L.	$39\frac{1}{2}$
do pref.	84
Harlem	258
N. J. Cen.	$112\frac{3}{4}$
N. Y., N. H., & H.	180
Union Pac.	$12\frac{3}{8}$

Express Companies.

Adams	148
Fargo	113

Industrials.

Am. Cot. Oil	$33\frac{3}{8}$
U. S. Cordage	$21\frac{1}{4}$

At Auction.

2 Merchants Nat. Bank	$159\frac{3}{4}$
1 Merrimac Mfg. Co.	1075
2 Mass. Cotton Mills	920
1 Kan. City Stock Yards Co.	134
\$1000 New Hampshire 6s., 1895	$101\frac{1}{4}$
\$2000 Boston 4s, 1923	112
\$1000 Duluth 4s, 1921	$95\frac{1}{2}$
\$1000 Minneapolis 4s, 1917	100

470. Examples for Written Work.

270. At the quotations above given find the cost of \$10,000 Atchison, Topeka, and Santa Fé R.R. 4 per cent bonds; \$5,000 Central Pacific R.R. First Mortgage Bonds; and \$5,000 Erie Railway Second Mortgage Bonds; brokerage $\frac{1}{4}\%$.

271. Find the cost of eight shares in the Boston and Albany R.R.; 16 shares in the preferred stock of the Cleveland, Cincinnati, Chicago, and St. Louis R.R.; and 20 shares in the New York, New Haven, and Hartford R.R.; brokerage $\frac{1}{4}$%.

272. Find the cost of twenty thousand dollars of the General Electric Company's 5 per cent bonds; $20,000 of the New Jersey Central R.R. 5 per cent bonds, payable in gold; $10,000 of the St. Louis and San Francisco R.R. General Mortgage Bonds; brokerage $\frac{1}{4}$%.

273. Among the securities coming into his hands the administrator of an estate finds certificates for 10 shares in the Harlem R.R.; 25 shares in the New Jersey Central; 15 shares in the Wells and Fargo Express Company; 12 shares in the Adams Express Company; 10 shares in the American Cottonseed Oil Company; 10 shares in the United States Cordage Company; 5 shares in the Merchants National Bank; 3 shares in the Merrimac Manufacturing Company; 5 shares in the Massachusetts Cotton Mills; and 10 shares in the Kansas City Stock Yards Company. At above quotations, what is this property worth?

274. At the prices above quoted, what per cent on the investment would be yielded annually by Wabash First Mortgage 5s? by General Electric 5s? by New Jersey Central gold 5s? by U.S. Government 5s?

275. At prices above quoted, compute the rate of income on money invested in U.S. 4% coupon bonds; in New Hampshire 6s; in Boston 4s; in Minneapolis 4s.

276. Which gives the larger return on the investment and how much larger: Atchison, Topeka, and Santa Fé 4s or City of Duluth 4s, as above quoted? Why, then, do the latter command a higher price in the market?

277. How much was gained on 500 shares Union Pacific bought at $11\frac{1}{4}$, and sold at $17\frac{5}{8}$, brokerage $\frac{1}{8}$% for each transaction?

278. At what per cent above par are the shares of the Merrimac Manufacturing Company selling, the par value being $1000? At what per cent below par are shares in the Massachusetts Cotton Mills selling, par value $1000? See quotations above.

279. When bonds with a face value of $5000 sell for $4625, at what per cent below par are they selling?

280. A broker sold a lot of stock for $5280, which was 12% below par. What was the par value?

281. A person bought 250 shares of Atchison, Topeka, and Santa Fé when the price was 47½ and sold it at 6¾. How much did he lose? What per cent on his investment did he lose?

282. How much is paid for a 5% bond if the return on the investment is 3%?

283. Which is the better investment, and how much, a 7% stock at 108, or a 6% stock at 105?

284. Wishing to secure 4½% a year for my money, how much can I afford to pay for a good 5% bond?

285. A broker deposits with a banker, as security for a loan, $10,000 Texas Pacific First Mortgage Bonds, and $5000 Texas Pacific Second Mortgage Bonds. How much money can he borrow on these securities if the banker is willing to lend 60% of their market value?

286. A merchant pledges 200 shares Boston & Albany R.R., 50 shares Harlem R.R., and 80 shares Adams Express Co., as collateral security for a loan of 80% of the market value of these stocks. How much did he borrow?

287. How much money can be borrowed on 500 shares Union Pacific R.R.; $5000 Union Pacific First Mortgage Bonds; and 500 shares Baltimore and Ohio R.R. as collateral security from a bank willing to advance 50% of the market value of these securities? See quotations above.

AVERAGE OR EQUATION OF PAYMENTS.

471. *Illustrative Example.* A debtor owes to one person the following sums at the dates specified: Oct. 1, \$262; Oct. 10, \$220; Nov. 6, \$250. At what date can he pay the whole debt without loss of interest to either party?

Interest Method.

WRITTEN WORK.

Due.	Items.	Days.	Interest.
Oct. 1,	\$262	0	
" 10,	220	9	\$0.33
Nov. 6,	250	36	1.50

1 day's int. of \$732 = 0.122) 1.83(15

Oct. 1 + 15 d. = Oct. 16. *Ans.*

EXPLANATION. — To do this example, we may suppose all the items to be paid at the earliest date at which any item becomes due, viz. Oct. 1.

This would involve a loss to the debtor of interest on \$220 from Oct. 1 to Oct. 10 (9 days), and on \$250 from Oct. 1 to Nov. 6 (36 days).

That no loss may result, the total of the items, \$732, should be paid as many days after Oct. 1 as will be required for \$732 at 6 % to gain \$1.83 of interest. To find this time, we divide \$1.83 by the interest of \$732 for 1 day at 6 %, and have for a quotient 15.

Fifteen days after Oct. 1 is Oct. 16. *Ans.* Oct. 16.

472. The process of finding the time when the payment of several items, due at different times, may be made at once, without loss of interest to either party, is **average, or equation of payments.**

473. The date at which several sums due at different times may be paid at once is the **average date** or **equated time of payment.**

474. From the foregoing operation may be derived a

Rule.

To find the average time for the payment of several sums due at different times:

1. *Select some convenient date; for example, the earliest date at which any item matures.*

2. *Compute the interest on each item from the selected date to the date of its maturity.*

3. *Add the interests thus found; divide their sum by the interest of the sum of the items for one day; the quotient will be the number of days from the selected date to the average date of payment.*

4. *Add this number to the selected date; the result will be the average date required.*

Proof.

Find the sum of the interests on all items due before *the average date, from the date at which they are severally due to the average date; also find the sum of the interests on all items due* after *the average date from that date to the dates at which they are severally due. If these sums are equal, or differ by less than half a day's interest on the sum of all the items, the result is correct.*

NOTE 1. — Any date may be selected from which to average an account. The last day of the month previous to that in which the earliest maturing item becomes due is a convenient date.

NOTE 2. — When any item contains cents, if less than 50, disregard them; if 50 or more, increase the units of dollars by $1.

475. Examples for Written Work.

288. What is the average date for paying three notes due as follows: March 31, $400; April 30, $300; May 30, $200?

289. What is the average date of maturity of three notes of $800 each, due respectively Nov. 5, Dec. 8, and Feb. 3?

290. What is the average date of maturity of the following items of account, viz.: $900, due Sept. 10; $2250.48, due Oct. 21; and $1049.65, due Oct. 28?

291. Find the equated time for paying $430, due in 5 months; $270, due in 9 months; and $300, due in 8 months.

292. Average the above, having the first item due in 3 months, the others in 9 months each.

293. A man purchased a farm for $3600, agreeing to pay $600 down, and the remainder in five equal semi-annual installments. What is the average date of the five installments?

294. Thomas Brown has borrowed sums of money from John Rich as follows: September 12, 1894, $500; December 22, 1894, $300; January 16, 1895, $600; and June 20, 1895, $400. What is the proper date for a note to cover all these loans?

295. What is the average date of the following account?

Mr. T. M. BAKER *To* **J. DOWNER & CO.,** *Dr.*

1896.		
April 10	*To Mdse on 30 days' credit . . .*	*$200*
May 16	*" " " 60 " " . . .*	*300*
June 3	*" " " 30 " " . . .*	*520*
" 18	*" Cash . . . .*	*250*

NOTE.—First find at what time each item falls due by adding the time of credit to the date of the item.

296. What is the equated date of maturity of the following items?

Mr. W. M. HAMMOND *To* **PENNSYLVANIA IRON CO.,** *Dr.*

1896.		
Mar. 11	*To Mdse on 30 days' credit . . .*	*$438*
" 29	*" " " 30 " " . . .*	*254*
Feb. 29	*" " " 60 " " . . .*	*145*
May 8	*" " " 60 " " . . .*	*300*
June 12	*" " " 30 " " . . .*	*159*

AVERAGE OF ACCOUNTS.

476. *Illustrative Example.* What is the average date of maturity of the following account kept by John Williams, creditor, against Charles Phelps, debtor?

Dr. **CHARLES PHELPS in Acct. with JOHN WILLIAMS.** *Cr.*

1895.		$	*1895.*		$
Mar. 18	*To Mdse*	*250*	*Apr. 1*	*By Cash . .*	*700*
" 30	*" "*	*600*	*" 20*	*" Real Estate*	*300*

WRITTEN WORK.

Dr. Due.	Items.	Days.	Interest.	*Cr.* Due.	Items.	Days.	Interest.
1895.				*1895.*			
March 18.	$250			April 1.	$700	14	$1.63
" 30.	600	12	$1.20	" 20.	300	33	1.65
	850		1.20		1000		3.28
					850		1.20
				1 day's int. of 150=0.025			2.08
							83

March 18 + 83 d. = June 9. *Ans.* June 9, 1895.

If interest at 6% were charged on all these items from the earliest date of any one, which is March 18, it would involve a loss to the debtor of $1.20, and to the creditor of $3.28, which is greater to the creditor than to the debtor by $2.08.

Now, as the balance of the account, $150, is owed by the creditor, he may be allowed to cancel this balance of interest lost, by paying the $150, a sufficient length of time *after* March 18 for the $150 to earn $2.08 of interest, which is 83 days. Eighty-three days after March 18 is June 9.

Observe that the balance of the account and the balance of the interest are on the *same* side of the account.

Suppose, for further illustration, that the first item in the foregoing account had been $500; what then would be the average date of its maturity?

In this case the balance of interest lost would be in favor of the creditor as before, but the balance of the account, $100, is owed by the debtor, and so he may be required to pay it a sufficient length of time *before* March 18 for the $100 to earn $2.08 of interest, which is 125 days. One hundred and twenty-five days before March 18 is Nov. 13. *Ans.* Nov. 13, 1894.

Observe in this case that the balances of the account and of the interest are on opposite sides of the account.

477. From these illustrations is derived the following

Rule.

To find the average or equated time for the settlement of an account when there are both debit and credit items:

1. *Find the interest on the several items of the account from the earliest date at which any item becomes due to their several maturities.*

2. *Find the balance of interest of the debit and credit sides of the account, also the balance of the items.*

3. *Divide the balance of interest by the interest of the balance of the items for one day. The quotient will be the time in days between the selected date and the average time of settlement.*

4. *Count this time* FORWARD *from the selected date, when the balances of the interest and of the account are on the same side of the account, and* BACK *when on opposite sides. The result will be the date of settlement.*

NOTE I. — When at the time of the averaging of an account the average time for paying the balance is already past, interest is allowed on the balance.

NOTE II. — Twelve per cent is a rate of interest much used in averaging accounts. It is convenient because it is equivalent to 1 per cent a month.

NOTE III. — It is usual to disregard cents in the items of the account, increasing the dollars by 1 if the cents disregarded are 50 or more.

478. Examples for Written Work.

297. At what date can the balance of the following ledger account be paid without loss to either party?

Dr. **DAVID G. SAUNDERS.** *Cr.*

		$	¢			$	¢
1895.				*1895.*			
April 1	*To Mdse . . . .*	*1000*	*00*	*April 14*	*By Mdse . . .*	*1393*	*00*
July 8	*" Cash . . . .*	*118*	*98*	*Aug. 10*	*" Real Estate.*	*94*	*33*

298. What is the average date of maturity for the following account?

Dr. **SLADE BROS. & CO.** *Cr.*

		$	¢			$	¢
1896.				*1896.*			
April 5	*To Sundries on 2 mo.*	*400*	*00*	*June 1*	*Mdse on 60 d. .*	*250*	*00*
Aug. 5	*" Mdse " 1 "*	*600*	*00*	*July 8*	*Mdse " 30 " .*	*700*	*00*
" 15	*" Mdse " 1 "*	*200*	*00*	*Aug. 13*	*Cash . . . .*	*200*	*00*

299. Find the average date of maturity of the following account:

Dr. **HENRY DRAPER & CO.** *Cr.*

		$	¢			$	¢
1894.				*1894.*	*[on 90 d.*		
Jan. 6	*To Mdse on 30 d. .*	*600*	*00*	*Jan. 1*	*By Real Estate*	*500*	*00*
Feb. 7	*" " " 60 " .*	*420*	*00*	*Mar. 16*	*" Cash . . .*	*300*	*00*

300. Average the following account:

Dr. **CHARLES BATES.** *Cr.*

		$	¢			$	¢
1896.				*1896.*			
Aug. 20	*To Mdse, 60 d. . .*	*173*	*15*	*Aug. 25*	*By Mdse, 30 d.*	*500*	*00*
Oct. 14	*" Cash . . . .*	*314*	*68*	*Sept. 12*	*" Mdse, 30 "*	*102*	*10*
" 18	*" Cash . . . .*	*230*	*00*				
" 30	*" Mdse, 1 mo. . .*	*81*	*25*				

301. Average the following account:

Dr.	WILLIAMS, RICE & CO.						*Cr.*
1895.		$	¢	*1895.*		$	¢
Jan. 6	*To Mdse, 3 mo. . .*	*339*	*42*	*Jan. 1*	*By Bal. of acct.*	*361*	*20*
" 25	*" Mdse, 30 d. . .*	*582*	*20*	*" 15*	*" Real Estate, 3 mo. . .*	*4000*	*00*
Feb. 21	*" Mdse, 3 mo. . .*	*85*	*12*				
May 29	*" Mdse, 2 " . .*	*2200*	*00*	*Feb. 7*	*" Mdse, 2 mo.*	*580*	*00*

479. Miscellaneous Examples.

302. Find the amount of a $400 note given the 26th of August, and paid the 29th of the next December, interest 8% per annum.

303. A trader borrowed $8000 at 5% per annum. He used this money in a way to yield him $1256.42 during the year. How much was the clear gain to him on the borrowed money?

304. A due-bill is given for $78.25 with interest at 8%. What will the bill amount to if it runs 72 days?

305. A note given May 7, 1895, on demand at 6% for $825, was indorsed July 6, 1895, $100; Oct. 9, 1895, $100. What sum remained due April 1, 1896?

306. Jan. 1, 1895, I gave a note for $712 at 5%. When will this note amount to $800?

307. The use of money being worth 5% a year, what is the present value of $4000 due four years hence?

308. If a merchant sells coal at $4.75 per ton, and has to wait 6 months for his pay, at what price could he afford to sell for cash, money being worth to him $1\frac{1}{2}$% a month?

309. Write a note on 60 days' time for such a sum that, when discounted by a bank at 6%, the proceeds may be $600.

310. What is the amount, at compound interest, of $100 from April 1, 1895, to Jan. 1, 1898, at 6% per annum, interest payable semi-annually?

311. A certain wealthy man is said to own $23,000,000 in U.S. 4 per cent bonds. What is his income per day from these bonds?

312. A man bought 6 per cent bonds at 90, received the interest for a year, and at the end of that time sold the bonds at 95, brokerage $\frac{1}{8}$% on the purchase and the same on the sale. What per cent did he make on his investment?

313. What should be the price of a thousand-dollar United States Government 4 per cent bond in order that the income received from it may be equivalent to a return of 3% semi-annually on the money invested?

314. From which investment would the larger returns be derived, from a stock paying 6 per cent purchased at 115, or from a stock paying 5 per cent purchased at 98?

315. What per cent must I make on an investment of $2000 to equal the income from six U.S. 4 per cent bonds of $500 each?

316. A owes B $4000, due Oct. 7. If he should pay $2500 of it on the 10th of the previous September, when should the balance be paid, that there may be neither gain nor loss of interest?

317. The value of 1 franc being $0.193, and the value of £1 being $4.8665, find the value of £1 in francs and centimes.

318. Find the value of 1 franc in English money.

319. Find the value of $1 in shillings and pence.

320. What is the value of $1 in francs and centimes?

321. I wish to remit £143 10 *s.* to London. Exchange is quoted at 4.87\frac{1}{4}$. What will my bill of exchange cost?

322. The franc being worth $0.193 and the mark $0.238, find the value of the mark in francs and centimes. Also find the value of the franc in decimals of a mark.

323. The quotation of exchange on Antwerp being 5.15$\frac{5}{8}$, what must be paid for a bill of exchange with which to remit 25,000 francs to that city?

SECTION XV.

PROPORTION.

480. *Illustrative Example.* Two persons bought cloth of the same kind. One bought 9 yards for 15 dollars, and the other 12 yards for 20 dollars. Were the amounts of money paid proportional to the quantities of cloth bought?

To answer this question we ask, first, what part 9 yards is of 12 yards; and then what part $ 15 is of $ 20. Finding that $ 15 is *the same part of* $ 20 that 9 yd. is of 12 yd., we say that the amounts of money paid were *proportional* to the quantities of cloth bought. We may say, too, that the quantities of cloth bought were proportional to the amounts of money paid.

481. If four numbers, as

9 yd. 12 yd. $ 15 $ 20,

are such that the first is the same part of the second that the third is of the fourth, the numbers are **proportional.** The last two numbers are proportional to the first two, and the first two to the last two.

482. To find what part one number is of another, the first number is divided by the second. The dividend and the divisor in such a case must be numbers of the same kind or denomination, and the quotient is called their **ratio.** This is the *measuring form* of division (Art. 130).

The **ratio** of two numbers is a number expressing the relative magnitude of one number as compared with or measured by the other.

In the above example, what is the ratio of 9 yd. to 12 yd.? What is the ratio of $ 15 to $ 20?

483. The **terms** of a ratio are the two numbers compared or measured the one with the other. One is the **antecedent** (dividend) and the other the **consequent** (divisor). They are usually written with a colon to indicate division; thus,

9 yd. : 12 yd. $15 : $20.

The terms of a ratio must be numbers of the same kind or denomination.

484. A **proportion** is an equality of ratios. The proportion which the two equal ratios above mentioned form is thus written:

9 yd. : 12 yd. = $15 : $20,

and is read, "Nine yards is to twelve yards as fifteen dollars is to twenty dollars."

Oral Exercises.

485. *a.* What is the ratio of 4 pounds to 14 pounds? Of 16 cents to 56 cents? What proportion, then, can be written?

b. Two men work, one 12 days for $21, and the other 4 days for $7. What is the ratio of 12 days to 4 days? Of $21 to $7? Is the pay proportional to the time? What proportion can be written to express this?

c. Read these proportions:

4 lb. : 14 lb. = 16¢ : 56¢.
12 days : 4 days = $21 : $7.
10 ft. : 30 ft. = 25 ft. : 75 ft.
$24.75 : $41.75 = 99 : 165.
30 miles : 75 miles = 6 hours : 15 hours.

d. How many terms are there in a proportion? Which of these are antecedents? Which are consequents?

e. Two men bought wheat. One bought 12 bushels for $8, and the other 30 bushels for $24. Were the amounts of money paid proportional to the quantities of wheat bought? Why not?

f. Can a ratio be found between 7 bushels and 21 days? Why not? Between 8 inches and 2 feet? Between 8 inches and 24 inches?

g. Write any two equal fractions, as $\frac{10}{15}$ and $\frac{16}{24}$, and tell whether the terms (numerator and denominator) of the one fraction are proportional to the terms of the other.

486. *The terms of equal fractions are proportional, and the fractions themselves are equal ratios.* Hence the equation

$$\frac{10}{15}=\frac{16}{24}$$

is another form, called the fractional form, of writing a proportion. The numerators are the antecedents and the denominators are the consequents of a proportion, which is read "10 is to 15 as 16 is to 24."

487. As the antecedent of a ratio is the dividend and the consequent the divisor, it follows that

When the antecedent is multiplied or the consequent is divided by any number,	the ratio is multiplied by that number.
When the antecedent is divided or the consequent is multiplied by any number,	the ratio is divided by that number.
When both terms of a ratio are multiplied or divided by the same number,	the value of the ratio is not changed.

488. Oral Exercises.

a. Multiply the ratio 2 : 3 by 2. *d.* Divide the ratio 4:7 by 2.

b. Multiply the ratio 5 : 6 by 3. *e.* Divide the ratio 5:2 by 3.

c. Multiply the ratio 3 : 4 by 4. *f.* Divide the ratio 5:3 by 5.

g. Change the ratio 18: 30 to an equal ratio in smaller terms.

h. Change the ratio 4: 9 to an equal ratio in larger terms.

i. In what respects is a ratio like a common fraction?

489. Of the four terms of a proportion, the first and last are called the **extremes**, and the second and third are called the **means**.

490. The product of the means and of the extremes.

Illustrative Example. Take any proportion:

(*Arithmetical.*)	(*Algebraic.*)
$3 : 5 = 12 : 20$	$a : b = c : d$
write it in the fractional form,	
$\frac{3}{5} = \frac{12}{20}$	$\frac{a}{b} = \frac{c}{d}$
multiply each member of the equation by 5,	by b,
$3 = \frac{12 \times 5}{20}$	$a = \frac{c \times b}{d}$
multiply each member by 20,	by d,
$3 \times 20 = 12 \times 5$	$a \times d = c \times b$

and the result in either case shows that:

In a proportion, the product of the extremes is equal to the product of the means.

NOTE. — This statement does not apply to a proportion whose four terms are all denominate numbers, unless denominations are disregarded. But it is usual to disregard denominations while working out a result from given denominate numbers. So with this understood, the statement may be taken as universal.

491. Given three terms of a proportion, to find the fourth.

Illustrative Example. Find the value of x in the proportion

$$3 : 4 = 9 : x.$$

WRITTEN WORK.

$$3\,x = 36$$
$$x = 12$$

The product of the extremes is 3 times x, written $3\,x$; and the product of the means is 36. Hence, $3\,x = 36$, and $x = 12$.

Oral Exercises.

492. In the following proportions find the value of the term represented by x.

a. $6 : 4 = 15 : x$.	*e.* 9 yd. : 12 yd. = \$15 : x.
b. $12 : 9 = x : 6$.	*f.* 2 lb. : 7 lb. = x : 56 cents.
c. $27 : x = 3 : 1$.	*g.* 12 days : x days = \$21 : \$7.
d. $x : 16 = 5 : 8$.	*h.* x : 4 hours = 15 miles : 12 miles.

NOTE. — In Exercises *e*, *f*, *g*, and *h*, disregard the denominations while finding the numerical value of x, and afterwards apply to this value its proper denomination.

493. Examples for Written Work.

1. Given $9 : 150 = 105 : x$, to find the value of x.
2. Given $850 : 75 = x : 65$, to find the value of x.
3. Given $375 : x = 225 : 255$, to find the value of x.
4. Given $x : 145 = 9 : 29$, to find the value of x.

Find the value and the denomination of x from the following proportions:

5. 35 hats : 200 hats = \$87.50 : x.
6. 750 acres : 3 acres = x : \$114.
7. 202 miles : x miles = 5.05 inches : 1.7 inches.
8. $3\frac{7}{8}$ inches : $4\frac{5}{8}$ inches = x : $13\frac{1}{8}$ inches.
9. \$1.00 : \$1.06 = x : \$500.
10. \$1.015 : \$1.00 = \$400 : x.

Applications of Proportion.

494. *Illustrative Example.* If 14 yards of cloth cost 98 cents, how much will 150 yards of the same cloth cost?

WRITTEN WORK.

14 yards : 150 yards = 98 cents : x cents

$$x = \frac{98 \times 150}{14} = 1050 \text{ cents}$$
$$= \$10.50.$$

Let x represent the cost of 150 yards. Then 98 and x are proportional to 14 and 150.

Now in the ratio 98 cents : x cents, the consequent is greater than the antecedent, because

150 yards of cloth cost more than 14 yards. Therefore the other ratio is to be made with the consequent greater than the antecedent, thus, 14 yd. : 150 yd. The value of x is found to be 1050 cents. *Ans.* $10.50.

495. From this example is derived the following

Rule.

To solve questions by proportion:

1. *Make the number that is of the same denomination as the answer the third term and the answer itself, represented by* x, *the fourth term.*

2. *Determine from the statement of the question whether the answer is to be greater or less than the third term.*

3. *Make the other two given numbers the first and second terms of the proportion, taking the greater number for the second term if the answer is to be greater than the third term, and the less number for the second term if the answer is to be less than the third term.*

4. *Multiply the third term by the second term, and divide the product by the first term.*

NOTE. — This rule is often called *The Rule of Three*, from the circumstance that *three* quantities are given to find a fourth.

496. Examples for Written Work.

11. If 17 yards of silk cost $ 76.50, how much will 24 yards cost?

12. How much wheat can be bought for $ 250, when 15 bushels can be bought for $ 10.20?

13. If $3\frac{1}{2}$ feet of lead pipe weigh $9\frac{3}{4}$ pounds, what is the weight of a piece measuring 84 feet?

14. A farmer raises $4\frac{1}{2}$ tons of hay on $2\frac{3}{4}$ acres of land. At the same rate of production, how many tons should he raise on $12\frac{3}{8}$ acres?

15. A father's age is to that of his son as 7 to 2, and the son is 14 years old. What is the age of the father?

16. If 400 pounds of coal are required to run an engine 12 hours, how many pounds will be required for 27 days of 10 hours each?

17. If 35 men can dig a trench in 14 days, how many men will it take to dig it in 5 days?

18. The amount of $1 on interest for 3 y. 9 mo. 12 d. being $1.227, how much money put on interest for the same time will amount to $1000?

19. A force of 220 laborers will clear the streets of snow in 8 hours. How many men will it take to do the same work in $5\frac{1}{2}$ hours?

20. Ralph and Arthur compare their speed in rowing, and find that Ralph can row 480 yards while Arthur rows 500 yards. If it takes Arthur $2\frac{1}{2}$ hours to cross a lake, how long will it take Ralph?

21. A has a horse that trots 8 miles while B's horse trots 7 miles. Both horses are driven over the same road, A's horse doing the distance in 2 h. 20 m. How long does it take B's horse?

22. The rates of two steamboats are as 11 to 13. If the faster boat makes her trip in 58 hours, in how many hours does the other make the same trip?

23. A contractor requires 64 days to pave a street when his men work 9 hours a day. How many days will he require when his men work 8 hours a day?

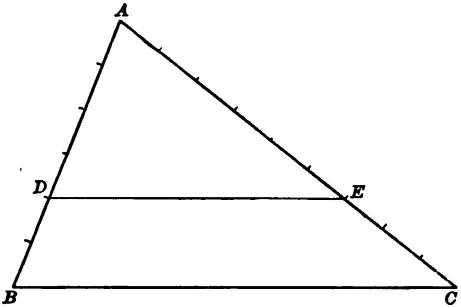
24. Turn to page 91, and do Examples 13 and 14 by proportion.

Additional examples which can be done by proportion may be found on pages 94, 110, and 259. See also page 143, Example 78, and page 145, Example 101.

497. Draw a triangle, *ABC*, and across it draw a line *DE* parallel to the side *BC*. Measure accurately *AD* and *AB*, also *AE* and *AC*. Find the ratio of *AD* to *AB*, also that of *AE* to *AC*. Are these two ratios equal? They should

be, for *a line drawn parallel to one side of a triangle across the other two divides them proportionally. That is,*

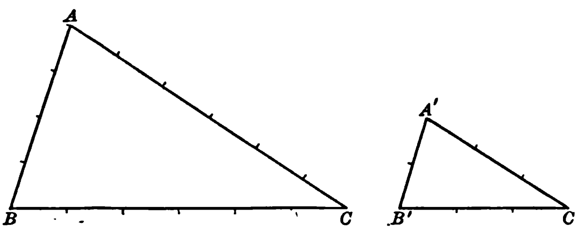
$$AD : AB = AE : AC.$$



25. Suppose the sides AB and AC of the triangle represented above measure 6 and 9 inches respectively, and the part AD measures 4 inches; what should the part AE measure?

26. Suppose the parts AD and AE measure $1\frac{3}{4}$ inches and $2\frac{5}{8}$ inches; how long is AB, if AC is $4\frac{7}{8}$ inches?

498. Draw two triangles like ABC and $A'B'C'$ represented below which shall be mutually equiangular; that is, which shall have the angle A equal to the angle A', $B = B'$, and



$C = C'$. Measure the sides of these triangles, and divide the length of each side of one triangle by the length of the corresponding side of the other. Are the three ratios thus

found equal? They should be, for *if two triangles are mutually equiangular their corresponding sides are proportional.* That is,

$$AB : A'B' = BC : B'C' = CA : C'A'.$$

These are similar triangles.

499. Draw two triangles, one with sides measuring, say, 18, 24, and 27 units, and the other with sides measuring 42, 56, and 63 units. Are the sides of these two triangles proportional? Measure the angles. Are the triangles mutually equiangular? They should be, for *if two triangles have their corresponding sides proportional, they are mutually equiangular.* These are similar triangles.

500. Similar triangles have these two properties:

(1) *Their corresponding angles are equal;*

(2) *Their corresponding sides are proportional.*

From what has just been said, it appears that triangles having one of these properties have also the other; so that similar triangles may be recognized either by their mutually equal angles, or by their proportional sides.

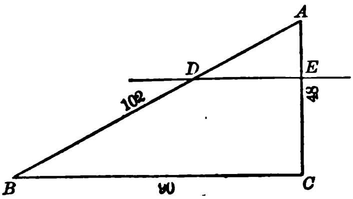
27. There are two similar triangles, one with sides measuring 11, 8, and 7 inches, and the other with its longest side measuring 33 inches. Find the other two sides. Draw these triangles to a convenient scale.

28. The longest side of a triangle is 87 ft., and the triangle is similar to another whose sides are 6 ft. 8 in., 7 ft., and 9 ft. 8 in. Find the other two sides of the first triangle.

29. The sides of a triangle are 39, 52, and 65 inches in length. The longest side of a similar triangle measures 95 inches. Find the other two sides.

30. A right triangle with base 90 in., perpendicular 48 in., and hypotenuse 102 in., is crossed by a straight line parallel

to the base which cuts off $\frac{3}{8}$ of the perpendicular. Find the two parts into which the hypotenuse is divided. Find the length *DE*, of the parallel line itself.



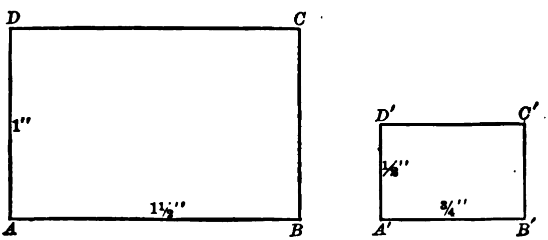
31. Is the triangle whose sides are 27, 36, and 45 similar to the triangle whose sides are 42, 56, and 70? Why?

32. The sun is shining at the same moment on two upright objects, a tree and a stake for example, which cast shadows on level ground. Imagine a line drawn from the top of each object to the end of its shadow. These lines represent the direction of the sun's rays, and are parallel. With the upright objects and their shadows they form two similar triangles. If the shadows measure $9\frac{1}{2}$ ft. and 90 ft., and the stake is 7 ft. high, how high is the tree?

33. What is the height of a church spire which casts a shadow 238 ft. long, if at the same moment a cane, 35 in. long, held vertically, casts a shadow 6 ft. 5 in. long?

501. Two plane figures are similar *if all the lines of one are proportional to the corresponding lines of the other.*

It is not enough that the two figures should have the same shape in the sense of having their angles equal each to each; they must have all



their corresponding lines proportional. Thus, a square and an oblong are not similar figures, for although each has four square corners, their

breadths are not proportional to their lengths. But two squares are similar figures. Why?

Two oblongs are similar if their breadths are proportional to their lengths; that is, if

$$AB : A'B' = AD : A'D'.$$

Their diagonals, AC and $A'C'$, are also proportional to their lengths or to their breadths.

34. An oblong 77 ft. long, 36 ft. wide, has a diagonal 85 ft. long. Find the length and the diagonal of a similar oblong which is 42 ft. wide?

35. The plan of a house shows a room which measures 16 in. in length by 10 in. in width. If the room itself is to be 32 ft. long, how wide is it to be? How long is the whole house to be, if it measures 42 in. on the plan? A chimney which is to be 32 in. square will be shown by how large a square on the plan?

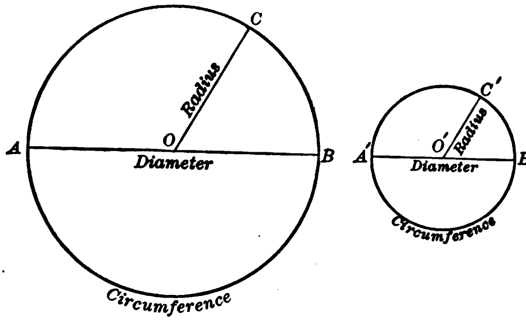
36. Two maps of the same region are similar plane figures. To test this take two maps of any region, say the New England States, one small, the other large. Measure the distances from Boston to Providence, from Providence to New Haven, from New Haven to Springfield, from Springfield to Montpelier, from Montpelier to Bangor, etc., first on the smaller map and then on the larger. Divide each distance measured on the larger map by the corresponding distance measured on the smaller map. You should find the same quotient (ratio) in each case.

37. On a city plan drawn to a scale of 100 ft. to the inch, what is the breadth of a street 80 ft. wide? What are the dimensions of a house lot fronting 40 ft. on the street and extending back 125 ft.?

38. On a township map drawn to a scale of one fourth of a mile to the inch, what are the dimensions of a farm 420 rods long and 200 rods wide?

39. On a map drawn to a scale of 40 miles to the inch, what would be the distance from Boston to Albany, which is about 200 miles?

502. Circles are similar figures; and *their circumferences are proportional to their diameters or to their radii.*



40. The diameters of two circles are as 24 to 39, and the circumference of the smaller circle is 50 ft. What is the circumference of the larger circle?

41. The diameters of two carriage wheels are as 3 to 5. How many times does the smaller wheel turn while the larger turns 1000 times?

503. The ratio of the circumference to the diameter of a circle cannot be expressed in numbers with absolute precision. A very useful approximation is 22 : 7.

42. What is the circumference of a cart wheel the diameter of which is 4 feet?

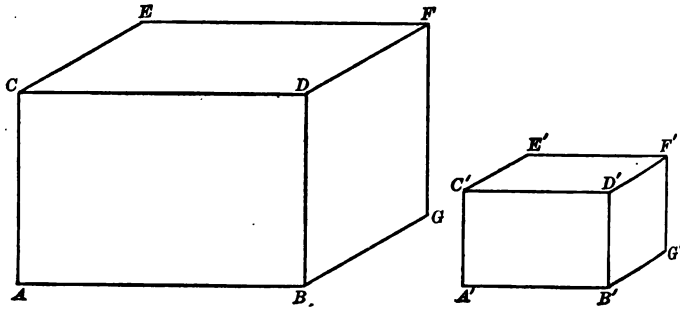
43. A string passed around the trunk of a large tree measures 56 ft. What is the diameter of the tree?

44. What is the diameter of a clock face the circumference of which is 8 feet?

45. With compasses opened $2\frac{5}{8}$ inches from point to point, draw a circle. Find the length of the circumference.

46. Another very close approximation to the value of the ratio of the circumference to the diameter is 355 : 113. Using this, find the diameter of a circular race track measuring one mile (5280 ft.) in circumference.

504. Two solids are similar *if all the lines* (edges, diagonals, etc.) *of one are proportional to the corresponding lines of the other.*



It is not enough that the two solids should have the same shape in the sense of having their corners alike, as a cube and a brick, but the corresponding lines must be proportional.

A cube and a brick are not similar, because the length, breadth, and thickness of the former are not proportional to the length, breadth, and thickness of the latter. But two cubes are similar solids. Why?

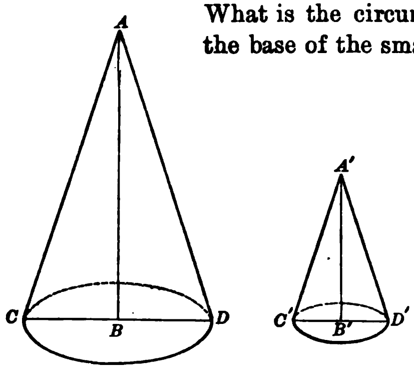
47. A common brick is 8 in. long, 4 in. wide, and 2 in. thick. Find the length and width of a stone 1 ft. 2 in. thick and similar to a brick in shape.

48. Is a rectangular block of marble with dimensions $4 \times 1\frac{1}{2} \times 1\frac{1}{4}$ ft. similar to one with dimensions $5 \times 1\frac{3}{4} \times 1\frac{1}{2}$ ft.? If not, change the width and thickness of the first block to make it similar to the second.

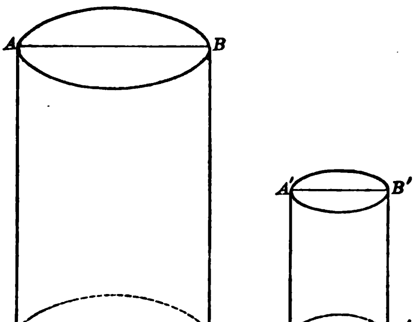
49. A monument is 180 feet high and 25 feet square at the base. If a model of the monument is made 15 inches high, how large is its base?

50. The three edges of a pyramid meeting at the top are each 15 in. in length. The base of the pyramid is a triangle, each side of which measures 6 inches. Find the corresponding lines of a pyramid similar to this, but higher in the ratio of 5 to 3.

51. The heights of two similar cones are as 4 to 2. The circumference of the base of the larger one is 14 inches. What is the circumference of the base of the smaller one?



52. The diameters of two similar cylinders are as 10 to 5. If the height of the larger is 24 inches, what is the height of the smaller?



53. Suppose the diameters, CD and $C'D'$, of the bases of two similar cones (see figures above) to measure 26 inches and 13 inches respectively, and suppose the slant height CA of the larger cone to measure 40 inches; what does the slant height $C'A'$ of the smaller cone measure?

PARTNERSHIP OR DISTRIBUTIVE PROPORTION.

505. *Illustrative Example.* Three men, A, B, and C, form a partnership, agreeing to share profits or losses in proportion to the parts of the capital put in by them. A put in $10,000, B put in $15,000, and C put in $20,000. They gained $ 9276.39. What was each partner's share?

WRITTEN WORK.

$$\$45{,}000 : \$10{,}000 = 9276.39 : \text{A's share.}$$
$$\$45{,}000 : \$15{,}000 = 9276.39 : \text{B's share.}$$
$$\$45{,}000 : \$20{,}000 = 9276.39 : \text{C's share.}$$
$$\text{A's share} = \frac{9276.39 \times 10{,}000}{45{,}000} = \$2061.42$$
$$\text{B's share} = \frac{9276.39 \times 15{,}000}{45{,}000} = \$3092.13$$
$$\text{C's share} = \frac{9276.39 \times 20{,}000}{45{,}000} = \$4122.84$$
$$\$9276.39$$

The proportions are formed on the principle that the whole capital is to the part put in by one partner as the whole gain is to that partner's share of the gain.

506. Examples for Written Work.

54. A and B formed a partnership, A putting in $ 6000, and B $ 8000. They gained $ 4431. What was the share of each?

55. Three merchants, A, B, and C, shipped goods from Calcutta to New York by the same vessel. The value of A's part of the cargo was $ 3650; of B's, $ 8265; and of C's, $ 9840. During a storm a part of the cargo valued at $ 6925 was thrown overboard. What was each merchant's share of the loss?

56. A bankrupt owed $ 950 to A, $ 875 to B, $ 1260 to C, and $ 1000 to D. His whole property was sold for $ 2584.80, of which $ 256.28 was used to pay the expenses of the sale and settlement. What was each person's share of the remainder?

57. Divide a line 10 inches long into parts proportional to the numbers 3, 4, and 5.

58. Divide 640 acres of land among four men in proportion to the numbers 3, 5, 7, and 9.

59. Divide the distance from A to B, which is 168 miles, into 5 day's journeys proportional to the numbers 4, 5, 6, 5, and 4.

60. A man left by his will an estate valued at \$ 35,000 to be divided among his four children in proportion to their ages, which were 14, 17, 19, and 20 years at the time of the division. How much should each child receive?

61. There were three policies of insurance in different companies on a stock of goods in a store: one for \$ 10,000, another for \$ 15,000, and another for \$ 12,000. The goods were partially destroyed or damaged by fire and water, causing a loss of \$ 11,500. How much of this loss should be borne by each insurance company?

62. A ship valued at \$ 90,000 was owned $\frac{1}{4}$ by A, $\frac{3}{8}$ by B, $\frac{5}{16}$ by C, and $\frac{1}{16}$ by D. It was insured for $\frac{3}{4}$ of its value, but was totally consumed by fire at sea. What was the loss to each owner?

507. *Illustrative Example.* H, B, and L formed a partnership. H put in \$ 2500 for 8 months; B \$ 3200 for 10 months; and L \$ 2000 for 12 months. They gained \$ 1852.50. What should be each partner's share of the gain?

WRITTEN WORK.

\$ 2500 × 8 = \$ 20,000	76 : 20 = \$ 1852.50 : H's share.
\$ 3200 × 10 = \$ 32,000	76 : 32 = \$ 1852.50 : B's share.
\$ 2000 × 12 = \$ 24,000	76 : 24 = \$ 1852.50 : L's share.
\$ 76,000	

The distribution is to be made on the double basis of money and time. The use of $2500 for 8 months is equivalent to the use of $20,000 for 1 month; the use of $3200 for 10 months is equivalent to the use of $32,000 for 1 month; and the use of $2000 for 12 months is equivalent to the use of $24,000 for 1 month. Therefore, the distribution should be made in proportion to $20,000, $32,000, and $24,000; or, what would be the same, in proportion to the numbers 20, 32, and 24.

Ans. H's share, $487.50; B's, $780; C's, $585.

63. X, Y, and Z engaged in a land speculation, agreeing to share the profits in proportion to the money invested and the time it remained invested. X put in $ 12,000 and withdrew it at the end of 3 years; Y put in $18,500 and withdrew it at the end of 4 years; and Z put in $ 22,500, which remained till the end of the 6th year, when a distribution of profits amounting to $ 48,256.18 was made. Find each man's share of the profits.

64. Two men, A and B, received $ 2500 for building a road. A furnished 10 laborers for 48 days and 5 horses for 42 days, and B furnished 8 men for 45 days and 4 horses for 30 days. Supposing a day's labor of a man as compared in value with that of a horse is as 7 to 5, what should be the share of the money for each contractor?

65. Four men hired a pasture the whole season for $500. The first put in 4 horses for 20 weeks, 3 cows for 12 weeks, and 10 calves for 24 weeks; the second put in 5 cows for 16 weeks; the third put in 2 horses for 15 weeks; and the fourth put in 6 cows for 18 weeks and 15 calves for 21 weeks. Supposing that a cow eats 4 times, and a horse 6 times, as much as a calf, how much should each man pay for the animals he pastured?

SECTION XVI.

POWERS AND ROOTS.

INVOLUTION.

508. *Illustrative Examples.* What is the product of the two equal factors 3×3? of 5×5? of 7×7? of 9×9? of 10×10?

What is the product of the three equal factors $2 \times 2 \times 2$? of $3 \times 3 \times 3$? of $4 \times 4 \times 4$? of $5 \times 5 \times 5$?

What is the product of the four equal factors $2 \times 2 \times 2 \times 2$? of $3 \times 3 \times 3 \times 3$? of the five equal factors $2 \times 2 \times 2 \times 2 \times 2$?

509. A **power** is a product of equal factors. The product of two equal factors is a **second power**; the product of three equal factors is a **third power**; the product of four equal factors is a **fourth power**; and so on.

What is the second power of 3? the third? the fourth? the fifth?
What is the third power of 5? the fourth? the fifth?
What is the second power of 10? the third? the fourth?

NOTE. — The names *square* and *cube* are often used instead of *second power* and *third power*, because the area of a square is found by multiplying together two equal factors (length and breadth), and the volume of a cube is found by multiplying together three equal factors (length, breadth, and height).

510. A power is indicated by writing one of the equal factors with a small figure, called the **exponent**, just above and to the right of it.

Thus, 5^3 indicates and is read "the third power of 5," or "5 to the third power," or "the cube of 5," or "5 cube."

511. **Involution** is the process of finding the power of a number, or raising a number to a power. It is performed by multiplication.

Oral Exercises.

512. *a.* Give the second powers (squares) of all the integral numbers from 1 to 10 inclusive.

b. Give the third powers (cubes) of all integral numbers from 1 to 10 inclusive.

c. What is the square of $\frac{1}{2}$? of $\frac{1}{3}$? of $\frac{2}{3}$? of $\frac{4}{5}$?

d. What is the square of 0.3? of 0.9? of 0.04?

e. What is the cube of $\frac{1}{2}$? of $\frac{1}{3}$? of $\frac{2}{3}$? of $\frac{3}{5}$?

f. What is the cube of 0.5? of 0.1? of 0.02?

g. What is the square of 20? of 40? of 90?

h. What is the cube of 10? of 30? of 50?

Examples for Written Work.

513. Find the powers of the following numbers as indicated:

1. 18^2.	**5.** 9^2.	**9.** $(0.17)^2$.	**13.** $(\frac{3}{10})^4$.	**17.** $(1.05)^3$.
2. 27^3.	**6.** 99^2.	**10.** $(10.8)^2$.	**14.** $(1.9)^4$.	**18.** $(1.03)^4$.
3. 145^3.	**7.** 999^2.	**11.** $(2\frac{1}{2})^3$.	**15.** $(0.1)^5$.	**19.** $(1.06)^2$.
4. $(\frac{7}{16})^2$.	**8.** 99^3.	**12.** $(0.125)^3$.	**16.** $(0.02)^5$.	**20.** $(1.06)^5$.

NOTE. — The results of the last four examples read as dollars are, respectively, the amounts of 1 dollar at compound interest at 5% for 3 years, at 3% for 4 years, at 6% for 2 years, and at 6% for 5 years. See the table (p. 262).

EVOLUTION.

514. One of the equal factors which multiplied together produce a number is a **root** of that number.

One of the two equal factors of a number is the **second root**, or **square root**; one of the three equal factors is the

third root, or **cube root**; one of the four equal factors, the **fourth root**; and so on.

Thus, 8 is the second root of 64, 5 is the third root of 125, and 2 is the fourth root of 16.

515. A root is indicated by the sign $\sqrt{\ }$, which is called the **radical sign.** But if the root is any other than a square root, a small figure, called the **index** of the root, is written over the opening of the sign.

Thus, $\sqrt{49}$ indicates and is read "the square root of 49"; $\sqrt[3]{64}$, "the cube root of 64"; $\sqrt[4]{81}$, "the fourth root of 81."

516. Evolution is the process of finding the root of a number. It is also called extracting the root.

Oral Exercises.

517. Find the roots indicated:

NOTE. — If the root is not readily seen, separate the given number into its prime factors, and arrange these into equal sets.

a. $\sqrt{16}$.	*f.* $\sqrt[3]{125}$.	*k.* $\sqrt[4]{16}$.	*p.* $\sqrt{.0064}$.
b. $\sqrt{81}$.	*g.* $\sqrt[3]{64}$.	*l.* $\sqrt[4]{81}$.	*q.* $\sqrt{.0049}$.
c. $\sqrt{100}$.	*h.* $\sqrt[3]{1000}$.	*m.* $\sqrt[5]{243}$.	*r.* $\sqrt{0.36}$.
d. $\sqrt{400}$.	*i.* $\sqrt[3]{27000}$.	*n.* $\sqrt[6]{64}$.	*s.* $\sqrt[3]{.008}$.
e. $\sqrt{3600}$.	*j.* $\sqrt[3]{64000}$.	*o.* $\sqrt[6]{1000000}$.	*t.* $\sqrt[3]{0.125}$.

Find

u. $\sqrt{1}$, $\sqrt[3]{1}$, $\sqrt[4]{1}$, $\sqrt[n]{1}$ (meaning *any* root of 1).

v. $\sqrt{0.01}$, $\sqrt[3]{0.001}$, $\sqrt[4]{0.0001}$, $\sqrt[5]{0.00001}$.

w. $\sqrt{121}$, $\sqrt{1.21}$, $\sqrt{144}$, $\sqrt{1.44}$.

x. $\sqrt{1\frac{7}{9}}$, $\sqrt{1\frac{9}{16}}$, $\sqrt{12\frac{1}{4}}$, $\sqrt{30\frac{1}{4}}$

SQUARE ROOT.

518. To find the number of figures required to express the square root of a number.

What is the smallest integral number expressed by one figure? What is the square of this number? What is the largest integral number expressed by one figure? What is the square of this number? Any number from 1 to 81 is expressed by how many figures? If, then, a root is expressed by *one* figure, by how many figures is its square expressed?

What is the smallest and what is the largest integral number expressed by two figures? What are the squares of these numbers? Any number from 100 to 9801 is expressed by how many figures? If, then, a root is expressed by *two* figures, by how many figures is its square expressed?

In a similar way it can be shown that if a root is expressed by *three* figures, its square is expressed by *five* or *six* figures; and that if the root is expressed by *four* figures, the square is expressed by *seven* or *eight* figures, and so on indefinitely.

In general, therefore, *the figures of a number are twice as many, or one less than twice as many, as the figures of its square root.*

Accordingly, *if the figures of a number are separated into periods of two figures each, beginning with the units' figure, there will be as many of these periods as the square root of the number has figures.*

NOTE. — The last (left-hand) period may have one or two figures. How many figures has the square root of 1156? of 334,084? of 21,025?

519. To find the parts of the second power of a number.

To find what parts the second power of a number is made up of, raise a number, 34 for example, to the second power thus:

$$
\begin{array}{rcrr}
34 & = & 30 + 4 & t + u \\
\underline{34} & = & \underline{30 + 4} & \underline{t + a} \\
136 & = & 120 + 16 & tu + u^2 \\
\underline{102} & = & \underline{900 + 120\phantom{{}+16}} & \underline{t^2 + tu\phantom{{}+u^2}} \\
1156 & = & 900 + 240 + 16 & t^2 + 2tu + u^2
\end{array}
$$

The work in the second form is done without carrying, and so shows all the partial products obtained by multiplying each term (Art. 32)

of the multiplicand by each term of the multiplier. The work at the right is in algebraic form, t and u standing for the tens and the units of any number whatever.

The product in either case shows that *the second power of any number consisting of tens and units is made up of three parts:*

(1) *the square of the tens,*
(2) *twice the product of the tens by the units,*
(3) *the square of the units.*

Formula: $$(t+u)^2 = t^2 + 2\,t\,u + u^2.$$

520. To extract the square root.

Illustrative Example. (1) What is the square root of 1156?

WRITTEN WORK.

$$\begin{array}{lrl}
t^2 + 2\,t\,u + u^2 = & 11'56 & (34 \\
t^2 \qquad\qquad = & 9 & \\
\quad 2\,t \qquad = 60 & \multicolumn{1}{|l}{256} & \\
\qquad\quad u = \underline{\;4\;} & \multicolumn{1}{|l}{} & \\
\quad 2\,t + u = \underline{64} & \multicolumn{1}{|l}{} & \\
\quad 2\,t\,u + u^2 = & \underline{256} & .
\end{array}$$

The figures of the given number being separated into periods of two figures each, show that the square root consists of tens and units. If the root is represented by $t+u$, its square, or the given number, will be represented by $t^2 + 2\,t\,u + u^2$; and the values of the several parts of this formula are to be sought in the given number.

Since *the square of the tens of the root*, t^2, is hundreds, the 11 hundreds of the given number contains the square of the tens of the root. The greatest square in 11 (hundreds) is 9 (hundreds), the square root of which is 3 (tens). That is,

$$t^2 = 900, \quad t = 30.$$

So the first figure of the root is 3.

Subtracting 900 from the given number, and t^2 from the formula, gives the equal remainders

$$2\,t\,u + u^2 = 256,$$

which, by separating the first into factors,* may be written

$$(2\,t + u)u = 256.$$

---

* It is easy to see that if $2\,t+u$ is multiplied by u, the product will be $2\,t\,u + u^2$.

If the value of the first factor, $2t + u$, were known, 256 divided by it would give the second factor, u. But $2t$ is much the larger part of the first factor and is known; so it can be used as a trial divisor for finding a probable value of u, which can be verified afterwards. Dividing 256 by $60(=2t)$ gives 4. So 4 is the second figure of the root, provided the remaining terms of the formula, $2tu + u^2$, do not exceed 256.

But	$2t = 60$
and	$u = 4,$
adding, the sum is	$2t + u = 64,$
which multiplied by	$u = 4$
gives	$2tu + u^2 = 256.$

So the value of u is verified. The true divisor, $64(=2t+u)$, multiplied by the quotient, $4(=u)$, gives a number not exceeding 256.*

Since there is no remainder, the given number is a perfect square, and 34 is its square root.

$$\sqrt{1156} = 34.$$ *Ans.* 34.

Examples for Written Work.

521. Find the square root of

21. 1681.	**24.** 1936.	**27.** 5625.	**30.** 8836.
22. 2704.	**25.** 3721.	**28.** 6561.	**31.** 9801.
23. 324.	**26.** 841.	**29.** 361.	**32.** 289.

Find the square root of the largest square in each of the following numbers; and find the remainder:

33. 730.	**37.** 1450.	**41.** 2450.	**45.** 2900.
34. 4300.	**38.** 5800.	**42.** 7760.	**46.** 1440.
35. 300.	**39.** 1770.	**43.** 9900.	**47.** 2620.
36. 250.	**40.** 360.	**44.** 490.	**48.** 8000.

* If the value of u had proved too large, it would have been necessary to diminish it, and repeat the process of verification.

522. *Illustrative Example.* (2) What is the square root of 190,794.24?

WRITTEN WORK.

$t^2 + 2tu + u^2 =$		19'07'94'.24	436.8
$t^2 =$		16	
$2t =$	80	307	
$u =$	3		
$2t + u =$	83		
$2tu + u^2 =$		249	
$2t =$	860	5894	
$u =$	6		
$2t + u =$	866		
$2tu + u^2 =$		5196	
$2t =$	8720	69824	
$u =$	8		
$2t + u =$	8728		
$2tu + u^2 =$		69824	

The square root of the largest square in 1907 is found, as in the first example, to be 43.

This 43 is now to be regarded as *tens relatively to the next term of the root.* Then twice the tens, $2t$, will be 860, which is contained 6 times in 5894, thus giving 6 for the next figure of the root. So the square root of the largest square in 190,794 is 436.

This 436 is now to be regarded as tens relatively to the next term of the root. Then twice the tens, $2t$, will be 8720, which is contained 8 times in 6982, thus giving 8 as the next figure of the root. Since there is no remainder, the given number is a perfect square, and $\sqrt{190794.24} = 436.8$. *Ans.* 436.8.

523. A condensed form of work is obtained by the omission of the formula and of unnecessary figures, thus:

WRITTEN WORK.

	19'07'94'.24(436.8
	16
83)	307
	249
866)	5894
	5196
8728)	69824
	69824

The process is expressed in the fewest words thus: Largest square in 19 is 16, root 4; 16 from 19 leaves 3, with which the next two terms of the given number make a dividend 307; twice the part of the root already found, 8; 8 in 30 is contained 3 times. Write 3 as next term of the root, also write it with the trial divisor 8, making true divisor 83; 3 times 83 is 249; 249 from 307 leaves 58, with which the next two terms of the given number make a dividend 5894; twice the part of the root already found, 86; and so on.

Examples for Written Work.

524. Find the square root of

49. 81225.	**52.** 38416.	**55.** 39526369.
50. 119025.	**53.** 767376.	**56.** 52345225.
51. 242064.	**54.** 327184.	**57.** 85155984.

525. From the foregoing examples and explanations may be derived the following

Rule.

To find the square root of a number:

1. *Beginning at the units' place, separate the figures of the given number into periods of two figures each.*

2. *Find the greatest square in the number expressed by the left hand period, and write its square root as the first term of the root.*

3. *Subtract this square from the corresponding part of the given number, and with the remainder unite the next two terms of the given number for a dividend.*

4. *Double the part of the root already found for a trial divisor, and by this, regarded as tens, divide the dividend, writing the quotient as the next term of the root, also adding it to the trial divisor to make the true divisor.*

5. *Multiply the true divisor by the quotient and subtract the product from the dividend. (If this product should be too large, make the quotient less and repeat the work.)*

6. *With the remainder last found unite the next two terms of the given number for the next dividend, take double the part of the root now found, regarded as tens, for a trial divisor and proceed as before.*

NOTE 1. — The last step is to be repeated until all the terms of the given number have been used, or until the root has been found to any desired number of decimal places. In the latter case two zeros are annexed to each successive remainder.

Note 2. — When a dividend does not contain a trial divisor, place a zero as the next figure of the root; place also a zero at the right of the trial divisor; for the next dividend unite the next two terms of the given number to the previous dividend, and proceed as before.

Examples for Written Work.

526. Find the square root of

58. 2039184.	**63.** 5241.76	**68.** 3610.8081.
59. 10361961.	**64.** 495.5076.	**69.** 645772.96.
60. 81432576.	**65.** 1618.4529.	**70.** 65.512836.
61. 94225849.	**66.** 0.651249.	**71.** 9040.2064.
62. 49112064.	**67.** 0.43652449.	**72.** 1.002001.

527. *Illustrative Example.* Find the square root of 15.

WRITTEN WORK.

```
      15|3.872983
       9
  68)600
     544
 767) 5600
      5369
 7742) 23100
       15484
  7744) 76160
        69696
         64640
         61952
          26880
```

After four figures of the root have been found by the rule, *three* more figures can be found by long division. This is because the trial divisor, $2t = 77440$, has now become so large in comparison with the quotient, $u = 9$, that the error committed by using the trial divisor instead of the true divisor rarely affects the quotient for at least three places.

Ans. 3.872983.

528. Find to seven places the square root of

73. 40.	**77.** 5.	**81.** 11.3.	**85.** 0.144.
74. 75.	**78.** 8.	**82.** 0.5.	**86.** 7.016.
75. 2.	**79.** 10.	**83.** 0.3.	**87.** 33.77.
76. 3.	**80.** 20.	**84.** 0.9.	**88.** 6.4.

529. The square root of a common fraction can be found by taking the square roots of both numerator and denominator.

NOTE. — If the denominator is not a perfect square, the fraction should be changed to an equivalent fraction the denominator of which is a perfect square. Thus, $\frac{2}{3}$ should be changed to $\frac{6}{9}$, and $\frac{4}{7}$ to $\frac{28}{49}$.

530. Find the square root of

89. $\frac{25}{81}$.	**92.** $\frac{36}{100}$.	**95.** $\frac{3}{4}$.	**98.** $12\frac{5}{9}$.
90. $\frac{64}{225}$.	**93.** $\frac{7396}{1000}$.	**96.** $\frac{6}{7}$.	**99.** $12\frac{8}{11}$.
91. $\frac{1444}{5929}$.	**94.** $\frac{15}{49}$.	**97.** $3\frac{2}{3}$.	**100.** $163\frac{28}{85}$.

531. Applications of Square Root.

101. A square garden contains 21,025 square feet. How many feet long is each side?

102. There are how many rods in each side of a square field containing 6724 square rods?

103. How many feet are there in each side of a square court covered by 16,384 paving blocks, each 8 inches square?

104. Find the side of a square that shall contain as many square feet as an oblong measuring 420 by 105 feet.

105. There are four oblongs measuring 24 by 216, 48 by 108, 54 by 96, and 27 by 192 feet respectively. Find the dimensions of a square which is equal to each of the oblongs, and of a square which is equal to all four of them together.

106. How many rods must one walk to go around a ten-acre lot of land laid out in square form?

107. There are $272\frac{1}{4}$ square feet in a square rod. How many feet are there in the side of a square rod?

108. Find how many feet of fence it requires to surround an acre of land laid out in square form. (Root to nearest 100th.)

109. Find how many feet of fence it requires to surround an acre of land laid out in oblong form, the length being to the breadth as 3 to 2. (Find the root to the nearest 100th.)

532. *Illustrative Example.* Find the value of x from the proportion

$$24 : x = x : 216.$$

Since the product of the means is equal to the product of the extremes,

$$x^2 = 24 \times 216 = 5184.$$

Then $x = \sqrt{5184} = 72.$ *Ans.* 72.

In cases like this, x is called the mean proportional between the two extremes.

The mean proportional between two numbers is the square root of their product.

Find the mean proportional between

110. 2 and 8.
112. 3 and 12.
114. 5 and 45.
111. 2 and 18.
113. 5 and 20.
115. 6 and 96.

116. Find the mean proportional between 126 and 504.

117. Find the mean proportional between 3528 and 14112.

118. Find the mean proportional between 4 and 8.

119. Find a mean proportional between 5 and 15.

120. Square the numbers 65 and 72, add the squares, and find the square root of the sum.

Do the same with the following numbers:

121. 27 and 36.
123. 36 and 105.
125. 39 and 80.
122. 72 and 96.
124. 36 and 77.
126. 48 and 55.

127. Square the numbers 95 and 57, subtract the smaller square from the larger, and find the square root of the difference.

Do the same with the following numbers:

128. 53 and 45.
130. 65 and 56.
132. 85 and 84.
129. 50 and 48.
131. 65 and 16.
133. 87 and 60.

CUBE ROOT.

533. To find the number of figures required to express the cube root of a number.

What is the smallest number expressed by one figure? The largest? What are the cubes of these numbers? Any number from 1 to 729 is expressed by how many figures? If, then, a root is expressed by one figure, by how many figures is its cube expressed?

What is the smallest number expressed by two figures? The largest? What are the cubes of these numbers? Any number from 1000 to 970299 is expressed by how many figures? If, then, a root is expressed by *two* figures, by how many figures is its cube expressed?

In general, *the figures of a number are thrice as many, or one or two less than thrice as many, as the figures of its cube root.*

Accordingly, *if the figures of a number are separated into periods of three figures each, beginning with the units' figure, there will be as many periods as the cube root of the number has figures.*

534. To find the parts of the third power of a number.

To find what parts the third power of a number is made up of, raise a number, 34 for example, to the third power, thus:

$$\begin{array}{rcrrrr}
34^2 = 1156 = & & & 900 + & 240 + & 16 \\
34 = & & & & 30 + & 4 \\ \hline
4624 = & & & 3600 + & 960 + & 64 \\
3468 = & & 27000 + & 7200 + & 480 & \\ \hline
39304 = & & 27000 + & 10800 + & 1440 + & 64
\end{array}$$

$$\begin{array}{rrrrr}
(t+u)^2 = & & t^2 + & 2tu + & u^2 \\
& & & t + & u \\ \hline
& & t^2u + & 2tu^2 + & u^3 \\
& t^3 + & 2t^2u + & tu^2 & \\ \hline
(t+u)^3 = & t^3 + & 3t^2u + & 3tu^2 + & u^3
\end{array}$$

The work in the second form is done without carrying, and so shows all the partial products obtained by multiplying each term of the multiplicand by each term of the multiplier.

The work in the third form is algebraic, t and u standing for the tens and units of any number.

The product in either case shows that *the third power of any number consisting of tens and units is made up of four parts:*

(1) *the cube of the tens,*
(2) *three times the product of the square of the tens by the units,*
(3) *three times the product of the tens by the square of the units,*
(4) *the cube of the units.*

Formula: $(t+u)^3 = t^3 + 3t^2u + 3tu^2 + u^3.$

535. To extract the cube root.

Illustrative Example. (1) What is the cube root of 39304?

WRITTEN WORK.

$$
\begin{array}{lrl}
t^3 + 3t^2u + 3tu^2 + u^3 = & & 39'304(34 \\
t^3 = & & 27 \\
3t^2 = & 2700 & \underline{12304} \\
3tu = & 360 & \\
u^2 = & \underline{16} & \\
3t^2 + 3tu + u^2 = & 3076 & \\
\underline{3t^2u + 3tu^2 + u^3 =} & & \underline{12304}
\end{array}
$$

The figures of the given number being separated into periods of three figures each, show that the cube root is expressed by two figures; that is, consists of tens and units.

If the root is represented by $t+u$, its cube, or the given number, will be represented by $t^3 + 3t^2u + 3tu^2 + u^3$, and the values of the several parts of this formula are to be sought in the given number.

Since *the cube of the tens*, t^3, is thousands, the 39 thousands of the given number contains the cube of the tens of the root.

The greatest cube in 39 (thousands) is 27 (thousands), the cube root of which is 3 (tens). That is,

$$t^3 = 27000, \text{ and } t = 30.$$

So the first figure of the cube root is 3.

Subtracting 27000 from the given number, and t^3 from the formula, gives the equal remainders,

$$3t^2u + 3tu^2 + u^3 = 12304$$

which, by separating the first into factors,* may be written

$$(3t^2 + 3tu + u^2)u = 12304$$

If the value of the first factor, $3t^2 + 3tu + u^2$, were known, 12304 divided by it would give the second factor, u. But $3t^2$ is much the largest part of the first factor, and is known; so it can be used as a trial divisor for finding a probable value of u, which can be verified afterwards.

Dividing 12304 by $2700 (= 3t^2)$ gives 4. So 4 is the second figure of the root, provided the remaining terms of the formula, $3t^2u + 3tu^2 + u^3$, do not exceed 12304.

But	$3t^2 = 2700$
and	$3tu = 360$
and	$u^2 = 16$
adding, the sum is	$3t^2 + 3tu + u^2 = 3076$
which multiplied by	$u = 4$
gives	$3t^2u + 3tu^2 + u^3 = 12304.$

So the value of u is verified. The true divisor, $3076 (= 3t^2 + 3tu + u^2)$, multiplied by the quotient, $4 (= u)$, gives a number not exceeding 12304.†

Since there is no remainder, the given number is a perfect cube, and 34 is its cube root. $\sqrt[3]{39304} = 34$. *Ans.* 34.

Examples for Written Work.

536. Find the cube roots of

134. 12167.	**137.** 226981.	**140.** 753571.
135. 32768.	**138.** 274625.	**141.** 592704.
136. 157464.	**139.** 389017.	**142.** 421875.

* It is easy to see that if $3t^2 + 3tu + u^2$ are multiplied by u, the product will be $3t^2u + 3tu^2 + u^3$.

† If the value of u had proved too large, it would have been necessary to diminish it, and repeat the process of verification.

537. *Illustrative Example.* (2) Find the cube
2433138.625.

$$
\begin{array}{lrl|l}
t^3 + 3t^2u + 3tu^2 + u^3 = & & 2'433'138.625 & \underline{134.5} \\
t^3 \hfill = & & 1 & \\
3t^2 \hfill = & 300 & 1433 & \\
\quad 3tu \hfill = & 90 & & \\
\quad\quad u^2 = & \underline{9} & & \\
3t^2 + 3tu + u^2 = & \underline{399} & & \\
3t^2u + 3tu^2 + u^3 = & & \underline{1197} & \\
\hline
3t^2 \hfill = & 50700 & 236138 & \\
\quad 3tu \hfill = & 1560 & & \\
\quad\quad u^2 = & \underline{16} & & \\
3t^2 + 3tu + u^2 = & \underline{52276} & & \\
3t^2u + 3tu^2 + u^3 = & & \underline{209104} & \\
\hline
3t^2 \hfill = & 5386800 & 27034625 & \\
\quad 3tu \hfill = & 20100 & & \\
\quad\quad u^2 = & \underline{25} & & \\
3t^2 + 3tu + u^2 = & \underline{5406925} & & \\
3t^2u + 3tu^2 + u^3 = & & \underline{27034625} & \\
\end{array}
$$

The cube root of the largest cube in 2433 is found, as in the first example, to be 13. This 13 is now to be regarded as *tens relatively to the next term of the root.* Then three times the square of the tens, $3t^2$, will be 50700, which is contained 4 times in the dividend 236138, thus giving 4 as the next figure of the root. The work now proceeds as before. There is no remainder; so the given number is a perfect cube, and 134.5 is its cube root. $\sqrt[3]{2433138.625} = 134.5$. *Ans.* 134.5.

538. From the foregoing examples and explanations may be derived the following

Rule.

To find the cube root of a number:

1. *Beginning at the units' place, separate the figures of the given number into periods of three figures each.*

2. *Find the greatest cube in the number expressed by the left-hand period, and write its cube root as the first term of the root.*

3. *Subtract this cube from the corresponding part of the given number, and with the remainder write the next three terms of the given number for a dividend.*

4. *Take three times the square of the part of the root already found for a trial divisor, and by this regarded as hundreds divide the dividend, writing the quotient as the next term of the root.*

5. *To the trial divisor add three times the product of the tens by the units and the square of the units of the root for the true divisor.*

6. *Multiply the true divisor by the quotient (units of the root) and subtract the product from the dividend. (If this product should be too large, make the quotient less and repeat the work.)*

7. *With the remainder last found write the next three terms of the given number for the next dividend, take three times the square of the part of the root now found, regarded as hundreds, for a trial divisor and proceed as before.*

NOTE 1. — The last step is to be repeated until all the terms of the given number have been used, or until the root has been found to any desired number of decimal places. In the latter case three zeros are annexed to each successive remainder.

NOTE 2. — When a dividend does not contain a trial divisor, place a zero as the next figure of the root, place also two zeros at the right of the trial divisor for the next trial divisor; for the next dividend unite the next three terms of the given number; and proceed as before.

Examples for Written Work.

539. Find the cube root of

143. 361400569867. **146.** 3436.115229. **149.** 27.216576512.
144. 171416328875. **147.** 573234910.443. **150.** 725845560803.
145. 95006081547. **148.** 740.480746823. **151.** 864161.578125.

540. *Illustrative Example.* Find the cube root of 15.

WRITTEN WORK.

	15 | 2.46621
	8
12	7000
24	
16	
1456	5824
1728	1176000
432	
36	
177156	1062936
181548	113064000
4428	
36	
18199116	109194696
18243468	38693040
	36486936
	22061040
	18243468

By the omission of the formula, and of unnecessary figures, the work of finding a cube root may be written in this condensed form.

After *four* figures of the root have been found by the rule, *two* more can be found by long division. This is because the trial divisor $3t^2 =$ 18243468 has now become so large in comparison with the quotient, that the error committed by using the trial divisor instead of the true divisor does not affect the quotient for at least two places. *Ans.* 2.46621.

541. Find to six places the cube root of

152. 40. **154.** 2. **156.** 1.25. **158.** 12.026.

153. 75. **155.** 4. **157.** 0.64. **159.** 2.16.

542. The cube root of a common fraction can be found by taking the cube roots of both numerator and denominator.

NOTE.—If the denominator is not a perfect cube, the fraction should be changed to an equivalent fraction, the denominator of which is a perfect cube. Thus, $\frac{2}{3}$ should be changed to $\frac{18}{27}$, and $\frac{8}{25}$ to $\frac{40}{125}$.

543. Find the cube root of

160. $\frac{8}{27}$. **163.** $\frac{1}{8}$. **166.** $3\frac{3}{8}$.

161. $\frac{729}{125}$. **164.** $\frac{8}{1000}$. **167.** $\frac{2}{3}$.

162. $\frac{1728}{3375}$. **165.** $\frac{27}{100}$. **168.** $\frac{5}{7}$.

544. Applications of Cube Root.

169. What is the length of one edge of a cube which contains 125,000 cubic feet?

170. How long, wide, and high is a cubical pile of wood containing 4 cords?

171. Find the dimensions of a cube which shall contain as much as a common brick measuring $8 \times 4 \times 2$ inches.

172. Find the dimensions of a cubical cistern which shall hold 3000 gallons.*

173. A bin whose length, breadth, and depth are to be proportional to the numbers 3, 2, and 1 is to be made capable of holding 500 bushels of grain. Find the dimensions.†

174. A cubic foot of water weighs 1000 ounces. Find the dimensions of a cubical cistern that would hold 10 tons (20,000 lb.) of water.†

175. Iron weighs 7.2 times as much as water. Find the dimensions of a cube of iron that would weigh a ton.*

176. A block of marble weighing a ton is as wide as it is long, and half as thick as it is long. Marble weighs 2.715 times as much as water. Find the dimensions of the block.*

177. The number of cubic inches in a sphere is found by cubing the diameter, and multiplying this by $\frac{1}{6}$ of the number 3.1416. Find how many inches there are in the diameter of a globe that holds just 1000 ounces of water.*

178. Gold weighs 19.35 times as much as an equal volume of water. Find the diameter of a sphere of gold that would weigh a ton (2000 lb.).*

179. Find the diameter of a sphere of iron that would weigh a ton.*

---

* Find the root to the 4th term.

† Find the root to the nearest 1000th.

SECTION XVII.

MENSURATION, OR PRACTICAL GEOMETRY.

545. The principles and rules of this section should be abundantly illustrated by objects. There should be always within reach pieces of paper, pasteboard, or thin wood cut in various forms and sizes to represent all kinds of triangles, quadrilaterals, and other polygons, which the pupils themselves will measure and find the areas of.

There should also be provided for the same purpose pieces of wood representing different forms and sizes of prisms, pyramids, cylinders, cones, spheres, and other solids. The pupils will be much interested in making pasteboard models of these solids. Clay also can be used with great advantage.

A few wooden boxes and tin dishes, a ruler, a square, a protractor, a pair of compasses, a sharp knife, a cutting board, some paste or glue, and some clean sand complete the outfit.

546. Mensuration, or practical geometry, teaches the measurement of lines, surfaces, and solids.

547. A **solid** is a portion of space separated from surrounding space by a surface or surfaces.

A solid has three **dimensions**, length, breadth, and thickness.

548. A **surface** is the boundary or limit of a portion of space. It has two dimensions, length and breadth, but no thickness.

549. Surfaces are either plane or curved. A **plane surface** is flat like the top of a table, the wall of a room, or the surface of still water. A **curved surface** is bent in one or more ways like a roll of paper, the covering of a ball, etc.

Find examples of surfaces, plane and curved, especially upon geometrical solids.

550. A **line** is the boundary or limit of a surface. It has one dimension, length, but no breadth and no thickness.

Notice the edges of a geometric solid. These are lines, bounding or limiting the faces (surfaces).

551. A **straight line** is the path of a point moving constantly in the same direction.

The point of your pencil moving against the edge of your ruler does not change its direction, and so makes a straight line.

A thread or a string held by the two ends and stretched tight is a good illustration of a straight line. How does a carpenter use a chalk line, and what for?

552. A **curved line** or a **curve** is the path of a point moving in a constantly changing direction.

The point of the pencil attached to one leg of your compasses when turning about the other leg moves in a constantly changing direction, and so makes the curve called the circumference of a circle.

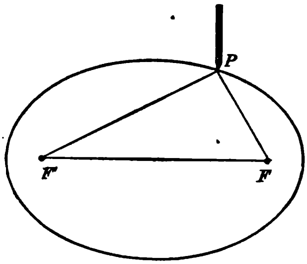


FIG. 1.

Another curve can be made in this way. Stick two pins through the paper securely into the board beneath, at the points *F* and *F'*. Take a piece of linen thread somewhat longer than twice the distance from *F* to *F'*; tie the ends together; and, with the point of a pencil at *P*, stretch the thread tightly into a triangular shape. Now move the pencil around, keeping the thread stretched tight. The pencil will mark out an ellipse.

Find examples of straight and of curved lines among the edges of geometric solids.

Cut a round piece of wood, like a broomstick or a roller, square across. What sort of a curve is the edge of the cut surface? Cut it obliquely. What sort of a curve is the edge of the cut surface?

553. A plane surface is touched by all points of a straight line laid upon it in any direction.

With your ruler test the top of your desk to see if it is a perfect plane. Test a warped board. How does the carpenter test the surface of a board he has planed?

554. Straight lines which have the same direction are **parallel lines.** Two parallel lines nowhere meet, however far extended either way.

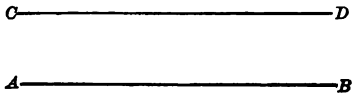


FIG. 2. — Parallel Lines.

555. When one straight line, *AB*, is met by another, *CD*, two **angles** are formed. If the angles are equal (Fig. 3)

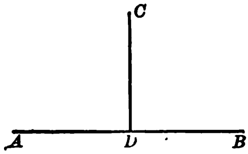


FIG. 3.

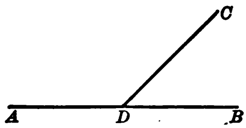


FIG. 4.

they are **right angles,** and the lines are said to be **perpendicular** to each other; if unequal (Fig. 4), they are oblique angles, the smaller, *BDC*, being an **acute** (sharp) angle, and the larger, *CDA*, an **obtuse** (blunt) angle.

Exercises.

556. *a.* Which is the larger, a right angle or an acute angle? A right angle or an obtuse angle? With a square draw a right angle (Fig. 5).

b. Draw a straight line, and another meeting it as in Fig. 4. With a protractor, (see p. 163), measure the angle *BDC*. Measure the angle *CDA*. What is their sum? 180° equals how many right angles?

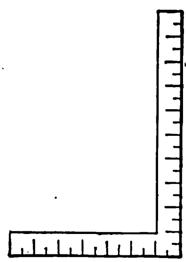


FIG. 5. — A Square.

c. Draw a straight line, *AB* (Fig. 6), and above it mark a point, *E*. With one edge of the square placed along the line *AB* measure the distance of the point *E* from the line *AB*.

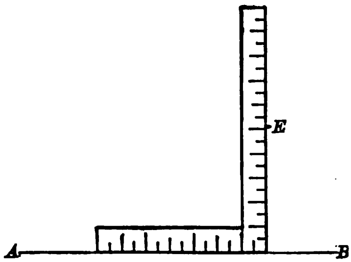


FIG. 6.

By distance from a point to a straight line is always meant the perpendicular distance.

d. From a given point P (Fig. 7), to draw a line perpendicular to a given line AB.

With one point of the compasses placed at P, draw arcs cutting the given line in two points C and D, conveniently far apart. Then with the compasses open somewhat more than half the distance from C to D, and from C and D as centers, draw two short arcs cutting each other at E.

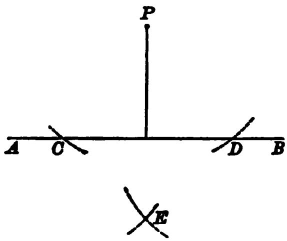


FIG. 7.

A straight line drawn through the points P and E with a ruler will be perpendicular to the given line. When the work is done, test its accuracy with a square.

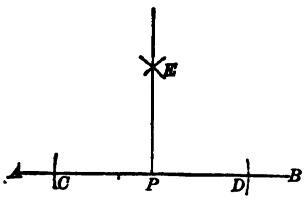


FIG. 8.

e. At a given point P, in a given line AB (Fig. 8), to draw a line perpendicular to the given line.

With compasses, place two points C and D at equal distances from P in the given line. From C and D as centers, and with the compasses open about as far as from C to D, draw two short arcs cutting each other at E. Draw with a ruler a line through the points P and E. This line will be perpendicular to the given line. When the work is done, test its accuracy with a square.

f. To bisect (that is, divide into two equal parts) a given straight line.

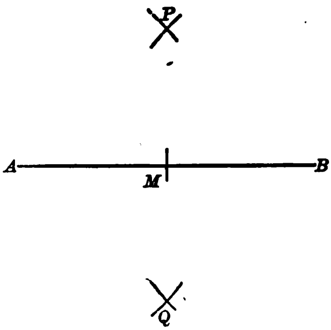


FIG. 9.

Let *AB* (Fig. 9) be the given straight line. With compasses open somewhat more than half the distance from *A* to *B*, and from *A* and *B* as centers, draw short arcs cutting each other at *P* and at *Q*. Lay the edge of a ruler on the points *P* and *Q* and draw a short line at *M*. This line bisects the given line.

g. To bisect a given angle.

Let *AOB* (Fig. 10) be the given angle. From *O* as a center, and with any convenient opening of the compasses, draw an arc *MN*. From *M* and *N* as centers, and with the compasses open about the distance from *M* to *N*, draw two short arcs cutting each other at *P*. Draw a straight line through *O* and *P*. This line bisects the given angle. It also bisects the arc *MN*.

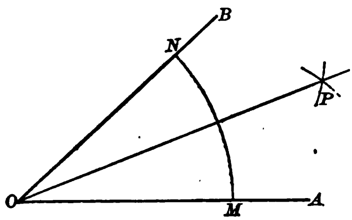


FIG. 10.

h. Through a given point to draw a straight line parallel to a given straight line.

With a square, measure the distance of the given point *P* (Fig. 11) from the given line *MN*. Place a point *Q* as far from *MN* as *P* is. A straight line drawn through *P* and *Q* is parallel to *MN*.

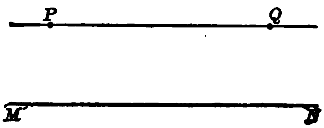


FIG. 11.

Two parallel lines are everywhere equally distant.

i. Draw a straight line; and, from two points M and N in it as centers (Fig. 12), and with the same opening of the compasses, draw two short arcs as represented in the figure. Place a ruler with its edge just touching these two arcs and draw a straight line. This line will be parallel to the first line.

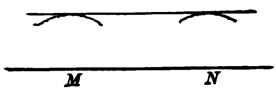


FIG. 12.

j. Draw two parallel lines, AB and CD (Fig. 13), and across them a straight line EF. How many angles are

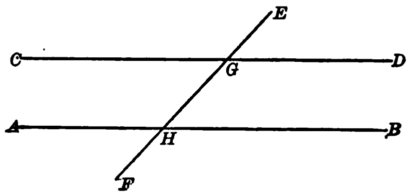


FIG. 13.

formed? How many are acute? How many are obtuse? Cut the paper along the lines AB, CD, and EF, and show, by laying one angle on another, that the four acute angles are equal to each other,

$$EGD = EHB = AHF = CGF;$$

and that the four obtuse angles are equal to each other,

$$DGF = BHF = AHE = CGE.$$

k. Lay a ruler, AB (Fig. 14), on the paper. Against it bring the wooden triangle GHI, and slide it so that the edge GH may have successively the positions CD, EF, GH, etc., which are marked by drawing a pencil against the edge GH in each position. As the ruler was not changed in direction, each side of the triangle, though

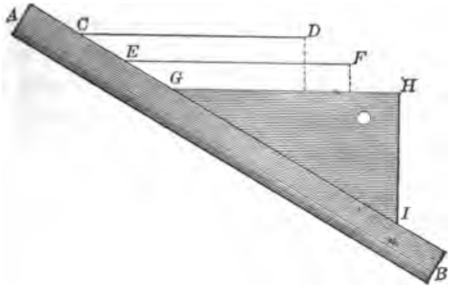


FIG. 14.

moved, kept the same direction; therefore the lines *CD*, *EF*, *GH*, etc., have the same direction, and are parallel.

557. A **plane figure** is a portion of a plane surface bounded by lines either straight, or curved, or both.

558. A **polygon** is a plane figure bounded by straight lines.

What is the least number of straight lines that will inclose a portion of a plane? What, then, is the least number of sides a polygon can have? How does the number of sides in any polygon compare with the number of angles?

559. Polygons are classified according to the number of their sides or angles, into **triangles**, having three angles (or sides); **quadrilaterals**, having four sides (or angles); **pentagons, hexagons, octagons, decagons, dodecagons**, etc., having respectively five, six, eight, ten, twelve, etc., angles (or sides).

TRIANGLES.

560. Triangles are classified according to their sides into **equilateral** triangles, having three equal sides; **isosceles** triangles, having two equal sides; and **scalene** triangles, having no equal sides.

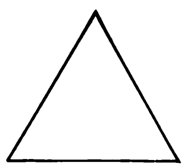


FIG. 15.

561. Triangles are also classified according to their angles into **acute-angled** triangles, having all three angles acute; **right-angled** triangles, having one right angle; and **obtuse-angled** triangles, having one obtuse angle.

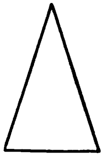


FIG. 16.

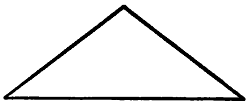


FIG. 17.

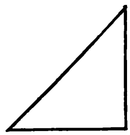


FIG. 18.

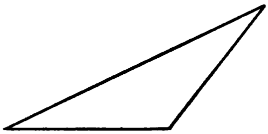


FIG. 19.

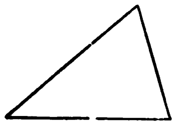


FIG. 20.

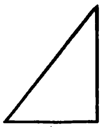


FIG. 21.

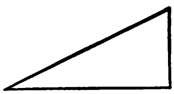


FIG. 22.

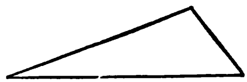


FIG. 23.

Exercises.

562. *a.* Draw triangles of various forms. Notice in each the position of the largest angle relatively to the longest side, and the position of the smallest angle relatively to the shortest side.

In any triangle the longer of two sides is opposite the larger angle, and the larger of two angles is opposite the longer side.

b. If two sides of a triangle are equal, what is true of the angles opposite them? Draw such a triangle on paper, cut it out, tear off the two angles that are opposite the equal sides, and compare them, thus learning that

The angles of an isosceles triangle which lie opposite the equal sides are equal.

c. Draw a triangle with three equal sides. What is true of the angles? Test with a paper triangle.

d. Draw any triangle on paper, and cut it out. Tear off

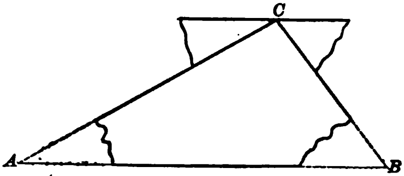


FIG. 24.

two of the angles and place them, as in Fig. 24, so as to show that

The sum of the three angles of a triangle is equal to two right angles.

e. How many degrees are there in each angle of an equilateral triangle?

f. If the two acute angles of a right-angled triangle are equal, how many degrees are there in each?

g. One angle of a triangle measures 110°, and the other two are equal. How many degrees are there in each of them?

h. One of the equal angles of an isosceles triangle measures 72°. What do the other two angles measure?

i. Draw any triangle on paper, measure the angles as accurately as possible with a protractor, and add the results. This sum should be how many degrees?

j. When two angles of a triangle are given, how do you find the third?

k. Draw a triangle that is both isosceles and acute-angled.

l. Draw a triangle that is both isosceles and obtuse-angled.

m. Draw a triangle that is both isosceles and right-angled.

n. Draw a triangle that is both scalene and acute-angled.

o. Draw a triangle that is both scalene and obtuse-angled.

p. Draw a triangle that is both scalene and right-angled.

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176. A block of marble weighing a ton is as wide as it is long, and half as thick as it is long. Marble weighs 2.715 times as much as water. Find the dimensions of the block.*

177. The number of cubic inches in a sphere is found by cubing the diameter, and multiplying this by $\frac{1}{6}$ of the number 3.1416. Find how many inches there are in the diameter of a globe that holds just 1000 ounces of water.*

178. Gold weighs 19.35 times as much as an equal volume of water. Find the diameter of a sphere of gold that would weigh a ton (2000 lb.).*

179. Find the diameter of a sphere of iron that would weigh a ton.*

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545. The principles and rules of this section should be abundantly illustrated by objects. There should be always within reach pieces of paper, pasteboard, or thin wood cut in various forms and sizes to represent all kinds of triangles, quadrilaterals, and other polygons, which the pupils themselves will measure and find the areas of.

There should also be provided for the same purpose pieces of wood representing different forms and sizes of prisms, pyramids, cylinders, cones, spheres, and other solids. The pupils will be much interested in making pasteboard models of these solids. Clay also can be used with great advantage.

A few wooden boxes and tin dishes, a ruler, a square, a protractor, a pair of compasses, a sharp knife, a cutting board, some paste or glue, and some clean sand complete the outfit.

546. Mensuration, or practical geometry, teaches the measurement of lines, surfaces, and solids.

547. A **solid** is a portion of space separated from surrounding space by a surface or surfaces.

A solid has three **dimensions**, length, breadth, and thickness.

548. A **surface** is the boundary or limit of a portion of space. It has two dimensions, length and breadth, but no thickness.

549. Surfaces are either plane or curved. A **plane surface** is flat like the top of a table, the wall of a room, or the surface of still water. A **curved surface** is bent in one or more ways like a roll of paper, the covering of a ball, etc.

Find examples of surfaces, plane and curved, especially upon geometrical solids.

diagonal. Lay one of the triangles thus made upon the other, and so learn that

1. *The opposite sides of a parallelogram are equal.*
2. *The opposite angles of a parallelogram are equal.*
3. *The diagonal of a parallelogram divides it into two equal triangles.*

d. Draw any quadrilateral and one of its diagonals. See whether the sum of all the angles of the two triangles is the same as the sum of the four angles of the quadrilateral. Then how many right angles (or degrees) is the sum of the four angles of a quadrilateral equal to?

The sum of the four angles of a quadrilateral is equal to four right angles.

e. One angle of a parallelogram is 60°. Find the other angles.

f. Draw a parallelogram and its two diagonals. Measure the parts into which the diagonals divide each other, and so learn that

The diagonals of a parallelogram bisect each other.

g. Draw a rhombus and its two diagonals. Cut out the rhombus and show by folding that

The two diagonals of a rhombus bisect each other at right angles, and divide the rhombus into four equal right-angled triangles.

h. Draw any polygon. Draw all the diagonals that can be drawn from one corner. This divides the polygon into how many triangles? How does the number of triangles compare with the number of sides? Try this with different polygons (four, five, six, etc., sides), and learn that, in general,

The number of triangles into which a polygon is divided by diagonals drawn from one corner is two less than the number of sides.

i. Taking any polygon divided into triangles, as in the last exercise, inquire whether the sum of all the angles of the triangles is equal to the sum of all the angles of the polygon. What, then, is the sum of all the angles of a pentagon? of a hexagon? octagon? decagon? Thus learn that, in general,

The sum of the angles of any polygon is equal to twice as many right angles as the polygon has sides less two sides.

j. If all the angles of a pentagon (hexagon, octagon, decagon, dodecagon) are equal, how many degrees are there in each angle?

REGULAR POLYGONS.

574. An equilateral polygon has all its sides equal. An equiangular polygon has all its angles equal.

A **regular polygon** is a polygon which is both equilateral and equiangular.

Which of the figures on pages 334, 335 are equilateral? Which are equiangular? Which are regular? Is there any kind of triangle which is a regular polygon? If so, what kind? Why?

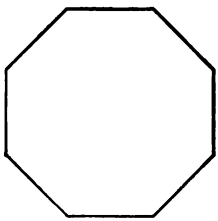

FIG. 33. — Regular Octagon.

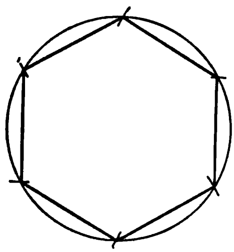

FIG. 34. — Regular Hexagon.

575. To draw a regular hexagon.

First draw a circle (Fig. 34); then, with the compasses kept at the same opening, lay off that distance six times on the circumference, and join the successive division points by straight lines. If the work is accurately done, the end of the last side will coincide with the beginning of the first side.

576. To draw a regular dodecagon.

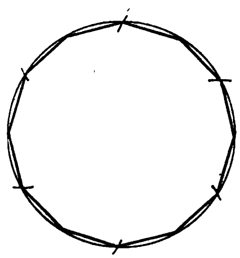

FIG. 35. — Regular Dodecagon.

First draw a circle and divide its circumference into six equal arcs as for a regular hexagon, and then divide each arc into two equal parts, making twelve parts in all. Join the successive division points by straight lines.

577. The sides of a polygon taken together form its **perimeter.**

Which differs less from the circumference of the circumscribed circle, the perimeter of the regular hexagon or the perimeter of the regular dodecagon?

If regular polygons of 24, 48, 96, etc., sides were to be drawn in the circle, the perimeter of which of these would come nearest to the circumference? If the number of sides of the polygon became infinite, the perimeter would come infinitely near the circumference. Hence,

A circle may be considered to be a regular polygon with an infinite number of sides.

AREA OF POLYGONS.

578. Parallelograms. — The area of any parallelogram is equal to that of a rectangle of the same base and height.

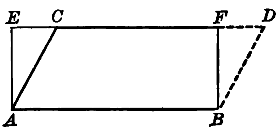

FIG. 36.

For if we cut off a right triangle from one end, and put it on at the other end, as in Fig. 36, we change the form to a rectangle having the same base and height as the parallelogram, though we do not change the area. But the area of a rectangle is equal to the product of its base and height. (Art. 108.) Hence,

To find the area of any parallelogram, multiply the base by the height.

579. Triangles. — A triangle is half a parallelogram of the same base and height.

For two equal triangles may be placed, as shown in Fig. 37, so as to make a parallelogram. Hence,

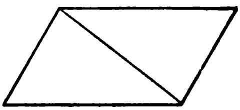

FIG. 37.

To find the area of a triangle, multiply the base by the height, and divide the product by two.

580. The following is a convenient rule for finding the area of a triangle when the three sides are given, but the reasons for it cannot be explained without more geometry than can well be introduced here:

To find the area of a triangle when the three sides are given: *Find half the sum of the three sides; from this half-sum subtract each side in turn; multiply together the three remainders and the half-sum, and extract the square root of the product.*

Exercises.

581. *a.* Cut out from paper, pasteboard, or a board, a parallelogram. Measure the base and the height and find the area.

b. Using the same parallelogram as in the last exercise, take one of the other pair of sides for a base, measure it and the height, and find the area. Compare the results. Should they be the same? Why?

c. Draw a parallelogram with one side measuring 8 inches, another 4 inches, and having an angle of 50 degrees. Find its area.

d. Draw a triangle with three sides of different lengths. Measure the three bases (sides) and the three heights, and with these find the area of the triangle in three different ways. Compare the three results. Should they agree? Why?

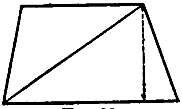

FIG. 38.

582. Trapezoids. — A trapezoid may be divided into two triangles having the parallel sides of the trapezoid for bases and the distance between them for their height.

Hence, *To find the area of a trapezoid, multiply half the sum of the parallel sides by the distance between them.*

FIG. 39.

583. Draw a trapezoid; find the middle point of each of the non-parallel sides; and measure the distance between these two points. How does this distance compare with half the sum of the parallel sides of the trapezoid?

The line joining the two middle points of the non-parallel sides of a trapezoid is parallel to the two parallel sides and in length is equal to half their sum. Hence,

To find the area of a trapezoid, multiply the distance between the parallel sides by the distance between the middle points of the non-parallel sides.

584. Trapezium. — A trapezium can be divided by a diagonal into two triangles; then the areas of these triangles can be found separately and added.

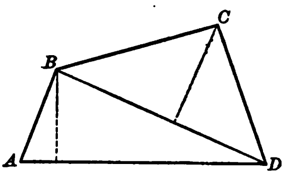

FIG. 40.

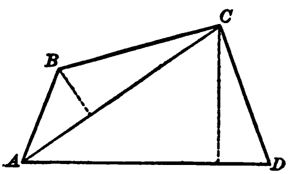

FIG. 41.

Draw a trapezium, $ABCD$ (Fig. 40), and divide it into two triangles by drawing the diagonal BD, and find their areas. Divide the same trapezium into two other triangles by drawing the diagonal AC (Fig. 41), and find their areas. Compare the sum of the areas of the first two triangles with the sum of the areas of the second two. Should these two sums be equal? Why?

585. Any plane figure bounded by straight lines can be divided into triangles; the areas of these triangles can be separately found, and the sum of these areas is the area of the whole figure.

NOTE. — It is well always to divide the figure into triangles in at least two different ways. The near approach to agreement between the results so obtained is proof of correctness in the work.

586. A regular polygon can be divided into as many equal isosceles triangles as the polygon has sides by drawing lines from the center to the corners of the polygon. Fig. 42.

587. The perpendicular drawn from the center to any side of a regular polygon is called the **apothem**. Fig. 42.

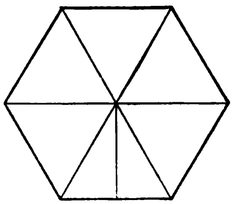

FIG. 42.

588. The area of one of the triangles is *half the product of the apothem by one side of the polygon,* and the area of the whole polygon (the sum of all the triangles) is *half the product of the apothem by the sum of the sides of the polygon* (the sum of the bases of the triangles). Hence,

To find the area of a regular polygon, multiply the perimeter by the apothem and divide the product by two.

589. Circles. — In a regular polygon, the greater the number of sides, the more nearly is the apothem equal to the radius of the circumscribed circle.

Draw a regular polygon of a small number of sides (say four). Draw one of a larger number of sides (say eight, sixteen, etc.). Draw the apothem in each. In which polygon is the apothem most nearly equal to the radius of the circumscribed circle?

In the circle itself (a regular polygon of an infinite number of sides), the apothem is equal to the radius. Hence,

To find the area of a circle, multiply the circumference by the radius, and divide the product by two. (Art. 318.)

590. Examples for Written Work.

1. How many square feet are there in a rhomboid whose base measures $4\frac{1}{2}$ feet, and whose height is $1\frac{3}{8}$ feet?

2. Find the area of a triangle, the base measuring 28 feet and the altitude $10\frac{1}{2}$ feet.

3. What is the area of a right-angled triangle, the sides which include the right angle measuring 140 and 160 feet?

4. Find the area of a rhombus, the diagonals of which measure 40 and 60 feet.

5. How many square feet are there in a board 14 ft. long, 18 in. wide at one end, and 26 in. wide at the other?

6. There is a house-lot with four straight sides, two of which are parallel, 90 feet apart, and measuring 118 and 129 feet. What is its value at 15 cents a square foot?

7. Draw a quadrilateral, $ABCD$; draw the diagonal AC, and draw perpendiculars to it from the points B and D. Compute the area, supposing $AC = 106$ feet, the perpendicular from $B = 62$ feet, and that from $D = 54$ feet.

8. Compute the area of a triangle whose sides measure 15, 18, and 23 yards. (Find the root to tenths.)

9. The four sides of a quadrilateral, $PQRS$, measure as follows: $PQ = 30$ yards, $QR = 28$ yards, $RS = 38$ yards, and $SP = 42$ yards. The diagonal, PR, measures 48 yards. Find the area. (Art. 580.)

10. Draw all the different plane figures mentioned in the above examples, of any size you please, make your own measurements, and compute the areas. Whenever you can see different ways of measuring and computing the area of the same figure, use them, and see how nearly the results agree. Perfectly accurate measurements would give equal results; but perfect accuracy in measurement is unattainable. Still, the more accurate your measurements the less will your results differ.

CIRCLES.

591. The ratio of the circumference to the diameter of a circle, expressed to the nearest ten-thousandth, is 3.1416. Hence the following

Formulas relating to the Circle.

1. $\text{Circumference} = \text{Diameter} \times 3.1416.$
2. $\text{Diameter} = \dfrac{\text{Circumference}}{3.1416}.$
3. $\text{Circumference} = 2\ \text{Radius} \times 3.1416.$
4. $\text{Radius} = \dfrac{\text{Circumference}}{2 \times 3.1416}.$

NOTE. — Instead of 3.1416, the value $3\frac{1}{7}$, or $\frac{22}{7}$, is much used. It is accurate enough for ordinary purposes.

592. The rule in Art. 589 may be expressed thus:

$$\text{Area} = \frac{\text{Circumference} \times \text{Radius}}{2}.$$

Putting in the place of the circumference its value as given in Formula 3, we have

$$\text{Area} = \frac{\cancel{2} \times \text{Radius} \times 3.1416 \times \text{Radius}}{\cancel{2}}.$$

Hence the formulas:

5. $\text{Area} = \text{Radius}^2 \times 3.1416 = \frac{1}{4}\ \text{Diameter}^2 \times 3.1416.$
6. $\text{Radius} = \sqrt{\dfrac{\text{Area}}{3.1416}}.$
7. $\text{Diameter} = \sqrt{\dfrac{\text{Area} \times 4}{3.1416}}.$

593. By using letters in place of words, and the Greek letter π (pronounced *pī*) to represent the ratio of circumference to diameter, these formulas may be written:

1. $C = \pi D.$
2. $D = \frac{C}{\pi}.$
3. $C = 2\pi R.$
4. $R = \frac{C}{2\pi}.$
5. $A = \pi R^2 = \frac{1}{4}\pi D^2.$
6. $R = \sqrt{\frac{A}{\pi}}.$
7. $D = \sqrt{\frac{4A}{\pi}}.$

594. Examples for Written Work.

11. What is the distance around a circular piece of ground which measures 200 feet across the middle?

12. Find how far a carriage has gone when one of its wheels, measuring 3 feet 8 inches in diameter, has made 1500 revolutions.

13. What is the distance across a circular piece of ground which measures 200 feet around?

14. Find the inner and the outer circumference of a walk $7\frac{1}{2}$ feet wide running around a circular grass plat that measures 150 feet in diameter.

15. There are two circles drawn from the same center; the circumferences measuring 88 feet and 132 feet respectively. Find the width of the ring.

16. What is the area of a circle 10 inches in diameter?

17. What is the area of a circle of 10 inches radius?

18. What is the area of a circle of 10 inches circumference?

19. How many square feet of land are inclosed by a circular race track measuring around the inner edge 1 mile?

20. Find the diameter of a circle whose area is 100 square feet. Of another, whose area is 50 square feet. Of another, whose area is 25 square feet. How do the diameters of the first and last compare?

21. How many feet of fence are required to surround an acre of land in the form of a circle?

595. Draw a straight line from one point of the circumference of a circle to another, but not passing through the center. Such a line is called a **chord.** A part of the circumference is called an **arc.** The chord joining the two ends of an arc is said to **subtend** the arc.

When a regular hexagon is inscribed in a circle (Art. 575), each side of it subtends an arc of how many degrees? When a square is inscribed in a circle, each side of it subtends an arc of how many degrees? Answer the same question for other regular polygons.

596. Draw a circle and two diameters at right angles with each other, and connect the extremities of the diameters by chords. How many chords have been drawn? What figure has been inscribed in the circle? The surface between a chord and the arc it subtends is called a **segment.**

How many segments are there in the figure you have drawn?

22. Suppose the radius of a circle measures 3 inches; find the area of the circle, the area of the inscribed square, and the area of each segment cut off by the sides of the square.

23. What is the diameter of a clock face, the minute spaces upon which measure each an inch?

NOTE. — It is the arc, not the chord, that measures 1 inch.

597. Draw a circle and two radii, making any angle you please with each other. The surface inclosed between two radii and the arc between them is called a **sector.** The area of a sector is the same part of the area of a circle that the arc of the sector is of the whole circumference. The angle between the two radii is the same part of 4 right angles (360°) that the arc between them is of the whole circumference. The arc between two radii is said to **subtend** their angle at the center, and to **measure** that angle.

24. In a circle whose radius is $1\frac{1}{2}$ yards, what is the area of a sector of 72°? of 22° 30′?

25. If eight radii are drawn from the center of a circle so as to form eight equal angles, what is the area of each sector if the circumference measures 100 feet?

26. In a circle whose radius is 15 inches, what is the length of an arc of 60°? 20°? 15°?

27. In a circle whose circumference is 40 inches, what angle at the center is subtended by an arc measuring 5 inches? 8 inches? 12 inches?

28. The radius of a circle is 10 inches. What angle at the center is subtended by an arc of 10 inches?

29. The radius of a circle is 10 inches. How long is an arc of 1°? of 1′? of 1″?

30. An arc of 1° on the circumference of a given circle measures 1 inch. Find the diameter.

31. An arc of 1° on the surface of the earth at the equator measures 69.16 statute miles. From this find the circumference and the diameter of the earth at the equator.

RIGHT TRIANGLES.

598. In a right triangle, the side opposite the right angle is the **hypotenuse,** and the other two sides are the **base and** the **perpendicular.** Fig. 43.

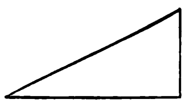

FIG. 43.

599. Suppose ABC (Fig. 44) is a right triangle, whose sides are 3, 4, and 5 feet respectively. A square formed upon the hypotenuse, AC, will contain how many square feet? A square formed upon the base, BC, will contain how many square feet? A square formed upon the perpendicular, AB, will contain how many square feet? Is the square upon the line AC equivalent to the sum of the two squares upon AB and BC?

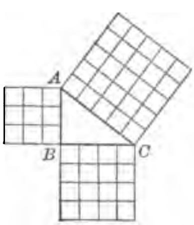

FIG. 44.

Exercises.

NOTE. — Figures drawn to a scale are recommended as a means of making the following exercises clear.

600. How does the square on the hypotenuse compare with the sum of the squares on the other two sides in each of the right triangles whose sides are as follows:

a. 6, 8, and 10?	*d.* 18, 24, and 30?
b. 20, 21, and 29?	*e.* 33, 56, and 65?
c. 39, 52, and 65?	*f.* 39, 80, and 89?

g. From one corner of a square floor, measure 6 feet along one side and 8 feet along the other side. Now measure across from the end of the 6 feet to the end of the 8 feet. How many feet is this last distance? It will be 10 feet if the corner of the room is exactly square and you have measured accurately.

h. Measure any distances you please along the two sides of the square corner of a floor; then measure across from the end of one of these distances to the end of the other. See if the square of this last distance is equal to the sum of the squares of the first two distances.

i. Draw on paper any right triangle you please, and measure the three sides as accurately as possible. Square the three sides, and see whether one of these squares is equal to the sum of the other two.

601. The foregoing are illustrations of the most celebrated and the most important of all the truths of geometry. It was taught nearly 2500 years ago by Pythagoras, and is commonly called

The Pythagorean Proposition.

The square upon the hypotenuse of a right triangle is equal to the sum of the squares upon the other two sides.

602. Hence we have the following rule:

1. To find the hypotenuse, having the base and perpendicular given: *Square the base and the perpendicular, add the squares, and extract the square root of the sum.*

2. To find the base or perpendicular, having the hypotenuse and the other side given: *Square the hypotenuse and the given side, subtract the latter square from the former, and extract the square root of the remainder.*

603. If the length of the hypotenuse is denoted by h, that of the perpendicular by a, and that of the base by b, the Pythagorean Proposition is expressed by this formula,

$$1.\quad h^2 = a^2 + b^2,$$

and the above rules by these formulas,

$$2.\quad h = \sqrt{a^2 + b^2},$$
$$3.\quad a = \sqrt{h^2 - b^2},$$
$$4.\quad b = \sqrt{h^2 - a^2}.$$

Examples for Written Work.

604. In the following examples two sides of a right-angled triangle being given, find the third (represented by one of the letters a, b, or h).

32. 48, 55, h.	**35.** 25, 60, h.	**38.** 48, 90, h.
33. 60, b, 87.	**36.** 36, b, 111.	**39.** 18, b, 82.
34. a, 84, 116.	**37.** a, 77, 85.	**40.** a, 110, 146.

In the following examples find the third side to two places of decimals:

41. 6, 10, h.	**43.** 5, b, 10.	**45.** a, 2, 3.
42. 1, 1, h.	**44.** $\begin{cases} a = b. \\ h = 1. \end{cases}$	**46.** $\frac{1}{2}$, b, 1.

47. Find the diagonal of a square, each side of which measures 10 inches.

48. If a ladder, 26 feet long, is placed with its foot 10 feet from a wall, how high up the wall will it reach?

49. Two boys start from the corner of a square field and walk along the two sides. How far apart will they be when one has walked 130 yards and the other 144 yards?

50. Two ships sail from a harbor, one 56 miles due east, and the other 90 miles due south in a day. How far apart are they at the end of the day?

51. A ship sails 72 miles on a northeast course, and then 21 miles on a southeast course. How far does this bring her from her starting point?

52. Find the diagonal of the floor of a room 24 feet long 18 feet wide. Using this diagonal as the base and the height of the room, 16 feet, as the perpendicular, find the distance from one corner of the floor to the corner of the ceiling diagonally opposite.

53. What is the length of the longest rod that, without bending, can be placed in a box measuring, inside, 16 inches long, 12 inches wide, and 21 inches deep?

54. The diagonal of a rectangular solid, like that of a room, passes from corner to corner *through* the solid. How many such diagonals can be drawn? Show that if a, b, and c represent the three dimensions of a rectangular solid, the length of each diagonal, d, is

$$d = \sqrt{a^2 + b^2 + c^2}.$$

55. Find the diagonals of the different faces of a common brick measuring 8 by 4 by 2 inches. Find also the diagonals passing through the brick itself.

56. Find the height and area of an isosceles triangle whose base is 78 feet and each of the other sides 89 feet.

57. Find the sides of a rhombus the diagonals of which are 24 and 70 inches.

58. Find the height and the area of an equilateral triangle, each side of which measures 10 inches.

59. Find how long the rafters must be for a barn 40 feet wide, the ridge-pole being 18 feet above the level of the eaves, and the eaves projecting 2 feet from the walls.

60. A regular hexagon is inscribed in a circle whose radius is 10 inches. Find the apothem. Find the ratio of the apothem to the radius. Find the area of the hexagon.

61. If six radii are drawn from the center of a circle so as to divide the circumference into six equal arcs, each arc subtends an angle of how many degrees at the center? If the chords are drawn to these six equal arcs, what kind of a polygon will be inscribed in the circle? How many triangles will be formed? What kind of triangles? If the radius of the circle is 10 inches, what is the area of each of these triangles? The area of each segment?

62. Given a rhombus the shorter diagonal of which is equal to one side, which measures 10 inches. Find the longer diagonal and the area of the rhombus.

63. An equilateral triangle is inscribed in a circle whose radius is 10 inches. Find one side of the triangle.

64. An equilateral triangle, each side of which is 10 inches, is inscribed in a circle. Find the radius of the circle.

Let $x =$ the radius. This and the following examples are designed to test pupils who have some aptitude in Geometry.

65. The corners of a square are cut off so as to make of it a regular octagon. The side of the square measuring 10 inches, what is the length of each side of the octagon?

66. Assuming any number you choose as the length of the base (side) of an equilateral triangle, find the height. Find also the area.

If a represents the length of one side of an equilateral triangle, its height is $\frac{1}{2} a \sqrt{3}$. See whether this is not so in the example you have chosen, or in any other example.

The base (side) of an equilateral triangle being represented by a and the height by $\frac{1}{2} a \sqrt{3}$, one half the product of these two dimensions, or $\frac{1}{4} a^2 \sqrt{3}$, is the area. See whether this is not so in the example you have chosen, or in any other example.

67. The area of an equilateral triangle being given as 100 square feet, find one side.

Let a represent the length in feet of the side to be found. Then $\frac{1}{4}a^2\sqrt{3} = 100$, from which the value of a can be found.

68. How many feet of fence are required to surround an acre of land in the form of an equilateral triangle?

69. How many feet of fence are required to surround an acre of land in the form of a regular hexagon?

Compare the result of Examples 68 and 69 with the results of Examples 108 and 109, page 316, and Example 21, page 345, and what do you learn? If you were to be given as much land as you could walk around in a day, what form of path would give you the most land?

SOLIDS.

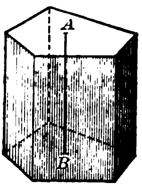

FIG. 45. — Prism.

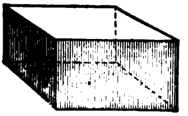
FIG. 46. — Rectangular Solid.

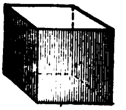
FIG. 47. — Cube.

605. A **prism** is a solid bounded by two equal and parallel polygons and three or more parallelograms.

The equal and parallel polygons are the *bases* of the prism, and the perpendicular distance between the bases is the *altitude* or *height.* The parallelograms taken together form its *convex surface.*

When the bases are regular polygons and the parallelograms are perpendicular to the bases, the prism is a *regular prism.*

606. A **rectangular solid** is a solid bounded by six rectangles.

607. A **cube** is a solid bounded by six equal squares.

Is a cube a prism? Is a rectangular solid a prism? How many faces of a rectangular solid can be squares, and yet the solid not be a cube?

Fig. 48.
Cylinder.

608. A **cylinder** is a prism the bases of which are circles. The parallelograms forming the convex surface are infinite in number and infinitely narrow, but taken together they form the *curved* convex surface. Fig. 48.

609. A **pyramid** is a solid bounded by one polygon, which is the base, and three or more triangles which terminate in one point at the top called the vertex. Fig. 49.

Fig. 49.
Pyramid.

The triangles form the *convex surface* of the pyramid. When the base of a pyramid is a regular polygon, and a line drawn from the vertex to the center of the base is perpendicular to the base, the pyramid is a *regular pyramid*.

Fig. 50.—Cone.

610. A regular pyramid the base of which is a circle is a **cone**. The triangles forming the convex surface are infinite in number and their bases are infinitely short; but these triangles taken together form the *curved* convex surface, and the bases of the triangles taken together form the circumference of the base of the cone.

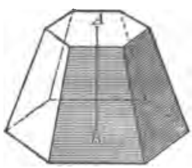

Fig. 51.—Frustum of a Pyramid.

Fig. 52.—Frustum of a Cone.

611. If the upper part of a pyramid or of a cone is cut off by a plane parallel to the base, the part that remains is a **frustum** of the pyramid or of the cone.

612. The **height** of any of the solids here defined is the perpendicular distance from the highest point above the base to the plane of the base. Thus, in Fig. 53, the line AB indicates the height of the solid.

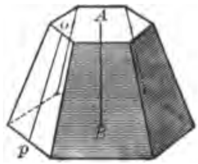

FIG. 53.

613. In a regular pyramid or in a cone, the shortest distance from the vertex to the perimeter (boundary) of the base is the **slant height.**

In the frustum of a regular pyramid or of a cone, the shortest distance between the perimeters of the two bases is the **slant height.** Thus, in Fig. 53, the line *op* indicates the slant height of the frustum. It is perpendicular to the two parallel lines between which it is drawn.

614. A **sphere** is a solid bounded by a curved surface, every point of which is equally distant from a point within, called the center.

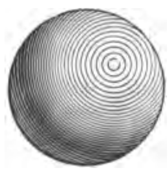
FIG. 54. — Sphere.

A circle which divides a sphere into two equal parts is a **great circle** of the sphere. The plane of a great circle passes through the center of the sphere. A circumference, a diameter, or a radius, of a great circle of a sphere is also a circumference, a diameter, or a radius of the sphere itself.

VOLUME AND SURFACE AREA OF SOLIDS.

615. A prism or a cylinder 1 inch high contains as many cubic inches as there are square inches in the base. If the height is increased to 2, 3, or any number of inches, the volume will be increased in the same proportion. Hence,

To find the volume of a prism or cylinder, multiply the area of the base by the height.

616. If a prism or cylinder is 1 inch high, its convex surface contains as many square inches as there are inches in the perimeter of the base. If the height is increased to 2, 3, or any number of inches, the convex surface will be increased in the same proportion. Hence,

To find the convex surface of an upright prism or cylinder, multiply the perimeter of the base by the height.

617. A pyramid or a cone is equivalent to $\frac{1}{3}$ of a prism or a cylinder of the same base and height. Hence,

To find the volume of a pyramid or cone, multiply the area of the base by the height, and divide the product by three.

The rule may be illustrated in this way. Make a pasteboard model of a prism and of a pyramid having its base of the same form and size and having the same height. Leave off the base of the pyramid and one base of the prism. Now fill the pyramid with sand and pour it into the prism. How many times can this be done before the prism is full?

In the same way, with pasteboard models, compare the volume of a cylinder with that of a cone.

618. The convex surface of a regular pyramid or cone is composed of triangles whose bases form the perimeter of the base of the solid, and whose height is the slant height of the solid. Hence,

To find the convex surface of a regular pyramid or cone, multiply the perimeter of the base by the slant height and divide the product by two.

619. The convex surface of the frustum of a regular pyramid is made up of trapezoids whose parallel sides form the perimeters of the bases, and whose height is the slant height of the frustum. The same statement applies to the frustum of a cone, because the bases, which are circles, are regular polygons of an infinite number of sides. Hence,

To find the convex surface of a frustum of a regular pyramid or cone, multiply the sum of the perimeters of the two bases by the slant height and divide the product by two.

620. It is proved by geometry that the surface of a sphere is equivalent to that of four great circles of the sphere. Hence,

To find the surface of a sphere, find the area of a great circle of the sphere, and multiply it by four.

NOTE. — The curved surface of a hemisphere has twice the area of the flat surface.

621. A sphere may be regarded as composed of pyramids whose bases taken together form the surface of the sphere, whose tops are all at the center, and whose height is the radius. Hence,

To find the volume of a sphere, multiply the convex surface by the radius and divide the product by three.

622. From the last two rules may be derived the following

Formulas relating to the Sphere.

1. Surface $= 4 \text{ Radius}^2 \times 3.1416 = \text{Diameter}^2 \times 3.1416.$
2. Volume $= \frac{4}{3} \text{ Radius}^3 \times 3.1416 = \frac{1}{6} \text{ Diameter}^3 \times 3.1416.$
3. Radius $= \sqrt{\dfrac{\text{Surface}}{4 \times 3.1416}} = \sqrt[3]{\dfrac{\text{Volume} \times 3}{4 \times 3.1416}}.$
4. Diameter $= \sqrt{\dfrac{\text{Surface}}{3.1416}} = \sqrt[3]{\dfrac{\text{Volume} \times 6}{3.1416}}.$

623. By using letters in place of words, and the Greek letter π to represent the ratio of the circumference to the diameter, these formulas may be written as follows:

1. $S = 4\pi R^2 = \pi D^2.$
2. $V = \dfrac{4\pi}{3} R^3 = \dfrac{\pi}{6} D^3.$
3. $R = \sqrt{\dfrac{S}{4\pi}} = \sqrt[3]{\dfrac{3V}{4\pi}}.$
4. $D = \sqrt{\dfrac{S}{\pi}} = \sqrt[3]{\dfrac{6V}{\pi}}.$

Frustum of a Pyramid or Cone.

624. The volume of the frustum of a cone can be found by supposing the part cut off to be restored, then finding the volume of the whole cone, the volume of the restored part, and the difference between the two. This difference is the volume of the frustum.

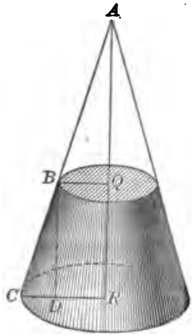

Fig. 55.

The heights of the two cones are found from the given dimensions of the frustum in this way. Draw (Fig. 55) the axis, AR, and a slant height, AC, of the whole cone, and between them the radii CR and BQ. Draw also the perpendicular BD, parallel and equal to QR. The two right-angled triangles ARC and BDC have the same angle at C and equal angles at A and B. Hence the triangles are similar and their sides are proportional. That is,

$$CD : DB = CR : RA.$$

The first three terms of this proportion are known; for CD is the difference between the given radii of the two bases of the frustum, DB is the given height of the frustum, and CR the given radius of the larger base.

The value of RA found from the proportion is the height of the whole cone. The height of the small cone, QA, is found by subtracting the given height of the frustum RQ from the height of the whole cone.

With the given radii of the two bases and the two heights RA and QA, the volumes of the two cones are found.

625. *Illustrative Example.* A common tin milk pan, measuring 12 inches in diameter on the bottom, 15 inches in diameter across the top, and $3\frac{1}{2}$ inches deep, contains how many cubic inches? How many quarts?

WRITTEN WORK.

$$CD : DB = CR : RA. \quad \text{(Fig. 55.)}$$
$$CD = CR - DR = CR - BQ = 7.5 - 6 = 1.5.$$
$$DB = RQ = 3.5.$$
$$CR = 7.5.$$

Then, $$RA = \frac{DB \times CR}{CD} = \frac{3.5 \times 7.5}{1.5} = 17.5.$$
$$QA = RA - RQ = 17.5 - 3.5 = 14.$$

Whole cone $= \frac{\pi}{3} CR^2 \times RA = \frac{3.1416 \times 7.5^2 \times 17.5}{3} = 1030.8375$

Small cone $= \frac{\pi}{3} BQ^2 \times QA = \frac{3.1416 \times 6^2 \times 14}{3} = 527.7888$

Frustum $=$ difference $= 503.0487$

Ans. The pan contains 503.0487 cu. in. or 8.7 qt.

626. The following is a convenient rule for finding the volume of a frustum of a pyramid or cone, but it cannot be explained without more geometry than can be given here.

Find the area of the lower base, the area of the upper base, and the mean proportional between the two. Multiply the sum of these three results by the height of the frustum and divide the product by three.

This rule can be used to verify the results obtained by the method explained above.

627. Examples for Written Work.

70. How many cubic inches are there in a triangular prism, each side of whose base measures 4 inches and whose height is 8 inches?

71. Find also the convex surface and the total surface of this prism.

72. How many cubic feet are there in a pyramid 2 feet high and having a base 1 foot square?

73. Find also the convéx surface of this pyramid.

74. The base of a pyramid is a square, and its faces are four equilateral triangles. Find the convex surface and the volume of such a pyramid, supposing each edge of it to measure 1 foot.

75. A regular hexagonal prism contains how many cubic inches if its height is 6 inches and each side of its base measures 2 inches?

76. Find the entire surface and the volume of a regular hexagonal pyramid, supposing its height to be 6 inches and each side of the base 2 inches.

77. How many square feet are there in the surface of a pyramidal roof covering a house 40 feet square, if the eaves project 2 feet beyond the walls of the house, and the apex of the roof is 16 feet above the level of the eaves?

78. There is a solid bounded by four equilateral triangles. Find the whole surface of such a solid, supposing each edge of it to measure 1 foot.

79. How many cubic inches are there in a right cylinder whose height is 6 inches and the diameter of the base 4 inches?

80. What is the volume of a right cone of the same base and the same height as the cylinder just described?

81. How many gallons of oil can be stored in a cylindrical iron vat 4 feet in diameter and 9 feet deep?

82. How many square feet of sheet iron are there in a piece of stove pipe 6 inches in diameter and 3 feet long, allowing 1 inch for lapping at the joint?

83. A section of cast-iron water pipe 12 feet long is of uniform thickness, 1 inch, throughout. How many cubic feet of material are there in the pipe, supposing the interior diameter to be 24 inches? What does it weigh, cast iron being $7\frac{1}{4}$ times as heavy as an equal bulk of water?

84. How many cubic inches are there in a grindstone 4 feet in diam., 4 inches thick, and having a hole at the center 4 inches square? What is the weight of this stone if it weighs 2.45 times as much as an equal bulk of water?

85. How many cubic inches are there in a bushel measure, cylindrical in form, 18.5 in. in diameter and 8 in. deep?

86. A two-quart measure is to be made of tin in cylindrical form, the diameter to be two thirds of the depth. Find the dimensions, one quart containing $57\frac{3}{4}$ cubic inches.

Let $x =$ the depth.

87. How many cubic inches are there in a box 12 inches square on the bottom, 16 inches square at the top, and 8 inches deep (interior measurements)? (See Article 626.)

88. A piece of granite 20 feet long, 4 feet square at one end and $2\frac{1}{2}$ feet square at the other, contains how many cubic feet? How much does it weigh, a cubic foot of granite weighing 162.5 pounds?

89. Measuring a common water pail, I find the diameter of the bottom to be $8\frac{3}{4}$ inches, diameter of the top $11\frac{1}{4}$ inches, and depth 8 inches. How many quarts does this pail hold, allowing 57.75 cubic inches to a quart?

90. Measure a common tumbler, tin dish, or other vessel made in the form of the frustum of a cone, and find the contents in cubic inches. Now measure the water which fills the vessel in quarts, and see how nearly the two results agree.

91. How many square inches are there in the surface of a ball 4 inches in diameter?

92. How many cubic inches are there in this ball?

93. Assuming the earth to be a sphere 7912 miles in diameter, how many square miles are there in its surface?

94. Suppose a cube, a cylinder, a sphere, and a cone all to have the same dimensions; that is, the edge of the cube is equal to the height and to the diameter of the cylinder; equal to the diameter of the sphere; and equal to the height and to the diameter of the base of the cone. If the edge of the cube measures 6 inches, what is the volume of each solid?

95. Show that if a cylinder, sphere, and cone have the same dimensions (see last example), their volumes are proportional to the numbers 3, 2, and 1.

96. Find the diameter of a sphere which contains 1000 cubic inches.

97. Find the diameter of a sphere whose surface contains 1000 square inches.

98. Find the volume of a sphere whose surface measures 1000 square inches.

99. Find the surface of a sphere which contains 1000 cubic inches.

100. If the height of a right cylinder is equal to half the diameter of its base, how does the area of its convex surface compare with the united area of its two bases?

101. The volume of a right cone is 1000 cubic inches. Find its height and the diameter of its base, the former being to the latter as 5 to 4.

SIMILAR POLYGONS.

628. Similar polygons are polygons which have the same shape, though they may not have the same size. (Art. 501.)

All the corresponding sides or other lines of similar polygons are proportional.

629. The areas of similar figures are thus compared:

In Fig. 56 are represented three similar triangles. The second has its sides *twice* as long, and the third has its sides *three times* as long, as the corresponding sides of the first. By dividing each side of the second triangle into two, and each side of the third into three equal parts, and drawing lines, we find that the second triangle is made

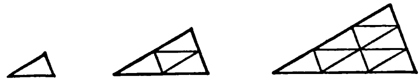

FIG. 56.

up of *four* and the third triangle of *nine* triangles, each equal to the first. The corresponding sides of these similar triangles are proportional to the numbers *one*, *two*, and *three*, while the areas are proportional to the numbers *one*, *four*, and *nine*, which are the squares of the former numbers.

In the same way squares (Fig. 57), which are similar polygons, if their sides are proportional to 1, 2, and 3, have areas proportional to 1, 4, and 9.

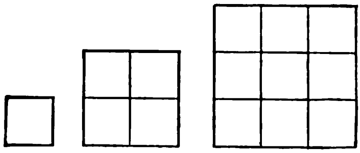

Fig. 57.

630. From these and similar illustrations we may learn that, in general,

The areas of similar plane figures are proportional to the squares of their corresponding lines.

Note. — This statement includes not only similar polygons, but similar plane figures of any kind. Two plans of the same house drawn on different scales, or two maps of the same country, one larger and one smaller, would be similar plane figures.

631. Examples for Written Work.

102. The side of a square is 10 inches. What is the side of another square of 9 times the area?

103. The sides of a triangle are 5, 6, and 7 inches. What are the sides of a triangle of 16 times the area?

104. There is a public park, in oblong form, 1320 feet in length, and containing 36 acres. What is the length of another park, of the same shape, containing 25 acres?

105. The sides of two regular hexagons are as 3 to 4. What is the ratio of their areas?

106. The sides of one square are equal to the diagonals of another square. Compare the areas of the squares.

107. The area of one circle is double that of another. What is the ratio of their radii? If the radius of the larger circle is 10 inches, what is that of the smaller?

108. A horse is tethered to a stake so that he can graze over 300 square feet of ground. Another horse tethered by a rope twice as long can graze over how much?

109. If the diameter of a circle is diminished by $\frac{1}{8}$, how will its area as then found compare with its original area? In what ratio will the area be diminished?

110. From a regular hexagon, each side of which measured 10 inches, was cut off a strip all around it wide enough to take off $\frac{1}{8}$ of the area. What is the length of each side of the hexagon that is left? How wide is the strip?

111. If a pipe 2 inches in diameter discharges 500 gallons of water in a given time, what amount of water will a pipe 5 inches in diameter discharge in the same time, the velocity of the running water being the same in the two pipes?

112. Draw four concentric circles, the inmost with a radius of 5 inches, and the others with such radii that the area of each ring shall be equal to that of the inmost circle. What are the lengths of these radii?

113. The altitude of a triangle is 10 inches. By a line drawn parallel to the base, the triangle is divided into two equivalent parts. What is the altitude of the triangle thus cut off?

114. By lines drawn parallel to the base of a triangle, its area is divided into three equivalent parts. Find the distance of each parallel line from the base, supposing the altitude of the whole triangle to be 10 inches.

SIMILAR SOLIDS.

632. Similar solids are solids which have the same shape, though they may not have the same size. (Art. 504.)

All the corresponding edges or other lines of similar solids are proportional.

The corresponding plane faces of similar solids are similar plane figures.

633. The volumes of similar solids are thus compared.

From the cubes, which are similar solids, represented in Fig. 58, we see that when the corresponding edges are proportional to the numbers *one*, *two*, and *three*, the volumes are proportional to the numbers *one*, *eight*, and *twenty-seven.*

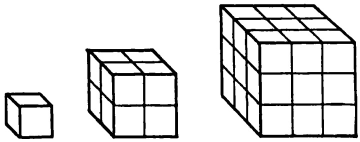

Fig. 58.

With clay make 27 cubes, all of the same size. Put 8 of these together to form a larger cube with edges double the edges of a single cube. Put the 27 cubes together, and what is formed?

Note.—This exercise may also be worked with wooden blocks; but the next one should be worked in clay.

Make 27 triangular pyramids all of the same shape and size. Put 8 of them together to make a similar pyramid with edges twice the edges of a single one. Put 27 pyramids together, and what is formed?

634. From these and similar illustrations we may learn in general that,

The volumes of similar solids are proportional to the cubes of their corresponding lines.

635. Examples for Written Work.

115. A prism 10 inches high contains 235 cubic inches. How many cubic inches does a similar prism contain which is 15 inches high?

116. A pyramid 10 feet high contains 125 cubic feet. How high is a similar pyramid containing 500 cubic feet?

117. The weight of a cube of copper each edge of which measures 4 inches is $20\frac{3}{5}$ pounds. What is the weight of a a cube of copper each edge of which measures 3 inches?

118. The weight of a lead ball 1 inch in diameter being 3.44 ounces, find the weights of lead balls 2, 3, and 4 inches in diameter.

119. A bushel measure is a cylindrical vessel 18.5 inches in diameter and 8 inches deep. Find the dimensions of a peck measure of similar form.

120. A farmer has three stacks of hay of similar form, their heights being respectively 15, 20, and 25 feet. How many tons of hay are there in all three stacks if the smaller one contains 8 tons?

121. The mean diameter of the earth being 7912 miles, and that of the moon 2160 miles, find how many bodies of the size of the moon could be made out of the earth.

122. The volume of Saturn is about 700 times, and that of Jupiter 1233 times, that of the earth. Find the ratio of the diameter of each of these planets to that of the earth.

123. A hemispherical bowl 16 inches in diameter holds a certain quantity of water. What is the diameter of a similar bowl that holds half as much water?

124. A cone is cut into two equivalent parts by a plane drawn through it parallel to the base. How far below the top of the cone is this plane drawn, if the height of the whole cone is 10 inches?

125. Gold, silver, and copper weighing respectively $19\frac{1}{3}$, $10\frac{1}{2}$, and $8\frac{9}{10}$, times as much as an equal volume of water, it is required to find the diameter of a sphere of silver, and another of copper, each to weigh as much as a sphere of gold 4 inches in diameter.

126. A cone 10 inches high is cut into three equivalent parts by planes drawn through it parallel to the base. Find the height of each frustum and of the small cone at the top.

636. Miscellaneous Examples.

1. The cost of a sidewalk was $288. One third of the cost was assessed upon the four abutters, whose lots extended respectively 80 feet, 125 feet, 66 feet, and 100 feet along the walk. How much should each pay?

2. When the rate of taxes is 1.4%, what amount of taxes should I pay on real estate valued at $5000 and personal property worth $2500?

3. In November, 1893, the Steamer Campania arrived in Queenstown, having made the voyage of 2890 miles from New York, in 5 days 12 hours 7 minutes. What was her average speed per hour?

4. In a certain manufactory in India, in 1893, the head man received 11 rupees a month. At 1*s.* 4*d.* per rupee in English money, how much were his wages per year in United States money, £1 being equal to $4.8665?

5. A difference of 15° of longitude makes 1 hour's difference of time as indicated by the sun. How many degrees east or west must a person go to find his watch 50 minutes fast? 80 minutes slow?

6. A man bought a lot of land for $1200, paying for half of it in stock reckoned at its par value which he had bought at 72% below par. How much did the land cost him?

7. The pyramids of Egypt are situated between 20° 17′ and 30° 4′ north latitude. How far north and south in common miles do they extend? (See Art. 276.)

8. It is said that the work of a healthy human heart equals the work of raising 194 pounds to the height of 1 foot in a minute. That would equal how many tons raised one foot in a day?

9. What is the difference between making a discount of 10% from the price of rubber goods with a further discount of 5% on the remainder, and making at once a discount of 15% from the price?

10. If Mr. White gets $35 a month for rent of a house worth $6000, on which he pays $70 a year for taxes, how much does he lose a year, money being worth 6%?

11. A man, wishing to get $400 from a bank for 3 months, added to the $400 the discount for 3 months 3 days at 6% and drew his note for the amount. If the note was immediately discounted, how much did the proceeds fall short of $400?

12. The rate of discount at a bank being $5\frac{1}{2}$%, what would be the total proceeds on the 14th of June, 1896, of the following notes:

$500, given for 6 months, dated May 17, 1896.
$750, given for 3 months, dated May 23, 1896.
$254, given for 4 months, dated May 31, 1896.
$435, given for 2 months, dated June 6, 1896.

13. A collector received $5.87 as his commission at 5% on the amount of his collections for 1 day. If the debtors were allowed 10% discount from the face of their bills, what was the face of the bills collected?

14. How much water must be mixed with a barrel of ink (31 gallons), which cost $34.10, that the mixture may be sold at $1.10 a gallon and 25% be gained?

15. A grocer loses 10% by bad debts, and pays $2\frac{1}{2}$% for having his bills collected. What per cent above cost must he charge for goods, that he may clear 10%?

16. A broker purchases stocks at an average of 9% below par, sells them at an average of $7\frac{3}{8}$% above par, and makes $300. What was the par value of the stocks?

17. 20% of a lot of barley, originally 5000 bushels, was destroyed by fire, the cost having been $$1\frac{1}{4}$ per bushel. What per cent will be gained on the lot by selling the remainder at $2 per bushel?

18. The rate of a clock is 0.000575 fast. How many seconds does it gain in a week?

19. I sell $\frac{1}{8}$ of a lot of goods for \$ 9, and thereby lose 25%. For what sum must I sell the remainder to make $8\frac{1}{3}$% on the whole?

20. What would be due May 1, 1896, on a note given for \$ 1000, at 5% interest, dated March 26, 1893, on which \$ 200 was paid at the end of each year from the date of the note?

21. How many cubic yards of earth must be removed in digging a cellar 10 feet deep which is to measure inside the walls 27 feet long and 15 feet wide, the wall to be 2 feet 6 inches thick?

22. What is the compound interest on a note for \$ 1000, dated September 14, 1896, and paid December 2, 1897, interest at 6% per annum, payable semiannually?

23. A water tank 3 feet in depth and 4 feet square is supplied by rain from the roof of a building 30 feet long and 20 feet wide. What depth of rain must fall to fill the tank?

24. A man agreed to pay \$ 1200 with interest at 6% per annum. Suppose the payment of interest due at the end of the first and second years was deferred, and that simple interest was allowed upon the deferred payments, what would be due at the end of the third year?

25. Mr. Lyon offered to sell his house for \$ 4500, but, finding no customer, he was obliged to keep it five years; he then sold it for \$ 8000. If he received \$ 200 per year for rent above what he paid for taxes and repairs on the house, money being worth 6% a year, how much did he gain by keeping the property? What per cent did he gain on the \$ 4500?

26. In North Eastland, near Spitzbergen, there is said to be a glacier from 2000 to 3000 feet deep. At the average depth of 2500 feet, what average pressure is exerted by the glacier upon a square foot of underlying earth, ice being 0.9 as heavy as water?

27. Suppose coal to be worth $6 a ton, and the net cost of manufacturing coal gas to be 15% of the price of the coal. If a ton of coal yields 12,000 cubic feet of gas, what is the cost of gas per thousand feet?

28. August 8, 1882, I insured my life, paying a premium of $121.30 a year. After I had paid my insurance for 11 years, the company went into bankruptcy, and on February 14, 1896, the sum of $162.12 was sent me in full for all demands on the company. Allowing the use of money to be worth 6%, how much money did I lose by the transaction?

29. It is a principle in mechanics that the lengths of the two arms of a lever are inversely proportional to the weights at their ends. What weight 6 feet from the fulcrum will balance a weight of 80 pounds 5 feet from the fulcrum? 2 feet from the fulcrum? 15 feet 6 inches?

30. How far from the support of a tilt must a boy weighing 120 pounds be placed to balance another boy weighing 96 pounds seated 10 feet from the support?

31. T. Banks & Co. imported from France 220 meters of cloth costing 15.75 francs per meter, paying 15¢ per yard for duties. If the cloth is sold at $4.50 per yard, how much is gained? (1 meter = $39\frac{3}{8}$ in.)

32. How many square yards of canvas are there in a circular tent 200 feet in diameter, the upright wall being 16 feet high, and the roof extending from the top of the wall to a point over the center 50 feet from the ground?

33. Find the first cost of an article of which 100 can be made with raw materials costing $350, labor $150, and other fixed charges $200; and find the selling price to gain 25%. How much would this price be affected by raising the wages of all the laborers 15%?

34. In the Centigrade thermometer the freezing point of water is marked 0° and the boiling point 100°. In the Fahrenheit thermometer the freezing point is marked 32° and the boiling point 212°. When the Centigrade ther-

mometer stands at 37°, at what degree will the Fahrenheit thermometer stand? What degree of the Centigrade thermometer corresponds to 92° Fahrenheit?

35. To meet the appropriations made in a town meeting \$158,400 must be raised by taxation. What must be the amount of the tax levy if a margin of 5% is allowed for uncollected taxes and a commission of $7\frac{1}{2}$% on taxes collected?

36. The cost of making a certain book is 38¢ per copy. What must be the retail price of the book that one third may be taken off for wholesale buyers, a further discount of 5% from the face of their bills allowed, and yet a profit of 50% remain for the publisher?

37. A salesman is allowed 12% on his sales. His employer makes a profit of 20% on the goods sold. What is the first cost of goods which are sold by the salesman at \$7.65?

38. How far from the wall of a house 24 feet high must the foot of a ladder 23 feet long be placed to reach within 5 feet of the top of the wall?

39. If from a point between two houses a ladder 32 feet 6 inches long will reach to a window 26 feet high in one house and to a window 30 feet high in the other, what is the distance between the houses?

40. An aeronaut ascends at the rate of $4\frac{1}{2}$ miles an hour for 40 minutes, after which he maintains the same elevation; if his balloon is driven east 7 miles during the first hour from the time of his starting, and in an opposite direction at the rate of 10 miles an hour for the remaining time, how far from his starting point in a straight line is he at the end of 5 hours?

41. How much can be paid on the dollar by a bankrupt having assets worth \$375,240 and liabilities amounting to \$681,426? How much more could he pay on the dollar if \$150,000 of these liabilities should fail to be proved by the supposed creditors?

42. A contractor has 20 days in which to do a piece of work. He hires 30 men, who after working 8 days strike for higher wages and remain idle 6 days, after which they return to their work. But for the strike the work would have been done at the end of the 18th day. How many more men must the contractor hire that he may finish his work within the time allowed by the contract?

43. A hot-air register in a schoolroom is 2 feet long by 1 foot 6 inches wide, and half the area is taken up by the grating. How much air per minute must pass through each square foot of the opening of this register into the room to supply each of 42 persons with 4 cubic feet of fresh air every minute?

44. A schooner beating against the wind sails S.E. 6 miles, then S.W. 12 miles, then S.E. 12 miles, then S.W. 12 miles, and finally S.E. 6 miles. How many miles is it in a straight course from the point she left to the point she arrives at?

45. What is the length of the edge of the largest cube that can be cut from a globe 9 inches in diameter?

46. What will a pine log weigh whose length is 18 feet, and which measures 3 feet across the larger end, and $2\frac{1}{2}$ feet across the smaller, pine being 0.6 as heavy as water, which weighs $62\frac{1}{2}$ pounds to a cubic foot?

47. What is the number of square feet in a walk surrounding a circular garden which is 25 yards across, the walk being $4\frac{1}{2}$ feet wide?

48. Find the cost of materials and labor for 100 square yards of lath and plaster work, as follows, and make out a bill for the same. Common lime, 4 casks @ $1.00; lump lime, $\frac{3}{4}$ cask @ $1.35; plaster of Paris, $\frac{1}{2}$ cask @ $1.50; laths, 2000 @ 20¢ per hundred; common sand, 7 loads @ 30¢; white sand, $2\frac{1}{2}$ bu. @ 8¢; nails, 13 lb. @ 5¢; hair, 4 bu. @ 15¢; mason's labor, $3\frac{1}{2}$ days @ $2.50; laborer, 3 days @ $1.25; cartage, $2.00.

49. An ancient cypress tree in Lombardy is 121 feet high. What must be the length of a cord that will reach from the top to the ground 50 feet in a horizontal line from the base?

50. The trunk of this tree is 23 feet in circumference at the base. If the trunk were a perfect cone to the top of the tree (121 feet), how many cords of timber would it contain?

51. The intensity of light from a given object varies inversely as the square of the distance. If your book is 27 inches from the light and mine is 6 feet 9 inches, how many times as great is the light on your book as on mine?

52. Pure iron weighs 7.79 times as much as water. A cubic foot of water weighs 1000 oz. Av. How many cubic inches are there in a cube of iron weighing 1 lb. Av.?

53. Find the diameter of a cast-iron ball weighing 9 pounds, supposing that the iron weighs 7.2 times as much as an equal bulk of water.

54. The largest known diamond in the world in its original state weighed 900 carats of $3\frac{1}{6}$ Troy grains each. What was its weight in Troy units?

55. The Kohinoor, now owned by Queen Victoria, once weighed $186\frac{1}{16}$ carats, but lost in cutting $82\frac{5}{16}$ carats. What is its present weight in Troy units? What per cent was lost in cutting?

56. Among the crown jewels of Russia is a diamond that weighs 194 carats, which was bought for $450,000 and an annuity of $20,000. What is the yearly expense of the jewel to Russia if money is worth 4% per annum?

57. In the temple at Baalbec is a stone 66 feet long, 12 feet wide, and 12 feet thick. If its weight is 2.6 times that of water, what is the weight of the stone?

58. The largest savings bank in the world is in Glasgow. In 1893, £4,622,000 belonged to 137,204 depositors. At $4.8665 per pound, what was the average in United States money belonging to each depositor?

59 A building was somewhat damaged by fire. The owner and the agents of the insurance companies agreed that \$4500 would cover the damage. There were three policies of insurance, one for \$5000 in the Royal Insurance Co., one for \$7500 in the North American, and one for \$2500 in the Farmers' Mutual. What amount should each company contribute towards making up the \$4500?

60. A party of emigrants bought a township of government land at \$1.25 an acre. Reserving 150 acres for public purposes and setting aside 200 acres which were worthless, they divided half of the remainder into farms which were sold at \$4 an acre, and the other half into farms sold at \$5 an acre. After selling all the farms, and paying for the township, how much money had they left?

61. James Fiske bought, June 8, 1894, 10 bales of cloth, 14 pieces in a bale, 43 yd. in a piece, at 8¢ per yd., for which he gave his note on interest at 6%. On the 4th of Nov., 1894, he sold 1 bale at 12¢ a yd., and gave the proceeds in part payment of his note. On the 3d of May, 1895, he sold 1 bale at 15¢, and paid on his note the amount he received. On the 17th of Sept., 1895, he sold the remainder at 16¢, and settled the note. How much did he gain?

62. A grocer buys coffee at \$29 per hundred pounds and chicory at \$11.75 per hundred pounds, and mixes them in the proportion of 2 lb. of chicory to 5 lb. of coffee. He sells the mixture at 38¢ a pound. What is his gain per cent?

63. What is gained by selling a hundred powder kegs at $16\frac{3}{4}$¢ apiece, the cost per hundred being \$1.87 for making and \$1.75 for hooping, eight hoops at \$0.045 per hundred being required for each keg, and the value of the other stock being 8¢ per keg?

64. When the shadow of a man 5 ft. 11 in. tall is found to measure 8 ft. 3 in., what is the height of a tree the shadow of which at the same moment measures 57 ft. 8 in.?

65. A person takes a note on 2 months for $ 110 in payment for a wagon. On getting the note discounted at a bank at 6%, he finds that he has lost 40% of what the wagon originally cost him. Find the cost.

66. In Lombardy, 60,000,000 cubic yards of water are daily distributed over 1,375,000 acres of land. If this water were equally distributed over the surface, what would be its depth in inches?

67. How many years of 365¼ days each would it take a body, moving at the rate of 40 miles per hour, to go from the sun to Jupiter, whose mean distance is 481,000,000 miles?

68. How many years would it take the body mentioned above to go from the sun to Neptune, 2,780,000,000 miles further from the sun than Jupiter is?

69. In 1800, the average of a day laborer's wages in the United States was $ 6 per month from May to November, and $ 5 per month from November to May, working 6 days in the week from sunrise to sunset. Allowing 52 weeks to the year, what was an average day's wages?

70. From the following record, find the average age of the pupils in the schools of M.; also find the average per cent of each school in Arithmetic, and the average of all the schools in the same study.

Schools.	No. of Pupils.	Average Age.		Arithmetic.				
		yr.	mo.	Add.	Sub.	Mult.	Div.	Av.
No. 1	18	14	0	67	72	61	39	?
" 2	9	13	8	100	78	44	56	?
" 3	25	13	11	88	92	72	36	?
" 4	4	14	7	75	75	50	75	?
" 5	8	14	2	75	88	75	38	?
" 6	23	14	1	83	96	70	52	?
	Average age ?			General Average in Arith.				?

71. A right pyramid, 20 inches high, has a base 10 inches square. What is the length of each edge of the pyramid? What is the slant height of each face? What is the area of each face?

72. A right pyramid, 40 inches high, has a regular hexagon for a base, each side of which measures 10 inches. What is the length of each edge of the pyramid? What is the slant height of each face? What is the area of each face?

73. A tennis court is 78 feet long and 36 feet wide. If a roller is 15 inches in diameter and 34 inches long, how many times must it revolve in going the length of the court, and what is the least number of times it must go lengthwise of the court to cover it?

74. What is the weight of a granite roller 5 feet in diameter and 6 feet long, granite being 2.62 times as heavy as water, and a cubic foot of water weighing $62\frac{1}{2}$ pounds?

75. What is the weight of a wedge which is 2 inches square on the head and 10 inches in length, measured on the slant surface, the iron being 7.778 times as heavy as water?

76. Find the weight of 15 steel treads of a winding stairway, each tread being $\frac{1}{2}$ an inch thick and 3 feet long, with a width of 10 inches at one end and of 5 inches at the other end, the steel being 7.816 times as heavy as water.

77. If the capital stock of a railway company is $ 9,000,000, the earnings for six months $ 570,618.75, and the expenses for the same period $ 165,892.40, how large a dividend may be declared, setting aside any fraction of 1% as a reserve? What amount would be thus set aside?

SUPPLEMENT.

Explanation of the Use of the Drill Tables.

(Pages 29, 44, 71, 89, 146, 159, and 181.)

1. The Drill Tables and Exercises provide for practice additional to what is furnished by the ordinary examples of the book. They may be used in part, or may be omitted altogether, at the discretion of the teacher. The class using them may do the same example, or each member may be assigned a separate example by the same dictation. Independence and accuracy can be secured by the latter plan.

When all the members of the class do the same example, the tables need no explanation. In most of the exercises, no copying of figures will be necessary; the pupil can work directly from the table, writing the result on a slip of paper placed beneath the figures used. Other exercises, as 120 to 131 in Drill Table No. 1, will need to be copied.

When each member of the class is assigned a separate example by the same dictation, the following suggestions will be helpful:

Drill Table No. 1. Page 29. — Exercises 85–104. The teacher gives out the exercise to the whole class, and directs the pupils to number themselves aloud, the first pupil saying "one," the next "two," and so on to twenty. If there are more than twenty pupils in the class, the remaining pupils will number themselves, beginning with "one" again.

At a given signal the pupils add, the first taking column 1; the second, column 2; the third, column 3, and so on. At another given signal they read their answers, the teacher verifying them by the key.

In Exercises 105–114 the pupils *letter* themselves *m*, *n*, *o*, etc., to *v*. The first pupil adds the double column *m*, the second adds *n*, the third adds *o*, etc.

In Exercises 120–131 the pupils letter themselves *a*, *b*, *c*, etc. The first pupil adds in line *a*, 2324, 2658, etc., to *zz*, the second adds in line *b*, 1222, 1355, etc., to *zz*, and so on. In this exercise the numbers may be either copied or added without copying.

Drill Table No. 2. Page 44. — In this table, Exercises 178–189, the pupils having lettered themselves *a*, *b*, *c*, etc., the first takes 23+24+26 from 100; the second takes 12 + 14 + 13 from 100, and so on.

In Exercises 238–243 the pupils letter themselves *a*, *c*, *e*, *g*, *i*, *k*. The first pupil takes 5542 from 5848; the second takes 1735 from 5641, and so on.

Drill Tables Nos. 3 and 4. Pages 71, 89. — The teacher who has used the preceding tables will need no explanation of the use of these. There are 12 examples in each exercise, and the pupils will letter themselves from *a* to *l*.

Drill Table No. 5. Page 146. — In each of these exercises the pupils will number themselves from 1 to 15.

Drill Table No. 6. Page 159. — Here the pupils will number themselves 1 to 14. In Exercise 242–255 the first pupil reads the numbers written in line 1, columns *E* and *G*, the second in line 2 and so on.

Drill Table No. 7. Page 181. — Needs no explanation.

Contractions in Multiplication. (Page 57.)

2. Some contractions in multiplication and division are given in Arts. 91, 141, 172, 237, and 238. Those in multiplication which follow may sometimes be useful.

Rules.

To multiply by any number whose terms are all 9's: *Annex as many zeros to the expression for the multiplicand as there are 9's in that of the multiplier, and from the number thus expressed subtract the multiplicand;* thus,

$$27 \times 99 = 2700 - 27 = 2673.$$

1. $28 \times 99 = ?$ **3.** $4868 \times 99999 = ?$

2. $92 \times 999 = ?$ **4.** $247 \times 998 = ?$

3. To multiply by a composite number: *Separate the multiplier into convenient factors, multiply the multiplicand by one of the factors, and that product by another factor, and so on, till all the factors have been used; the last product is the answer:* thus, $41 \times 25 = 41 \times 5 \times 5$.

5. Multiply 374 by 45; by 108.

6. Multiply 369 by 144. **7.** Multiply 453 by 132.

4. To multiply when the number of tens is the same in the multiplicand and multiplier, and the sum of the units is ten: *Multiply the number of tens by the number of tens plus one; write the product as hundreds; at the right express the product of the units by the units.*

$$\begin{array}{r} 27 \\ \underline{23} \\ 621 \end{array}$$

8. $42 \times 48 = ?$ **10.** $35 \times 35 = ?$ **12.** $64 \times 66 = ?$

9. $51 \times 59 = ?$ **11.** $27 \times 23 = ?$ **13.** $102 \times 108 = ?$

5. To square a number consisting of an integer and $\frac{1}{2}$: *Multiply the integer by the integer plus one, and to the product add* $\frac{1}{4}$.

$$\begin{array}{r} 7\frac{1}{2} \\ \underline{7\frac{1}{2}} \\ 56\frac{1}{4} \end{array}$$

14. $8\frac{1}{2} \times 8\frac{1}{2} = ?$ **15.** $11\frac{1}{2} \times 11\frac{1}{2} = ?$ **16.** $16\frac{1}{2} \times 16\frac{1}{2} = ?$

The Divisibility of Numbers. (Page 106.)

6. 1. *Divisibility of numbers by* 2, 4, 5, *or* 8.

Ten is divisible by 2, so any number of tens is divisible by 2. Hence ***a number is divisible by* 2 *if the number of its units is divisible by* 2.**

For a similar reason, ***a number is divisible by* 5 *if the number of its units is divisible by* 5.**

One hundred is divisible by 4, so any number of hundreds is divisible by 4. Hence ***a number is divisible by* 4 *if its tens and units together are divisible by* 4.**

One thousand is divisible by 8, so any number of thousands is divisible by 8. Hence ***a number is divisible by* 8 *if its hundreds, tens, and units together are divisible by* 8.**

2. *Divisibility of numbers by* 9.

ILLUSTRATION.

$$3483 = \left\{ \begin{array}{rl} 3000 = & 333 \times 9 \quad + 3 \\ + \ 400 = & 44 \times 9 \quad + 4 \\ + \ 80 = & 8 \times 9 \quad + 8 \\ + \ 3 = & \qquad\qquad\ 3 \end{array} \right.$$

$$3483 = (333 + 44 + 8) \times 9 + (3 + 4 + 8 + 3)$$

The number 3483 is now separated into two parts, one of which is divisible by 9, and the other equals the sum of its digits. Every number can be so separated. Hence *a number is divisible by* 9 *if the sum of its digits is divisible by* 9.

3. *Divisibility of numbers by* 3. Any number that is divisible by 9 is divisible by 3. Therefore, when a number has been separated into two parts, as shown above, the first part is divisible by 3. The other part is equal to the sum of its digits. Hence *a number is divisible by* 3 *if the sum of its digits is divisible by* 3.

4. *Divisibility of numbers by* 11.

ILLUSTRATION.

$$86427 = \begin{cases} 80000 = 7272 \times 11 + 8 \\ 6000 = 546 \times 11 - 6 \\ 400 = 36 \times 11 + 4 \\ 20 = 2 \times 11 - 2 \\ 7 = +7 \end{cases}$$

$$86427 = (7272+546+36+2) \times 11 + (8+4+7) - 6 - 2$$
$$(8+4+7) - (6+2) = 19 - 8 = 11$$

The number 86,727 is now separated into two parts, one of which is divisible by 11, and the other equals the sum of one set of alternate digits, less the sum of the other set. If the sums are equal, or if their difference is divisible by 11, the entire number must be divisible by 11.

Greatest Common Factor of Two or more Numbers. (Page 113.)

7. *Illustrative Example.* Find the g. c. f. of 52 and 91.

WRITTEN WORK.

```
52)91(1
   52
   39)52(1
      39
      13)39(3
         39
```

Divide the greater number by the less, and then divide the less number by the remainder, if there is any. Continue dividing the last divisor by the last remainder until nothing remains. The last divisor will be the g. c. f. sought.

To find the g. c. f. of more than two numbers, *find the g. c. f. of any two of them and then of that common factor and a third number, and so on till all the numbers are taken.*

This method of finding the greatest common factor of numbers depends on the following principles:

I. *Any factor common to two numbers is also a fa*
their sum and of their difference.

Thus 3, which is a common factor of 24 and 18, is a factor of $42(=24+18)$ and of $6(=24-18)$. Since 24 is equal to a certain number of 3's, and 18 to a certain other number of 3's, their sum must be a number of 3's, and their difference must be a number of 3's.

II. *The greatest common factor of two numbers is equal to the greatest common factor of the smaller of them, and the remainder obtained by dividing one of them by the other.*

$$\begin{array}{r} 52)143(2 \\ \underline{104} \\ 39 \end{array}$$

Thus, the g. c. f. of 143 and 52 is equal to the g. c. f. of 52 and 39.

Since $39 = 143 - 52 - 52$, all factors common to 143 and 52 are also factors of 39, and hence common factors of 52 and 39. Therefore the g. c. f. of 143 and 52 is *a common factor* of 52 and 39, and cannot be greater than the g. c. f. of 52 and 39.

Again, since $143 = 52 + 52 + 39$, all factors common to 52 and 39 are also factors of 143, and hence common factors of 143 and 52. Therefore the g. c. f. of 52 and 39 is *a common factor* of 143 and 52, and cannot be greater than the g. c. f. of 143 and 52.

The g. c. f. of 143 and 52 and the g. c. f. of 52 and 39 are, then, two numbers, neither of which can be greater than the other; they are therefore equal.

Least Common Multiple of Two or more Numbers. (Page 118.)

8. *Illustrative Example I.* What is the least common multiple of 6, 9, and 15?

WRITTEN WORK.

$$6 = 2 \times 3$$
$$9 = 3 \times 3$$
$$15 = 3 \times 5$$
$$\text{l. c. m.} = 2 \times 3 \times 3 \times 5 = 90$$

Explanation. — The least multiple of 6 is 6, which may be expressed in the form 2×3.

The least multiple of 9 is 9, which may be expressed in the form 3×3. But in 6 we have already one of the factors (3) of 9; hence if we put with the prime factors of 6 the remaining factor (3) of 9, we shall have $2 \times 3 \times 3$, which are all the factors necessary to produce the l. c. m. of 6 and 9.

The least multiple of 15 is 15, which may be expressed in the form 3×5. In the l. c. m. of 6 and 9 we have one of the prime factors (3) of 15; hence if we put with the prime factors of 6 and 9 the remaining factor (5) of 15, we shall have $2 \times 3 \times 3 \times 5$, which are all the prime factors necessary to produce the l. c. m. of 6, 9, and 15.

The product of these factors is 90, which is the l. c. m. sought.

NOTE. — In finding the least common multiple, the factors of the given numbers seldom need to be expressed, and the written work may be greatly reduced. Thus, in this example the written work may be simply l. c. m. $= 2 \times 3 \times 3 \times 5 = 90$.

Rule.

To find the least common multiple of two or more numbers: *Take the prime factors of one of the numbers; with these take such prime factors of each of the other numbers in succession as are not contained in any preceding number, and find the product of all these prime factors.*

Addition of Mixed Numbers. (Page 119.)

9. Example 91 (page 119). $25\frac{1}{3} + 67\frac{3}{5} + 14\frac{7}{25} = ?$

l. c. m. $= 75$

$25\frac{1}{3}$	25
$67\frac{3}{5}$	45
$14\frac{7}{25}$	21

Ans. $107\frac{16}{75}$ $\quad \frac{91}{75} = 1\frac{16}{75}$

This method of writing the work has the advantage of brevity, as the new numerators are expressed without the denominators.

Division of Fractions. (Page 132.)

10. *Illustrative Example.* What is the quotient of $\frac{4}{5} \div \frac{2}{3}$?

WRITTEN WORK.

$$\frac{\overset{2}{\cancel{4}} \times 3}{5 \times \cancel{2}} = \frac{6}{5} = 1\frac{1}{5}.$$

Ans. $1\frac{1}{5}$.

EXPLANATION 1. — The quotient of $\frac{4}{5}$ divided by 1 is $\frac{4}{5}$; of $\frac{4}{5}$ divided by 2 is $\frac{1}{2}$ of $\frac{4}{5}$, or $\frac{4}{5 \times 2}$; of $\frac{4}{5}$ divided by $\frac{2}{3}$ (which is $\frac{1}{3}$ as large as 2) is 3 times $\frac{1}{2}$ of $\frac{4}{5}$, or $\frac{4 \times 3}{5 \times 2}$, which equals $1\frac{1}{5}$. *Ans.* $1\frac{1}{5}$.

EXPLANATION 2. — The quotient of $\frac{4}{5}$ divided by 1 is $\frac{4}{5}$; of $\frac{4}{5}$ divided by $\frac{1}{3}$ (which is $\frac{1}{3}$ as large as 1) is 3 times $\frac{4}{5}$ or $\frac{4 \times 3}{5}$; and of $\frac{4}{5}$ divided by $\frac{2}{3}$ is $\frac{1}{2}$ of 3 times $\frac{4}{5}$ or $\frac{4 \times 3}{5 \times 2}$, which equals $1\frac{1}{5}$. *Ans.* $1\frac{1}{5}$.

Circulating Decimals. (Page 150.)

11. We have seen (Art. 251) that in expressing $\frac{1}{3}$ decimally (0.333...) the figure 3 is repeated again and again. So in expressing $\frac{3}{7}$ decimally (0.428571428571...) the figures 428571 are repeated again and again.

Decimal fractions that are expressed by the same figures repeated again and again are called **repeating** or **circulating decimals.**

NOTE. — Circulating decimals arise from the reduction of common fractions whose denominators contain prime factors other than 2 and 5.

12. The repeating figures of a circulating decimal are called a **repetend.**

A repetend is marked by placing dots over the first and last of the figures that repeat.

Thus, $\frac{11}{37} = 0.297297\ldots = 0.\dot{2}9\dot{7}$; $3\frac{1}{6} = 3.166\ldots = 3.1\dot{6}$.

13. A decimal expressed wholly by a repetend, as $0.\dot{2}9\dot{7}$, is a **pure circulating decimal.** A decimal expressed only in part by a repetend, as $0.1\dot{6}$, is a **mixed circulating decimal.**

Change the following fractions to decimals till the figures repeat, and mark the repetends:

17. $\frac{5}{6}$.	**19.** $\frac{5}{9}$.	**21.** $\frac{7}{11}$.	**23.** $1\frac{2}{13}$.
18. $\frac{3}{7}$.	**20.** $\frac{11}{12}$.	**22.** $\frac{1}{24}$.	**24.** $3\frac{5}{37}$.

14. To change a circulating decimal to a common fraction.

Illustrative Example. Change $0.\dot{6}\dot{3}$ to a common fraction.

WRITTEN WORK.

$$
\begin{aligned}
0.\dot{6}\dot{3} \times 100 &= 63.6363\ldots \\
0.\dot{6}\dot{3} \times 1 \quad &= \underline{\ \ 0.6363\ldots} \\
0.\dot{6}\dot{3} \times 99 \ \ &= 63.
\end{aligned}
$$

$$0.\dot{6}\dot{3} = \frac{63}{99} = \frac{7}{11}. \quad \textit{Ans.}$$

EXPLANATION. — As the repetend consists of two figures, we multiply the given circulate by 100, and find that the decimal part of the product is precisely the same as the given circulate. Hence, if we subtract the given circulate from this product there will be *no decimal fraction*

in the remainder. Thus we find that 99 times the given circulate equals 63; therefore *once* the given circulate is $\frac{63}{99}$, or $\frac{7}{11}$. Hence the

Rule.

To change a circulating decimal to a common fraction: *Take the repetend for the figures of the numerator, and for the figures of the denominator as many 9's as there are figures in the repetend. Change the fraction to its smallest terms.*

Change the following to common fractions in their smallest terms:

25. $0.\dot{3}$	**27.** $0.\dot{3}\dot{9}$	**29.** $0.\dot{6}2\dot{1}$	**31.** $0.\dot{4}2857\dot{1}$
26. $0.\dot{4}\dot{2}$	**28.** $0.\dot{6}4\dot{8}$	**30.** $0.\dot{1}0\dot{8}$	**32.** $0.8\dot{2}\dot{7}$

NOTE. — $0.8\dot{2}\dot{7} = 0.8\frac{27}{99} = 0.8\frac{3}{11} = \frac{8\frac{3}{11}}{10} = \frac{91}{110}$. *Ans.*

33. $1.8\dot{6}$	**35.** $0.0\dot{3}\dot{3}$	**37.** $0.01\dot{6}$	**39.** $2.0\dot{7}67\dot{1}$
34. $2.7\dot{3}$	**36.** $0.0\dot{2}\dot{7}$	**38.** $0.0\dot{4}\dot{2}$	**40.** $7.16\dot{1}88\dot{1}$

Compound Subtraction. (Page 171.)

15. To find the difference of time between two dates.

Illustrative Example. What is the time in years, months, and days from May 11, 1897, to August 7, 1899?

1899 y.	8 mo.	7 d.
1897	5	11
2 y.	2 mo.	26 d.

August being the 8th month and May the 5th, subtract 1897 y. 5 m. 11 d. from 1899 y. 8 m. 7 d., allowing 30 days for 1 month, and 12 months for 1 year.

Annual Interest. (Page 249.)

16. Simple interest, taken upon the principal, and upon each year's interest of the principal due and unpaid, is **annual interest.**

Rule.

To compute annual interest: *Compute simple interest on the principal for the time it is on interest. Also, on one year's simple interest for a period of time equal to the sum of the times for which the yearly interests severally remain unpaid. Add the results.*

41. What is the annual interest of $118.50 for 5 yr. 3 mo. 18 da., at 6%?

The Vermont Rule for Partial Payments.

17. 1. *Compute annual interest upon the principal to the end of the first year in which any payments are made; also compute interest upon the payment or payments from the time they are made to the end of the year.*

2. *Apply the amount of such payment or payments first to cancel any interests that may have accrued upon the yearly interests, then to cancel the yearly interests themselves, and then towards the payment of the principal.*

3. *Proceed in the same way with succeeding payments, computing, however, no interest beyond the time of settlement.*

The New Hampshire Rule.

18. This is the same as the Vermont Rule, with the following provision:

If at the time of any payment no interest is due except what is accruing during the year, and the payment or payments are less than the interest due at the end of the year, deduct such payment or payments at the end of the year without interest added.

The Connecticut Rule for Partial Payments.

19. This is the same as the United States Rule except as follows:

1. *When* LESS *than a year's interest has accrued at the time of a payment, except it be the last payment, find the difference between the amount of the principal for an* ENTIRE *year, and the amount of the payment for the balance of a year after it is made; this difference will form the new principal.*

2. *If the interest which has arisen at the time of a payment exceeds the payment, compute interest upon the principal only.*

TABLES OF DENOMINATE NUMBERS.

Measures of Extension.

20. Long Measure.

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	= 1 rod (rd.).
320 rods or 5280 feet	= 1 mile (mi.).

The following measures are also used:

1 hand = 4 inches (used to measure the height of horses).
1 span = 9 inches (from the end of the thumb to the end of the little finger extended).
1 cubit = 18 inches (from the elbow to the end of the middle finger).

Note 1. — The standard unit of length is the **yard**, which is identical with the imperial yard of Great Britain.

Note 2. — The yard is also divided into halves, fourths, eighths, etc.

21. Square Measure.

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ sq. ft.	= 1 square rod (sq. rd.).
160 square rods	= 1 acre (A.).
640 acres	= 1 square mile (sq. m.).

22. Cubic Measure.

1728 cubic inches	= 1 cubic foot (cu. ft.).
27 cubic feet	= 1 cubic yard (cu. yd.).
128 cubic feet	= 1 cord (cd.), *used in measuring wood.*

Wood is generally cut for the market into sticks 4 feet long, and laid in piles, so that the length of the sticks becomes the width of the pile. A pile 4 feet wide, 4 feet high, and 8 feet long, contains 1 cord.

One eighth of a cord is called 1 **cord foot**. 1 cord foot contains 16 cubic feet.

23. Surveyors' Long and Square Measure.

Surveyors, in measuring, use a chain called Gunter's chain (ch.), which is 4 rods or 66 feet long. The chain is divided into one hundred links (l.).

24. Surveyors' Long Measure.

7.92 in. = 1 link.
100 l. = 1 chain.
80 ch. = 1 mile.

25. Surveyors' Square Measure.

10 sq. ch. = 1 A.
640 A. = 1 sq. m.
1 sq. m. = 1 section.
36 sec. = 1 township.

Note 1. — 25 links equal 1 rod.

Note 2. — In the measurement of public lands, a township is 6 miles square and contains 36 square miles or *sections*.

26. Mariners' Measures.

Mariners, in measuring short distances at sea, use cable-lengths and fathoms.

6 feet	= 1 fathom (used in measuring depths at sea).
120 fathoms	= 1 cable-length.
7⅓ cable-lengths	= 1 common mile.

Longer distances at sea are estimated in nautical or geographical miles (Art. 277).

3 nautical miles = 1 marine league.

27. Circular and Angular Measures.

60 seconds (″)	= 1 minute (′).
60 minutes	= 1 degree (°).
360 degrees	= 1 circumference.

Note 1. — A degree of the circumference of the earth at the equator is about 69.16 common miles in length.

Note 2. — A minute of the circumference of the earth at the equator is a **geographical** or **nautical mile**, and equals about 1.15 common miles.

In astronomical calculations $30° = 1$ sign; there are therefore 12 signs in a circle.

Measures of Weight.

28. In weighing groceries and most common goods **Avoirdupois** weight is used. **Troy** weight is used in weighing silver, gold, precious stones, and articles that require great accuracy in weighing.

29. Avoirdupois Weight.

16 ounces (oz.) = 1 pound (lb.).
2000 lb. = 1 ton (T.).

Note 1. — In weighing some articles, as iron and coal at the mines, and goods on which duties are paid at the United States custom-houses, the *long ton* of 2240 lb. is used. In this weight

Note 2. — A cubic foot of water weighs 62½ lb., or 1000 oz. avoirdupois.

28 lb. = 1 quarter (qr.).
4 qr. = 1 hundredweight (cwt.).
20 cwt. = 1 T.

30. Troy Weight.

24 grains (gr.) = 1 pennyweight (pwt.).
20 pennyweights = 1 ounce (oz.).
12 ounces = 1 pound (lb.).

31. Comparison of Weights.

175 lb. Troy = 144 lb. Av.
175 oz. " = 192 oz. Av.
7000 gr. " = 1 lb. Av.
5760 gr. = 1 lb. Troy.

Note. — The standard unit of weight is the **Troy pound.** From this the other units of weight are derived.

32. The following denominations are also used:

14 pounds = 1 stone.
100 pounds of grain or flour = 1 cental.
100 pounds of dried fish = 1 quintal.
100 pounds of nails = 1 keg.
196 pounds of flour = 1 barrel.
200 pounds of beef or pork = 1 barrel.
280 pounds of salt = 1 barrel.

Apothecaries' Weight.

33. In mixing medicine, apothecaries use the Troy pound divided into ounces (oz. or ℥), drams (dr. or ʒ), scruples (sc. or ℈), and grains. They also use the fluid ounce (f. ℥), fluid dram (f. ʒ), minims or drops (♏).

34. Weight.

20 grains	= 1 scruple.
3 scruples	= 1 dram.
8 drams	= 1 ounce.
12 ounces	= 1 pound.

35. Fluid Measure.

60 minims	= 1 fluid drachm.
8 fluid drachms	= 1 fluid ounce.
16 fluid ounces	= 1 pint (O).
8 pints	= 1 gallon (Cong).

Apothecaries buy and sell in large quantities by avoirdupois weight.

Measures of Capacity.

36. Liquid Measure.

4 gills (gi.)	= 1 pint (pt.).
2 pints	= 1 quart (qt.).
4 quarts	= 1 gallon (gal.).

37. Dry Measure.

2 pints	= 1 quart (qt.).
8 quarts	= 1 peck (pk.).
4 pecks	= 1 bushel (bu.).

Note 1. — A pint of water weighs about a pound avoirdupois.

Note 2. — The standard unit for liquid measure is the gallon.

Note 3. — The standard unit for dry measure is the bushel.

The gallon contains 231 cu. in.; the bushel, 2150.42 cu. in.

Note 4. — In buying and selling grain and many other kinds of produce, the bushel is reckoned at a certain number of pounds. Thus, potatoes have 60 lb. to a bushel and corn has 56 lb. to a bushel.

Note 5. — In determining the capacity of reservoirs, etc., 31½ gallons are considered a barrel and 2 barrels, or 63 gallons, a hogshead.

38. Time.

60 seconds (sec.)	= 1 minute (min.).
60 minutes	= 1 hour (h.).
24 hours	= 1 day (d.).
7 days	= 1 week (w.).
365 days or 52 weeks 1 day	= 1 common year.
366 days	= 1 leap year.
100 years	= 1 century.

Note. — In most business transactions, 30 days are considered a month, and 12 months a year.

39. Leap Year.

The earth revolves around the sun in 365 days 5 hours 48 minutes and 50 seconds nearly, but we call 365 days a year. It will be seen that what we call a year is nearly 6 hours less than the true year, and 4 such years nearly 1 day less than 4 true years. To rectify this error, 366 days are allowed to every fourth year. The year of 366 days is called a *leap year*.

The addition of a day in every fourth year is too much by a number of minutes, which in one hundred years amounts to about three fourths of a day. To balance this error, only 365 days are allowed to the final year of a century in three centuries out of every four.

Hence any year is a leap year, *when the number denoting the year is divisible by* 4 *and not by* 100, *and when it is divisible by* 400.

40. Numbers.

12 units = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 units = 1 score.

41. Paper.

24 sheets = 1 quire.
20 quires = 1 ream.
2 reams = 1 bundle.
5 bundles = 1 bale.

42. Books.

A book formed of sheets folded in

2 leaves is a folio.
4 leaves is a quarto.
8 leaves is an octavo.
12 leaves is a duodecimo, or 12 mo.
16 leaves is a 16 mo.

Measures of Value.

43. United States Money.

10 mills (m.) = 1 cent (ct. or ¢).
10 cents = 1 dime (d.).
10 dimes or 100 cents = 1 dollar ($).
10 dollars = 1 eagle.

The standard unit of value is the dollar. Its standard weight in gold is 25.8 grains.

44. The coins of the United States in common use are:

Gold.		*Silver.*	
Double-eagle	= $20.00	Dollar	= $1.00
Eagle	= 10.00	Half-dollar	= 0.50
Half-eagle	= 5.00	Quarter-dollar	= 0.25
Quarter-eagle	= 2.50	Dime	= 0.10

Nickel 5-cent piece, and bronze 1-cent piece.

NOTE 1.—The gold coin is hardened by an alloy of 1 tenth copper and silver (the silver not to exceed 1 tenth of the whole alloy). The silver coin is hardened by 1 tenth copper. The bronze cent has 95 parts of copper to 5 parts of tin and zinc. The 5-cent piece has 75 parts of copper to 25 parts of nickel.

NOTE 2.—All gold coins are legal tender for any amount; silver coins *less* than $1 are legal tender for any amount not exceeding $10 in any one payment; nickel and bronze coins, for any amount not exceeding 25 cents at any one payment.

NOTE 3.—Bank bills and United States Treasury notes are largely used in place of coins. These represent the value of $1, $2, $5, $10, $20, $50, $100, $500, and $1000.

45. English Money.

4 farthings (qr.)	= 1 penny (*d.*).
12 pence	= 1 shilling (*s.*).
20 shillings	= 1 pound (£).

The coins of Great Britain in general use are,

Gold. Sovereign (= 1 pound), and half-sovereign.

Silver. Crown (= 5 shillings), half-crown, florin (= 2 shillings), shilling, 6-penny and 3-penny pieces.

Copper. Penny and half-penny.

The standard unit of value is the pound, which equals $4.8665 in United States gold.

46. Other Foreign Money.

French Money	100 centimes	=1 franc (fr.)	=$0.193	U.S.	money.
German Money	100 pfennigs	=1 mark	=$0.238	"	"
Russian Money	100 kopecks	=1 rouble	=$0.772	"	"
Austrian Money	. . . . .	1 crown	=$0.203	"	"
The Netherlands	. . . . .	1 florin	=$0.402	"	"

The Metric System of Weights and Measures.

47. The Metric System of Weights and Measures is a decimal system based upon the meter and used in the greater part of Europe and for scientific calculations in the United States.

NOTE 1.—It was intended that the meter should be one ten-millionth of the distance from the equator to the poles, but later calculations have shown it to be a little less than that.

NOTE 2.—The word **meter** means a *measure*. The standard meter is a certain bar of platinum carefully preserved at Paris. The standard meter of the United States is a copy of this bar kept at Washington. The meter-sticks made for ordinary use are copies of the standard meter.

48. The multiples of the meter are designated by the Greek prefixes **Deca** (10), **Hecto** (100), **Kilo** (1000), and **Myria** (10,000); the decimal parts are designated by the Latin prefixes **deci** ($\frac{1}{10}$), **centi** ($\frac{1}{100}$), and **milli** ($\frac{1}{1000}$). The abbreviations of these prefixes are their initial letters, those of the multiples being in capitals, and those of the decimal parts, in small letters.

49. The standard units of the several measures with their values are as follows:

MEASURES OF	STANDARD UNIT.	ABBREVIATIONS.	ORIGIN.	LEGAL U. S. EQUIVALENTS.
Length	Meter	m.	0.0000001 Eq. to P.	39.37 in.
Surface	Sq. meter	sq. m.	m. × m.	1.196 sq. yd.
Land	Are*	a.	10 m. × 10 m.	3.954 sq. rd.
Solids	Cu. meter	cu. m.	m. × m. × m.	1.308 cu. yd.
Wood	Stere*	s.	1 cu. m.	0.2759 cd.
Capacity	Liter*	l.	1 cu. dm.	0.908 dry qt. 1.0567 liq. qt.
Weight	Gram	g.	1 cu. cm. of water	0.03527 oz. av. 15.432 gr.

* *Are*, pronounced *air;* stere, pronounced *stair;* liter, pronounced *leeter*.

50. The metric system being a decimal system, units of any denomination in the system can be changed to those of a higher or lower denomination by moving the decimal point.

In all but the square and cubic measures, the units increase and decrease by a scale of tens.

The units of square measure increase and decrease by a scale of hundreds, and those of cubic measure by a scale of thousands.

In the following tables the standard units are printed in capitals and the other units usually employed in business or science are indicated by full-faced type.

51. LONG MEASURE.

10 mm. = 1 **cm.**
10 cm. = 1 dm.
10 dm. = 1 METER.
10 m. = 1 Dm.
10 Dm. = 1 Hm.
10 Hm. = 1 **Kilometer.**
10 Km. = 1 Mm.

52. SQUARE MEASURE. **53.** LAND.

100 sq. mm. = 1 sq. cm.
100 sq. cm. = 1 sq. dm.
100 sq. dm. = 1 SQ. METER = 1 centare (ca.).
100 sq. m. = 1 sq. Dm. = 1 ARE (a).
100 sq. Dm. = 1 sq. Hm. = 1 hectare (Ha.).
100 sq. Hm. = 1 sq. Km.
100 sq. Km. = 1 sq. Mm.

54. SOLID MEASURE.

1000 cu. mm. = 1 cu. cm.
1000 cu. cm. = 1 cu. dm. = 1 liter.
1000 cu. dm. = 1 CU. METER = 1 stere.

55. WOOD MEASURE.

10 ds. = 1 STERE. (1 cu. m.)
10 s. = 1 decastere.

56. CAPACITY.

10 ml. = 1 cl.
10 cl. = 1 dl.
10 dl. = 1 LITER (1 cu. dm.)
10 l. = 1 **Dl.**
10 Dl. = 1 **Hl.**
10 **Hl.** = 1 **Kl.** (1 cu. m).

57. WEIGHT.

10 mg. = 1 cg.
10 cg. = 1 dg.
10 dg. = 1 GRAM = { wt. cu. cm. of water.
10 g. = 1 Dg.
10 Dg. = 1 Hg.
10 Hg. = 1 **Kg.** = { wt. cu. dm. of water.
10 Kg. = 1 Mg.
10 Mg. = 1 quintal (Q.).
10 Q. = 1 **Tonneau** (T.). = { wt. cu. m. of water.

Practical Measurement and Construction of the Metric Units.

58. Measure off upon a straight stick ten times the length of the decimeter represented in the margin. You have now the standard of length. Indicate upon the stick **decimeters** and **centimeters**. From these all the other metric measures may be obtained.

Upon any convenient floor measure off a square meter; upon a level plat of ground, measure off the **are** (ten meters square); in the corner of a room, measure off the **stere**, one meter each way. A liter, a cubic centiliter, may be easily formed out of pasteboard or tin, also a cubic centiliter, which filled with water would weigh a gram.

Or, for the gram, take a strip of sheet lead that weighs exactly a nickel and divide it into five equal parts. One of these parts weighs a gram.

Exercises.

59. *a.* Measure off 10 meters on a string, with knots to indicate the meters.

b. Find the length and breadth of the school yard, of a garden, etc.

c. Measure 10 decameters and pace it off. How many of your paces make a decameter?

d. How many of your paces make a kilometer?

e. How many kilometers is it from your home to school?

60. Approximate Equivalents.

The equivalents in United States measures here given are accurate enough for most purposes, and are easy to remember.

A decimeter	=	4 inches.	A stere	=	$\frac{1}{4}$ of a cord.
A meter	=	3 ft. $3\frac{3}{8}$ in., or $1\frac{1}{10}$ yards.	A liter	=	1.06 liquid qt., or $\frac{9}{10}$ of a dry qt.
A decameter	=	2 rods.	A decaliter	=	1 peck and 1 qt.
A kilometer	=	$\frac{5}{8}$ of a mile.	A hectoliter	=	$2\frac{5}{6}$ bushels.
An are	=	4 sq. rods, or $\frac{1}{40}$ of an acre.	A gram	=	$15\frac{1}{2}$ grains.
			A kilogram	=	$2\frac{1}{5}$ pounds av.
A hectare	=	$2\frac{1}{2}$ acres.	A metric ton	=	2200 pounds av.

61. Examples for Written Work.

42. Express as meters and add 582^{cm}, 6428^{dm}, and 495^{mm}.

43. Express as meters and add 369^{Dm}, 4073^{Hm}, and 5^{Km}.

44. Add 48.06^{m}, 709.63^{m}, 3708.9^{m}, 800.9^{m}, and express the answer in kilometers.

45. The distance round a certain park is 2.58^{Km}. How many meters will a man go who rides around it six times?

46. Ellen's hoop is 3.6^{m} around. How many times will it turn in rolling a distance of 1.08^{Km}?

47. Express the following in ares and add them: 1.3 hectares, 155.5 ares, 43 hectares, 26 centares.

48. In a piece of land 15^{m} long and 14.5^{m} wide, how many square meters or centares are there? how many ares?

49. A had 6 Ha., 7 a., 9 ca. of land, and sold 0.2 of it at \$54 an are. How much did he receive for what he sold?

50. Express the following in cubic meters and add them: 7 cu. meters, 40 cu. decimeters; 5 cu. meters, 3 cu. decimeters, 19 cu. centimeters; 25 cu. centimeters.

51. If I burn 27 steres of wood in the three winter months, what must be the length of a pile 1 meter wide and $\frac{3}{4}$ meter high to last a month, and what will it cost at \$2.25 a stere?

52. How many hectoliters of oats can be put into a bin that is 2^{m} long, 1.3^{m} wide and 1.5^{m} deep?

53. What must be the length of a bin 1 meter wide and 1 meter deep, to contain 4500 liters of grain?

54. At \$11 per T. for coal, what will the coal cost to keep fire a week if 30 kilos (kilograms) are burned each day?

55. What is the weight of $10^{cu\,cm}$ of mercury, mercury being 13.5 times as heavy as water?

56. If marble is 2.7 times as heavy as water, what is the weight of a pedestal 1 meter square at each end and 2 meters high?

57. What weight of water in kilos may be contained in a cistern 4^{m} deep, 1.5^{m} long, and 1.2^{m} wide?

ADDITIONAL EXAMPLES FOR OCCASIONAL USE.

62. Integral Numbers and United States Money.

1. If I can buy cloth at one place for 35 cents a yard and at another place for 28 cents a yard, how much money do I save by buying 25 yards at the latter place?

2. When the introduction price for readers is 50 cents, and the exchange price is 36 cents, what is the cost of a new set of readers in a town requiring 675 new readers, and giving 250 old readers in exchange?

3. Nine hundred and sixty people struck for higher wages, and were out of work 6 weeks in consequence. If their average earnings were $1.35 a day, what was their entire loss for the time?

4. Of a barrel of beef (200 pounds) which cost 17 dollars, 136 pounds were sold @ 12 cents a pound and the remainder @ 8 cents a pound. How much was gained?

5. What is the average cost of milk per month for a family whose bills for the year are as follows: $4.34, $3.92, $4.34, $4.20, $3.72, $3.60, $3.72, $3.72, $3.60, $3.72, $4.20, $4.34?

6. A man bought a barrel of beef for $16.10. Including 55 cents for freight and 35 cents for carting, how much did the beef cost him a pound?

7. Mr. Breck received $175 for his month's salary; of this he paid for rent $20, for help $12, meat bill $9.63, grocer's bill $13.41, milk $2.79, fuel $8.25, clothing $43.18, and $11.14 for other expenses. How much of his salary remained unexpended?

8. A dealer bought 328 bushels of wheat @ 87¢ a bushel, 745 bushels of oats @ 56¢ a bushel, and gave in part payment $425, drawing a check for the balance. For how much was the check drawn?

9. Mr. Goss and Mr. Smith went on a journey, agreeing to share the expenses equally.

Mr. Goss paid:	Mr. Smith paid:
Fares, $25, $42, $36.19;	Fares, $11.94, $0.88, $1.72;
Hotel bills, $5.25, $3.12, $16;	Carriage hire, $1.25, $1.80;
Refreshments, $1.50, $0.25, $0.47.	Porter, $0.50, $0.75.

Which is in debt to the other, and how much?

10. Of 745 bushels of oats which cost 34¢ a bushel, 326 bushels, being damaged, were sold @ 25¢ a bushel, and the remainder @ 45¢ a bushel. Was there, on the whole, a gain or loss and how much?

11. I started out shopping with \$32.22, and bought with the money 2 collars @ 10¢ apiece, 6 collars at the rate of 3 for 25¢, 2 pairs of cuffs @ 20¢, 4 yd. gingham @ 17¢ and 2 yd. @ 33¢, 14 yd. linen @ 33¢, one and a half yd. lace @ 18¢, two and a half yd. edging @ 22¢. How much money should I have left?

12. Mr. Phipps kept 42 bushels of cranberries through the winter, but found in the spring that a third of them were decayed. He then sold the remainder for \$4.50 a bushel and received in payment the price which he might have received for the 42 bushels in the fall. What was the price per bushel in the fall?

13. Mr. Plympton had \$2145 on deposit in a bank. He drew out \$572 of his deposit at one time, \$67 at another, and gave a check for the balance towards the payment of a debt of \$1700. What was the amount of the check he gave, and how much of his debt remained unpaid?

63. Bills.

In the following examples, supply dates, etc., when wanting.

14. Make out a bill against yourself from the N.Y. Central R.R. Company for the freight of 1 carload household goods, 12000 lb.; 3 boxes goods, 1450 lb.; 1 piano, 1200 lb.; freight, 24¢ a hundred.

15. Make out a bill against Mr. Edwin Train for the following lots of coal sold him @ \$6.25 per ton of 2000 lb.: 2100 lb., 2540 lb., 2100 lb., 2400 lb., 2480 lb., 2430 lb., 2460 lb.

16. James Otis sold to Harrison Grey 57 bushels of corn @ 88¢ a bushel, and 29 bushels of turnips @ 45¢ a bushel, and received \$28.38 in part payment. Find how much was still due Mr. Otis and make out the bill.

17. Two customers ordered the following at a restaurant: roast beef 40¢, veal pie 35¢, 2 tumblers milk @ 5¢, pudding 10¢, pie 20¢, pickles 5¢, sauce 5¢. Make a statement of the above with amount due; also tell the proper bills and coins to return if \$10 should be given in payment.

64. Fractions.

18. To what must you add the difference between $11\frac{8}{15}$ and $5\frac{3}{4}$, that the sum may be $16\frac{1}{3}$?

19. How many jackets, each jacket requiring $1\frac{3}{8}$ yards of cloth, can be made from $43\frac{1}{2}$ yards, and how much cloth will be left?

20. Divide 52 into two such parts that $\frac{1}{3}$ of one part shall equal $\frac{3}{4}$ of the other.

21. A train leaving M at 10 o'clock, P.M., reaches N at $5\frac{1}{2}$ o'clock the next morning. The distance from M to N by rail is $175\frac{1}{2}$ miles. What is the average rate of the train per hour?

22. At 10 A.M. Mr. Brice starts at Springfield for Worcester, distant 54 miles, and travels at the rate of 8 miles an hour. At the same time Mr. Smith starts at Worcester for Springfield, and travels at the rate of $9\frac{1}{4}$ miles an hour. How far apart will they be at the end of an hour? In how many hours will they meet?

23. Two boys are to have $1.75 for hoeing a field of corn; if one hoes 5 rows while the other hoes 7 rows, till the work is done, how much should each boy receive?

24. If 42 lb. of maple-sugar can be made from 150 gal. of sap, how many pounds can be made in three weeks from the sap of 128 trees, the average daily yield per tree being $7\frac{1}{2}$ gal.?

25. At 7 o'clock, A.M., A started on a journey, and traveled at the rate of 7 miles an hour; at 9 o'clock B started from the same place, and traveled in the same direction at the rate of $9\frac{1}{2}$ miles an hour. In how many hours did he overtake A?

26. A man owned $\frac{7}{8}$ of a hotel which cost $12000; he sold $\frac{1}{4}$ of his share to a third party at cost. What part of the hotel did he then own, and how much did he receive for what he sold?

27. At 8¢ per yard for calico, and 33¢ per pound for cotton, what is the cost of materials for a pair of calico bed comforters, each being in length $2\frac{1}{2}$ yards, and in width equal to $2\frac{1}{2}$ breadths of calico, and requiring 3 pounds of cotton?

28. A flagstaff 58 feet long is fastened to a building in such a way that $\frac{3}{10}$ of what is above the roof equals $\frac{2}{3}$ of what is below. How much is above the roof?

65. Decimals.

29. How many bushels of wheat, each 60 pounds, must be used to make a barrel of flour, 0.28 of the wheat being lost in the making.

30. January 1, a gas meter registered 11,800 cubic feet of gas consumed; April 1, it registered 14,100 cubic feet. At $1.70 per thousand cubic feet, what was the cost of gas used in the intervening time?

31. How many hours will it take a person to travel 40.5 miles at the rate of $5\frac{5}{8}$ miles per hour?

32. A and B are walking the same way along a road. At noon A was 3.5 miles ahead of B. When will B overtake A, if A walks 3.75 miles an hour, and B 4.125 miles an hour?

33. What part of \$9.50 is \$0.125?

34. To 5.49 add 0.7 of 8.65; subtract the sum from 18; multiply the remainder by $\frac{4}{9}$ of 9.18, and divide the product by 0.007.

35. From what must you take the difference between 4.25 and $3\frac{5}{16}$, that the remainder may be 7.63?

36. If 7.75 ounces of gold are divided among 3 men and a boy, the boy receiving half as much as a man, how much gold will each person have?

66. Compound Numbers.

37. A floor $14\frac{1}{2}$ feet by 12 feet 8 inches was covered by Japanese matting a yard wide laid crosswise of the floor, at a cost of 75¢ per yard complete. What was the cost?

38. The Magna Charta was signed June 19, 1215. Find the time in years, months, and days from that date to January 1, 2000.

39. How many quarts of currants will a basket hold that is 8 inches square and $4\frac{1}{2}$ inches deep?

40. How many quarts of water may be held in a dish of the same dimensions as the basket named above?

41. What does it cost a week to keep a horse, if he eats in a year 3 tons of hay worth \$19 a ton, 4 quarts of corn and 4 quarts of oats a day, the corn being 60¢ a bushel and the oats 45¢?

42. The Washington Elm in Cambridge has been estimated to bear 7,000,000 leaves annually, averaging 4 square inches of surface each. What is the aggregate surface of the leaves in feet? in acres?

43. A speculator bought a field of $3\frac{1}{2}$ acres; from this he made 8 house lots of 9650 sq. ft. each, and the rest he divided equally into 5 house lots. How many sq. ft. were there in each of the 5 house lots?

44. A schoolroom 32 feet long by 28 feet wide and 13 feet high is occupied by 35 pupils. How long will it take to use up the air in the room, if each pupil uses up 4 cubic feet in a minute?

45. In the United States 20,000,000,000 matches are manufactured yearly. If 50 matches are made from a cubic inch of wood, how many cords of wood are required for these matches, no allowance being made for waste?

46. The force of waves against a sea wall in a heavy storm is sometimes $2\frac{1}{2}$ tons to a square foot. At this rate, what is the total force exerted upon a sea wall 20 feet in height and $\frac{1}{4}$ of a mile long?

47. A water tank 6 feet long, $2\frac{1}{2}$ feet wide, and 3 feet deep is filled by 1242 strokes of a force pump. How many gallons does the tank contain? How much water is raised by each stroke of the pump?

48. Before Lake Haarlem was drained it was 15 miles in length and covered 45,000 acres. What was its average width?

49. I have a rectangular field 80 rods long and 6 rods wide. If this field is divided into 10 equal rectangular house lots, having their fronts on the longest side of the field, what will be the cost of fencing to inclose and separate the lots @ $6 a rod?

50. A street 3 rods wide and 200 rods long is on an average 10 inches above grade; how many cubic yards of earth must be removed to bring the street to grade?

51. What is the weight of the air in a room 25 ft. long, 20 ft. wide, and 12 ft. high, water weighing 770 times as much as air, and a cubic foot of water weighing 1000 ounces?

52. When you can get 3 gills of water by melting a quart of snow, dry measure, how much snow must you melt to fill a 10-gallon tub full of water? [Answer in bushels, pecks, and quarts.]

53. In 100 parts by weight of air there are 22 parts of oxygen. What is the weight of the oxygen in a room 20 ft. long, 18 ft. wide, and 10 ft. high?

54. If, when flour is $5 per barrel, a 4-cent loaf weighs 3 oz., what should a 6-cent loaf weigh when flour is $4.50 per barrel?

67. Miscellaneous.

55. When stock is quoted at $104\frac{5}{8}$, how many shares can be bought for $25,454.25, brokerage $\frac{1}{8}$%?

56. It takes 4 men 5 days to build 16 rods of fence; how long will it take to build 90 rods if 4 boys help, each boy doing half as much work as a man does?

57. If the weight of a block of sandstone 3 ft. long, 2 ft. wide, and 1 ft. thick, is 840 lb., what is the weight of another block of sandstone 6 ft. long, 2 ft. 6 in. wide, and 2 ft. thick?

58. Suppose all the water falling upon the surface of New York State during a rainfall of 1 in. should be gathered into one cubical vat just large enough to hold it; what would be the dimensions of the vat, the area of New York State being 49,170 square miles?

59. Two contractors have finished work for which they have been paid $12,500. One of them employed 40 laborers for 125 days of 9 hours each; the other 25 laborers for 110 days of 8 hours each. Divide the money between them in proportion to the labor each furnished.

60. The distance around a field containing one acre being 52 rods, what is the distance around a field of similar shape containing 9 acres?

61. How many pounds of flour must a baker use to make 100 lb. of bread, if the bread weighs 30% more than the flour used?

62. From a bill amounting to $1140, there is thrown off $40; that is equivalent to a discount of what per cent?

63. I bought a lot of wool at 20% below the asking price, and sold it for $2600, and by so doing I gained 30% on the cost. What was the asking price?

64. I built a house costing $5000 upon a lot which cost $800. The house being burned, the insurance company paid me 60% of the cost of the house. I then sold the land for $1200; did I gain or lose by the transactions, and what per cent?

65. A marine insurance company took a risk of $35,000 at $2\frac{1}{2}$%, and reinsured $\frac{3}{4}$ of their risk in another company at $2\frac{1}{4}$%. Should the property be lost at sea, how much would the first company lose?

66. Water weighs $62\frac{1}{2}$ lb. to the cubic foot, and milk $64\frac{1}{2}$ lb. By what per cent is milk heavier than water?

67. What is the interest of 1 cent for 1 day at 1% a year (360 days)?

68. Find the amount at 7% of $820 from March 10 to December 4 of the same year.

69. Find the difference between the accurate interest of $75,000 at 6% and the interest by the common method, the time being from May 24 to October 15 of the same year.

70. What per cent on my investment do I save by buying, April 1, coal for $60 for which I should pay $96 the first of the following December, money being worth 6% a year?

71. A merchant sold goods amounting to $6500, one half on 4 months' credit, the rest on 6 months, and got the notes discounted at a bank at 6%. How much was realized, allowing for grace?

72. A person holding three promissory notes dated January 1, one for $600 payable in 2 months, another for $800 payable in 4 months, and another for $750 payable in 6 months, exchanges them for a single note for the sum total. When should this note be made payable?

73. If the side of a square inscribed in a circle is 40 feet, what is the area of the circle?

74. Find the convex surface of the frustum of a regular octagonal pyramid, the slant height of the faces being 8 inches, the length of each side of the upper base 5 inches, and that of each side of the lower base 7 inches.

75. If $x=\sqrt{0.49+12\frac{1}{4}\times 7\frac{1}{9}}$, what is the simplest value of x?

76. How many cubic feet are there in a granite monument 15 feet high, 40 inches square at the base, and 24 inches square at the top?

77. Find the entire surface of a cube each of whose edges measures $4\frac{1}{2}$ inches.

78. How many cubic inches of water can be contained in a globe whose diameter inside is 15 inches?

79. What is the area of a triangle whose sides measure 20, 28, and 32 inches? What is the length of each side of a similar triangle containing one half the area?

80. Wishing to find the number of cubic inches in an irregular piece of stone, I sunk it in a cylindrical glass of water and found it raised the water $2\frac{3}{8}$ inches. The glass was 5 inches in diameter. How many cubic inches were there in the stone?

81. A wheel 6 ft. in diameter is turning at the rate of 800 revolutions a minute. At the rate of how many miles an hour is a point on the rim of the wheel moving?

82. If a square contains $342\frac{1}{4}$ square feet, what is the length of one of its sides?

83. In a rectangular courtyard there are laid 12,800 paving stones, the number in the length being twice the number in the width. What is the number each way?

84. Two parallel sides of a quadrilateral field are 42 chains and 34 chains respectively, the distance between them is 21.6 chains. How many acres are there in the field?

85. The sides in a quadrilateral field taken in order around the field measure 25 chains, 21.7 chains, 13.4 chains, and 20.3 chains respectively; the length of a diagonal drawn from the beginning of the first to the end of the second side is 26.5 chains. How many acres are there in the field?

86. How many square inches are there in the surface of the largest sphere that can be put into a cubical box whose dimensions inside are 12 inches each?

87. A father has settled on his daughter $1500 a year. What sum must he invest in government 4 per cents at $113\frac{3}{4}$ to produce that amount of income?

88. If by investing in 5 per cent bonds I get an annual income of $4\frac{3}{8}$% on my money, at what price were the bonds bought?

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